



External geophysics, climate

Investigation of power-law correlations within daily total ozone time series and search for a stable node through phase portrait

Investigation sur les corrélations de loi de puissance dans des séries temporelles d'ozone total journalier et recherche d'un nœud stable dans une configuration de phase

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ABSTRACT

In the study reported in this paper we have dealt with the intrinsic complexity of the daily total ozone time series over Mumbai, India (19°23'N, 72 15'00'E) by means of detrended fluctuation analysis and phase portrait analysis. From the detrended fluctuation analysis of first order it is revealed that the said time series is characterized by persistent power-law correlation which is stronger in shorter time scale than in the longer time scale. The phase portrait analysis is based on a non-linear autonomous system generated from regression equations; infinitely many stable nodes are found along the straight line $x = 273.556$ and from the behaviors of the contours, it has been revealed that the daily total ozone concentration is unlikely to go beyond 315 DU (approximately) and it is almost stable.

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R É S U M É

L'étude présentée dans cet article a concerné la complexité intrinsèque des séries temporelles quotidiennes d'ozone total au-dessus de Mumbai, Inde (19°23'N, 72 15'00'E), au moyen d'une analyse de fluctuations redressées et de portrait en phase. À partir de l'analyse de fluctuations redressées, il apparaît que la série dite temporelle est caractérisée par une corrélation en loi de puissance persistante, qui est plus forte à l'échelle de temps courts qu'à celle de temps plus longs. L'analyse de portrait de phase est basée sur un système autonome non linéaire, généré par des équations de régression; de très nombreux nœuds stables sont répertoriés le long de la ligne droite et, d'après les comportements des contours, il s'avère que la concentration journalière en ozone est peu encline à baisser au-dessous de 315 DU (approximativement) et est presque stable.

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1. Introduction

Discovery of the springtime ozone hole over Antarctica in the mid-1980s (Farman et al., 1985) shed light on ozone dynamics, especially over high and polar latitudes. The intrinsic spatial and temporal irregularity of emission

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concentrations, the influence of weather conditions, and uncertainties associated with initial and boundary conditions have made it a difficult task to model, calibrate and validate ozone variations (Chattopadhyay and Chattopadhyay, 2009). Sources of complexity within the total ozone have been summarized in many reports (e.g., WMO, 2010). In this regard, and studies (e.g. Chattopadhyay and Chattopadhyay-Bandyopadhyay, 2007) as follows:

- total ozone comprises tropospheric and stratospheric ozone. Both atmospheric layers contribute degrees of complexity to the corresponding ozone layer;
- formation of ozone is a complex photochemical reaction;
- formation of ozone depends upon a number of meteorological variables that may have their own complexities.

Chen et al. (1998) carried out a multidimensional phase space analysis of the observed ozone concentration and developed a powerful predictive model after investigating the trajectories on a phase space reconstructed for hourly ozone time series. Several non-linear approaches, till date, have been proposed to develop predictive models for tropospheric ozone based on the dealing with the intrinsic complexities. Some remarkable works in this direction include the studies of Jorquera et al. (1998), Koçak et al. (2000), Novara et al. (2007), Sousa et al. (2007), Heo and Kim (2004) and Feng et al. (2011). The total ozone (TO) is the integral of the ozone concentration with respect to height (Bandyopadhyay and Chattopadhyay, 2007). Association between climate and TO has been investigated by Reid et al. (1994), Varotsos et al. (1994), Efstathiou et al. (2003), Kondratyev and Varotsos (2000, 2002) and De et al. (2011). The necessity of considering the intrinsic complexity of the ozone dynamics has been emphasized in references like Kondratyev and Varotsos (1995, 2001), Varotsos et al. (2001) and Brandt et al. (2003).

In recent years, there has been growing evidence indicating that many physical systems have no characteristic length scale and exhibit long-range power-law correlations (Chen et al., 2002). Traditional approaches such as the power-spectrum and correlation analysis are suitable for stationary time series. However, many time series, which pertain to complex geophysical systems are non-stationary. This means that the mean, standard deviation and higher moments, or the correlation functions are not invariant under time translation (Stratonovich, 1981). To realize the essential dynamics of a given system, it is imperative to analyze and correctly interpret the associated time series. One of the frequent challenges is that the scaling exponent is not always constant (independent of scale) and the value of the scaling exponent fluctuates for different ranges of scales (Chen et al., 2002; Ivanov et al., 1999). Non-stationarity, which is an essential aspect of a complex variability, is often related to different trends in the signal or heterogeneous segments with different local statistical properties (Ivanov et al., 1999). The detrended fluctuation analysis (DFA) addresses this problem to accurately quantify long-range power-law correlations embedded in a non-stationary time series.

