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Dynamic regimes in planetary cores: τ–ℓ diagrams

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1. τ–ℓ Python package

In the attached tau-ell_programs.zip archive, we provide short Python scripts used to draw our article's figures.

- read_parameters.py: document the parameters of the natural object, or numerical simulation, or Lab experiment, for which one wishes to draw τ–ℓ regime diagrams (or templates). The objects can be fluid full spheres (as in the article) or fluid spherical shells.
- plot_object.py: plot τ–ℓ diagram of the chosen object. You select a number of options, including the force balance you want to test, and the program calls plot_template and plot_scenario.
- plot_template.py: plot the template for the chosen object (common to natural objects, DNS and experiments).
- plot_scenario.py: overlay the τ–ℓ regime diagram produced by scenarios built upon various force balances (CIA, QG-CIA, QG-VAC, MAC, QG-MAC, QG-MAC_JA, IMAC). More scenarios can be added.
- plot_DNS.py: plot τ–ℓ diagrams of numerical simulations, given their spherical harmonic degree n-spectra.
- plot_experiment.py: plot τ–ℓ diagrams of Lab experiments, given their wavenumber k-spectra.
- plot_Kolmogorov.py: plot τ–ℓ diagram of Kolmogorov's universal turbulence, as an example.
- plot_convection_onset.py: plot dynamo onset parameters for rotating and non-rotating convection (full sphere).
- tau_ell_lib.py: library gathering τ–ℓ conversion rules from spectra, graphical functions, and other common functions.

The numerical simulation and Lab experiment data used for the examples shown in our article are available in folders:

- DNS_Guervilly: u and ρ spectra of Guervilly et al. [2019]'s 3D rotating convection simulation at $\mathrm{Ek} = 10^{-8}$.
- DNS_Schaeffer: u, b, and ρ average spectra of Schaeffer et al. [2017]'s S2 numerical geodynamo simulation.
- DNS_Dormy: u, b, and ρ average spectra of weak and strong dynamo numerical simulations proposed by Dormy [2016].
- experiment_DTS: u and b wavenumber spectra of a composite run of the DTS liquid sodium experiment [e.g., Brito et al., 2011].

This package is also available at https://gricad-gitlab.univ-grenoble-alpes.fr/natafh/shell_tau-ell_programs.

2. τ–ℓ regime diagram of the DTS liquid sodium experiment

We think that laboratory experiments can also provide a better perspective when translated into τ–ℓ

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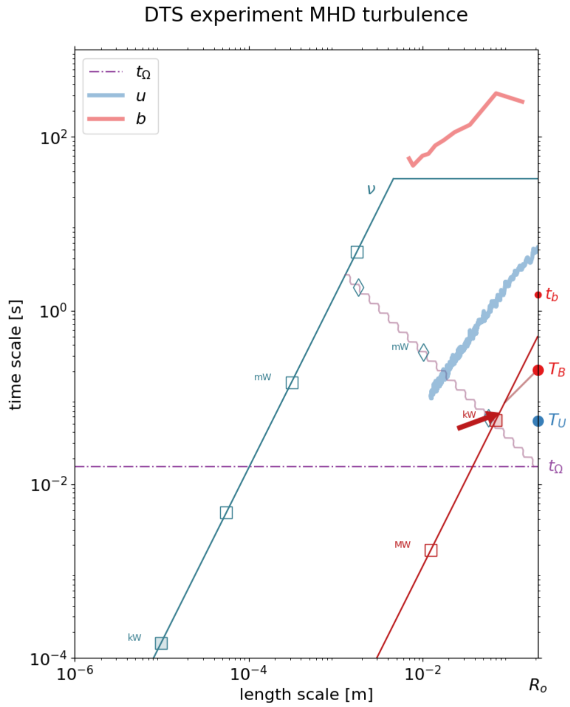
regime diagrams. As an illustration, we present in Figure S1 the composite τ–ℓ diagram of a representative run of the DTS experiment. The DTS experiment is a magnetized spherical Couette experiment. Fifty liters of liquid sodium are enclosed in a spherical container (R_o = 0.21 m) that can rotate around a vertical axis. A central inner sphere (R_i = 0.074 m) can rotate independently around the same axis, and hosts a strong permanent magnet producing an axial dipolar magnetic field. See Nataf et al. [2008], Brito et al. [2011] for more details.

We draw τ_ν and τ_η lines from properties of liquid sodium at 130 C ($\nu = 6.5 \times 10^{-7}$ m^2·s^{-1}, $\eta = 8.8 \times 10^{-2}$ m^2·s^{-1}). Power markers along these lines are deduced from liquid volume and density (ρ = 930 kg·m^{-3}). From the energy of applied dipolar magnetic field, we deduce time T_B and draw the $\tau_{\rm Alfven}(\ell)$ red wavy line of Alfvén wave propagation. We pick a run with outer shell rotation rate f_o = 10 Hz, which yields horizontal line t_Ω and wavy line $\tau_{\rm Rossby}$.

We consider an inner sphere differential rotation rate Δf = 10 Hz, for which a power dissipation $\mathscr{P}_{\rm diss} \simeq 700$ W is measured. Cabanes et al. [2014] reconstructed time-averaged flow and induced magnetic field by a joint inversion of a comprehensive set of flow velocity profiles and magnetic field measurements. Their Table III provides the energies of these fields, which we convert into vortex overturn time at integral scale T_U, and the corresponding time for induced magnetic field t_b. Note that their analysis if for $f_o = 0$, but should approximatively apply to our case.

The three components of the induced magnetic vector are measured every 6° along a meridian between latitudes −57° and +57°. We thus computed a k-spectrum of magnetic energy density from a set of 60s-long records, which was converted into line $\tau_b(\ell)$ according to Equation (42) of the article's Appendix A. No velocity measurement was available for that run, but a nice profile of angular velocity was measured using Ultrasound Doppler Velocimetry for another run with f_o = 5 Hz and Δf = 10 Hz. Using Equation (61) of Appendix A, we extract the frequency power spectral density of a 40s-long record at fluid mid-depth, and convert it to a kinetic energy density k-spectrum using Equation (62) with U deduced from the same profile, yielding line $\tau_u(\ell)$ drawn in Figure S1. Note that this spectrum might be contaminated by instrumental noise. The resulting τ–ℓ diagram suggests that velocity fluctuations are mostly quasi-geostrophic because line τ_u plots above Rossby line. Magnetic energy fluctuations are in a strongly dissipative region, and are almost three orders of magnitude smaller that kinetic energy fluctuations, in agreement with observations



Supplementary Figure S1. Composite representative τ–ℓ diagram of DTS magnetized spherical Couette laboratory experiment. Outer shell rotation frequency is 10 Hz, while inner sphere spins at 20 Hz. Time T_B (large red dot) is obtained from the energy of the dipolar magnetic field applied by the inner sphere magnet, while time t_b (small red dot) represents the time-averaged induced magnetic field. Time T_U (blue dot) is deduced from the time-averaged kinetic energy. Red horizontal dotted line marks dissipated power (700 W). Thick red line $\tau_b(\ell)$ is obtained from a k-spectrum of magnetic fluctuations measured at the outer shell surface. Thick blue line $\tau_u(\ell)$ is obtained from a frequency-spectrum of flow velocity fluctuations measured at mid-depth in the fluid using Ultrasound Doppler Velocimetry.

of Figueroa et al. [2013]. Under the combined influence of strong rotation and strong imposed magnetic field, energy fluctuations of both types are two to three orders of magnitude smaller than time-averaged energies (tagged by T_U and t_b in Figure S1), as noted by Nataf and Gagnière [2008], and Kaplan et al. [2018].

The short red wavy line $\tau_{\mathrm{Alfven}}(\ell)$ indicates that geostrophic Alfvén waves can propagate but are severely damped, as analyzed by Tigrine et al. [2019]. Dissipation is dominated by Ohmic dissipation of the time-averaged flow.

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