A classical approach is to use qualitative differential equations to characterize the orientation flow field by considering it as the velocity field of a particle in a dynamical system (Cohen and Herlin, 1996, 1999). Modeling the orientation flow field by a dynamical system allows us to characterize the flow field through the particles trajectories and their stationary points. Dynamical system approach is very common in bio-mathematical modeling (e.g. Jiang et al., 2007 and references therein Kelleher et al., 2003) and cosmological modeling (e.g. Jamil and Debnath, 2011; Szydlowski and Krawiec, 2003 and references therein). However, limited works are available in the area of geophysical studies, where dynamical system approach has been adopted to deal with the intrinsic complexity. We carried out a survey of the literature considering geophysical problems in the view of dynamical system. Guro and Singer (1982) studied the kinetics of ozone decomposition in aqueous solution under dynamic conditions; ozone decomposition is a second-order reaction with respect to ozone concentration. A remarkable approach towards time series modeling of ozone concentration from dynamic point of view is by Chen et al. (1998), who emphasized the complexity of ozone dynamics due to the inherent spatial and temporal variability of emission concentrations, influence of meteorological concentrations, and uncertainties associated with the initial and boundary conditions. In Chen et al. (1998), a phase space reconstruction was presented to model and predict hourly ozone concentration time series. Jayawardena and Gurung (2000) adopted non-linear dynamical systems and linear stochastic approaches to analyze synthetic and hydrometeorological time series, where the analysis by dynamical systems approach included computation of the correlation dimension, noise reduction and non-linear prediction. Ruzmaikin et al. (2004) presented a simple dynamic model of solar variability influence on climate based on three ordinary differential equations controlled by two parameters. Different techniques and concepts of chaotic theory were adopted by Ng et al. (2007) to enhance our understanding of the phenomena of outliers for daily extreme hydrological observations.

Purpose of the present paper is twofold. We can specifically state the aims and motivation behind this study as follows: this work deals with the daily total ozone (TO) concentration over Mumbai, India. Mumbai is situated about midway on the western coast of India (19°23'N, 72°15'00'E) and is a peninsular city joined to the mainland at its northern end (Venkataraman et al., 2002). Mumbai has a tropical monsoon climate with diurnal land and sea breezes of daytime onshore winds from the west and northwest and nighttime offshore winds from the north, east and northeast. An extensive analysis of various meteorological parameters and aerosol characteristics over Mumbai is available in Venkataraman et al. (2002). Two noteworthy studies by Srivastava (2004) and Srivastava et al. (2006) show the significant dominance of volatile organic compounds over Mumbai. Association between formation of ozone and VOC is discussed in Carter (1994), Kondratyev and Varotsos (2001) and Varotsos (2002). Due to the radiative and dynamic coupling between the stratosphere and troposphere, any change

in the climate influences the evaluation of ozone layer through multiple changes in transport, chemical composition and thermal character (Pal, 2010). In an attempt to shed light on aforementioned complexity, scaling analysis of atmospheric data certainly is felt an important approach. Consideration of the decay of correlations in time has often been used to look into the dynamics of complex systems. A randomly forced first-order linear system with memory has an autocorrelation function decaying exponentially with lag time, but a higher order system will tend to have a different decay pattern. However, direct calculation of the autocorrelation function is affected by noise superimposed on the data (Varotsos and Kirk Davidoff, 2006). Following Varotsos and Kirk Davidoff (2006) we investigate the pattern of the decay of autocorrelation within the daily TO with increasing time lag by means of detrended fluctuation analysis (DFA). Afterwards, we have generated an autonomous system from regression equations and considered the daily TO as a dynamic system. Sources of complexity in TO has already been discussed in the works of Varotsos and Kirk Davidoff (2006), Chattopadhyay and Chattopadhyay-Bandyopadhyay (2007, 2008), De et al. (2011). This approach have been adopted because we have tried to shed light into the intrinsic complexity of the time series of daily TO by means of a phase portrait analysis.

2. Investigation of power-law correlation

2.1. Overview of detrended fluctuation analysis

Temporal correlations within the climate system are of physical and practical interest. Daily and seasonal correlations enable weather and climate prediction, and correlations on longer time scales characterize the interaction of climate components. Therefore, the analysis of correlations within the different compartments of the climate system and its realistic physical modeling are fundamental in climate research (Fraedrich and Blender, 2003). Detrended fluctuation analysis (DFA) (Peng et al., 1993) has recently been extensively implemented in the various physical processes. Several studies have reported that the DFA measure provides information that could not have been otherwise obtained, or that it is intrinsically a better measure to existing techniques (Heneghan and McDarby, 2000). The DFA is a scaling analysis method used to estimate long-range power-law correlation exponents in a time series. This method has been described in detail in the papers like Hu et al. (2001), Kantelhardt et al. (2002) and Chen et al. (2002). A brief overview of DFA is presented in this section. Peng et al. (1994) originally proposed DFA as a technique for quantifying the nature of long-range correlations. In recent years this method has become a widely used technique for the detection of long-term correlations in noisy, non-stationary time series (e.g. Bunde et al., 2002, Chen et al., 2002, Kantelhardt et al., 2006). In DFA, the data are first converted to non-stationary by generating the non-stationary profile $Y(i)$ generated by $Y(i) = \sum_{k=1}^i \phi_k, i = 1, 2, 3, \dots, N$ (Varotsos and

Kirk Davidoff, 2006), for the data series ϕ_k of size N . Thus, the reconstructed time series has the entries:

$$\begin{aligned} y(1) &= [\phi_1 - \phi_{average}] \\ y(2) &= [\phi_1 - \phi_{average}] + [\phi_2 - \phi_{average}] \\ y(3) &= [\phi_1 - \phi_{average}] + [\phi_2 - \phi_{average}] + [\phi_3 - \phi_{average}] \\ &\vdots \\ y(N) &= \sum_{k=1}^N [\phi_k - \phi_{average}] \end{aligned} \quad (1)$$

The integration:

- exaggerates the non-stationarity of the original data;
- reduces the noise level;
- generates a time series corresponding to the construction of a random walk that has the values of the original time series as increments (Varotsos and Kirk Davidoff, 2006).

The new time series, however, still preserves information about the variability of the original time series. In the next step, we divide the reconstructed time series $Y(i)$ into $N_l = \text{int}(N/l)$ non-overlapping segments of equal length l . The length N of the series being not a multiple of the considered timescale l , a small part at the end of the reconstructed time series may remain outside the segments generated above. In order not to disregard this part of the series, Kantelhardt et al. (2006) proposed to repeat the same procedure starting from the opposite end. Hence, $2N_l$ segments are obtained altogether. For each of the $2N_l$ blocks by fitting a polynomial of order n to the data we determine the variance for each block as (Kantelhardt et al., 2006)

$$\sigma(v, l) = \frac{1}{l} \sum_{i=1}^l [Y((v-1)l+i) - p_v(i)]^2 \quad (2)$$

where v varies from 1 to $2N_l$ and $p_v(i)$ is the fitting polynomial representing the local trend in the segment. The local trend in each block is defined to be a linear least-squares fit to the samples in that block. Linear, quadratic, cubic, or higher-order polynomials can be used in the fitting procedure. When linear polynomials are used, the fluctuation analysis is called DFA1, for quadratic polynomials we have DFA2. In the present work, we have experimented DFA1. A detrended signal is defined for each block as the difference between the original signal and the local trend for that block and the detrended fluctuation function is derived as (Kantelhardt et al., 2006)

$$F^2(l) = \frac{1}{2N_l} \sum_{v=1}^{2N_l} \sigma(v, l) \quad (3)$$

The detrended fluctuation function is computed for different values of l . It has been shown by Buldyrev et al. (1995) that $F(l)$ varies as a power-law in l , for sequences with power-law long-range correlations. This means that if data series ϕ_k is characterized by long-term power-law correlation, $F(l)$ increases, for large values of l , by a power-law, notably $F(l) \propto l^\alpha$ (Kantelhardt et al., 2006). When the

Table 1Nature of the series self-correlations based on least square fit to the log-log graph of l against $F(l)$.**Tableau 1**Nature des auto-corrélations basées sur l'ajustement des moindres carrés au graphique log-log de l en fonction de $F(l)$.

Range of l	Straight line fit	Value of α	Nature of the series self-correlations
2 to 15	$y = 1.4436x + 1.4378$ ($R^2 = 0.9587$)	1.4436	Persistent long-range power-law correlations
2 to 30	$y = 1.0238x + 1.7741$ ($R^2 = 0.8006$)	1.0238	Persistent long-range power-law correlations
2 to 45	$y = 0.8158x + 1.9676$ ($R^2 = 0.3242$)	0.8158	Persistent long-range power-law correlations

autocorrelation function decreases faster than $1/l$ in time, we asymptotically have $F(l)\alpha^l$. Conventionally, an exponent $\alpha \neq 1/2$ in a certain range of l values implies the existence of long-range correlations in that time interval. In Varotsos (2005), the types of correlation present in a time series implied by the value of α is thoroughly discussed. A straight line fit to the log-log plot of the log-log graph indicates statistical self-affinity expressed as $F(l)\alpha^l$. The scaling exponent α is calculated as the slope of a straight line fit to the log-log graph of l against $F(l)$ using least-squares. If $0.5 < \alpha \leq 1$, then persistent long-range power-law correlations exist in the time series, if $0 < \alpha < 0.5$, then the time series is characterized by power-law anticorrelations (antipersistence) and when $1 < \alpha < 1.5$, then there are correlations again. A Brownian noise is indicated by $\alpha = 1.5$ (Varotsos, 2005).

2.2. Discussion on the data under consideration

In the present work, we have dealt with the daily total ozone (TO) concentration over Mumbai, India. The data are derived from the measurements made by the Earth Probe Total Ozone Mapping Spectrometer (EP/TOMS) and are available at the website http://ftp://jwocjy.gsfc.nasa.gov/pub/eptoms/data/overpass/OVP206_epc.txt. For executing DFA we have considered the daily TO spanned over 1999–2000. Based on the discussion presented in the previous section we have first detrended the time series of daily TO and then divided the data series into several non-overlapping boxes of different sizes. The box size l has varied from 2 to 50. For each box size we have removed the local trend by means of removing the first order as well as second order trend. This means that, in Eq. (2) we have used $pv(i)$ in the linear form. Thus, DFA1 is executed. For different ranges of the box length we fit straight lines to the log-log graph of l against $F(l)$ by least square method. Results are presented in the Table 1. It is observed from Table 1 and from the Fig. 1(a-c) that the scaling exponent α is always above 0.5 and never becomes equal to 1.5. Hence, it is concluded that for the time interval ranging from about 2 days to more than one month, the time series of daily TO over Mumbai is characterized by persistent power-law correlations. However, from the decreasing behavior of the R^2 it may be interpreted that the power-law correlation is stronger in shorter time scale than in the longer time scale. Moreover, no prominent periodicity is observed in this power-law correlation. Based on the observation of Varotsos and Kirk Davidoff (2006) this may be interpreted as an indicator of existence of dynamical links between long and short time-scale behavior. In the subsequent section, we shall consider the said daily TO

time series as a dynamic system by generating a set of differential equations and subsequently studying the behaviors of the critical points within the phase portrait.

3. Dynamical system approach

Two systems of first order autonomous differential equations are said to be topologically equivalent if an invertible mapping exists between one phase portrait onto the other while preserving the orientation of the trajectories. Phase portraits are created using isoclines, vector fields, and eigenvectors. A non-linear autonomous system is considered in the form (Lynch, 2010):

$$\dot{x} = P(x, y) \quad (4a)$$

$$\dot{y} = Q(x, y) \quad (4b)$$

where at least one of P and Q is non linear in x and y . The critical points (u, v) is obtained by solving $\dot{x} = 0 = \dot{y}$. The non-linear system is linearized by computing the following Jacobian matrix at every critical point (u, v) :

$$J_{(u,v)} = \begin{pmatrix} \frac{\partial P}{\partial x} & \frac{\partial P}{\partial y} \\ \frac{\partial Q}{\partial x} & \frac{\partial Q}{\partial y} \end{pmatrix} \quad (5)$$

In the present work, we shall view the daily TO time series as a dynamical system. To do the same, it is needed to generate two-dimensional linear differential equations in the forms presented in (1a) and (1b). Now we consider polynomial regressions as:

$$x = \xi(t) \quad (6)$$

$$y = \eta(x) \quad (7)$$

where $\xi(t)$ and $\eta(x)$ denote second degree polynomials in t and x respectively, t denotes time, x denotes the observations corresponding to t , and y denotes the time series lagged by 1 time point. By means of least square regression we estimate the regression constants and coefficients as: $a = 250.91$, $b = 0.1346$, $c = -0.0002$, $d = -283.66$, $e = 3.2806$ and $f = -0.0045$.

Subsequently, the non-linear autonomous system is generated as:

$$\frac{dx}{dt} = [b^2 - 4ac + 4cx]^{1/2} \quad (8)$$

$$\frac{dy}{dt} = \left[b^2 e^2 + 4ce^2(x - a) + 4b^2 f(y - d) \right]^{1/2} + 16cf(-a + x)(-d + y) \quad (9)$$

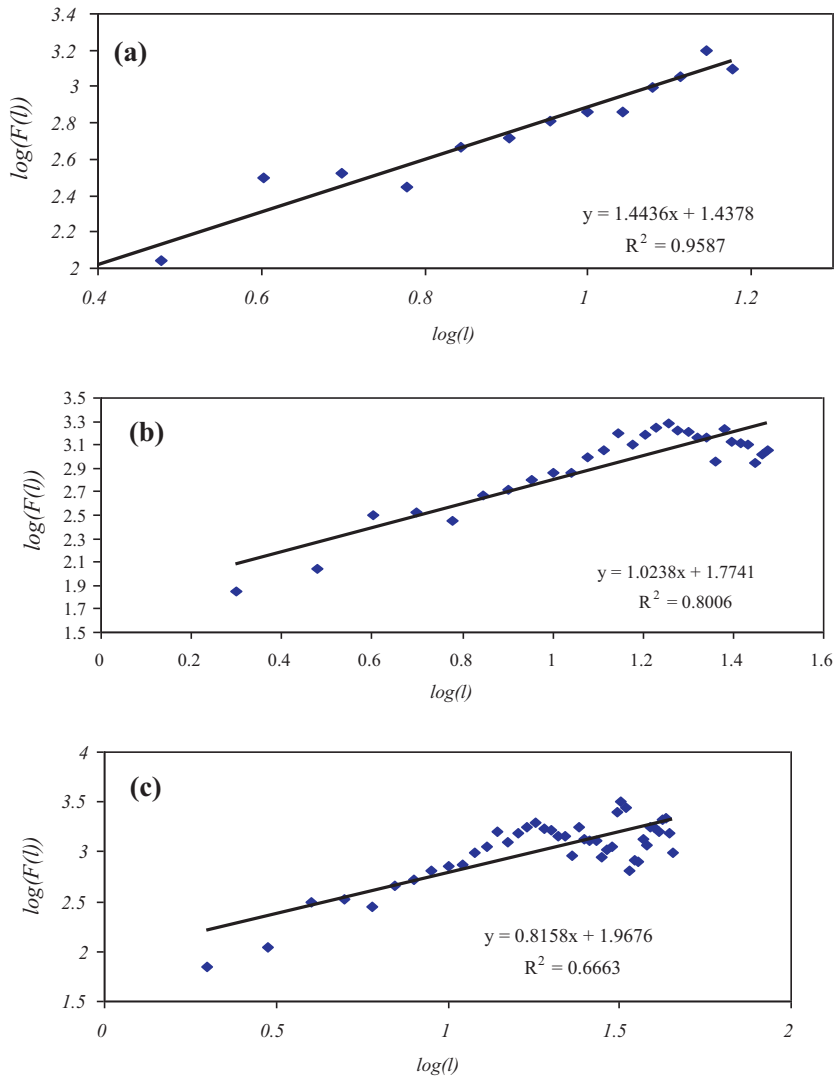


Fig. 1. Straight line fits to various ranges of l . The first panel corresponds to the range 2 to 15, the second panel corresponds to the range 2 to 30 and the third panel corresponds to the range 2 to 45.

Fig. 1. La ligne droite est ajustée aux différentes amplitudes de l . Le premier panneau correspond à la gamme de 2 à 15, le deuxième à la gamme de 2 à 30 et le troisième à la gamme de 2 à 45.

$\frac{dx}{dt} = 0$ occurs when $x = 273.556$. With this value of x , the $\frac{dy}{dt}$ happens to be 0 for any value of y . Thus, we get infinitely many critical points along the straight line $x = 273.556$. Since we are considering density of TO, we shall consider y to be positive. The Jacobian matrix comes out to be:

$$J = \begin{pmatrix} \frac{2c}{\sqrt{b^2 - 4ac + 4cx}} & 0 \\ \frac{2c\sqrt{e^2 - 4df + 4fy}}{\sqrt{b^2 - 4ac + 4cx}} & \frac{2f\sqrt{b^2 - 4ac + 4cx}}{\sqrt{e^2 - 4df + 4fy}} \end{pmatrix} \quad (10)$$

Using the values of the regression constants and parameters the eigenvalues are:

$$\lambda_1 = -\frac{0.0004}{\sqrt{0.218845 - 0.0008x}} \quad (11)$$

$$\lambda_2 = -\frac{0.009\sqrt{0.218845 - 0.0008x}}{\sqrt{5.65646 - 0.015y}} \quad (12)$$

Since both of the eigenvalues are real and negative along for all of the critical points, we conclude that the eigenvectors represent stable situations that the system converges towards, and the intersection is a stable node. For a system of n first-order ordinary differential equations (or more generally, Pfaffian forms), the $2n$ -dimensional space consisting of the possible values of $(x_1, \dot{x}_1, x_2, \dot{x}_2, \dots, x_n, \dot{x}_n)$ is known as its phase space (Weisstein, 2011). For a function with 2 degrees of freedom, the 2-dimensional phase space that is accessible to the function or object is called its phase plane. A phase portrait is a plot of multiple phase curves corresponding to

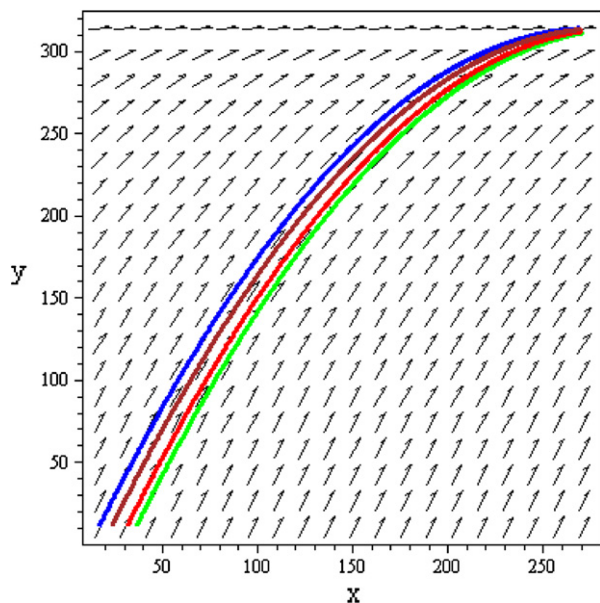


Fig. 2. The phase portrait. Here, the horizontal axis corresponds to x_t and the vertical axis corresponds to x_{t+1} . For convenience, these are denoted as x and y respectively.

Fig. 2. Portrait de phase. L'axe horizontal correspond à x_t et l'axe vertical à x_{t+1} . Par commodité, ces axes sont appelés x et y respectivement.

different initial conditions in the same phase plane (Weisstein, 2011). For the present dynamical system, the phase portrait is presented in Fig. 2. We find that all the trajectories are directed towards the line $x = 273.556$ and the situation being stable it may be interpreted that the daily TO over Mumbai has the tendency to be in the vicinity of 273 DU. Now we consider the contours within the phase portrait. These contours pertain to different initial conditions. From the nature of the contours we see that all of them are converging towards the point (273, 310). The values of y are first increasing with increase in x . This means that the concentration of daily TO is increasing as well as the concentration at time point lagged by 1 is also increasing. In the vicinity of (273, 315), both variables get confined and thus it may be interpreted that the concentration of the daily TO concentration is unlikely to go beyond 315 DU (approximately) and it is almost stable.

4. Concluding remarks

In the study explained in the previous sections, we have dealt with the intrinsic complexity of the daily total ozone time series over Mumbai, India by means of detrended fluctuation analysis and phase portrait analysis. In the introductory part of the paper we have thoroughly discussed the motivation behind adopting the above methodologies to deal with the daily total ozone over Mumbai, where the atmospheric environment is dominated by volatile organic compounds. The detrended fluctuation analysis has been implemented by using first order polynomials to the non-overlapping blocks. It is revealed that for the time interval ranging from about 2 days to more than one month, the time series of daily total ozone

over Mumbai is characterized by persistent long-range power-law correlations. However, from the decreasing behavior of the coefficient of determination with block length it has been understood that the power-law correlation is stronger in shorter time scale than in the longer time scale. In the next part of the study, we have created an autonomous system by means of ordinary differential equations based on non-linear regression equations fit to the time series first with time t as the independent variable and the corresponding observation x_t as the dependent variable and then by x_t as the independent and x_{t+1} as the dependent variable. Subsequently we have created the phase portrait where it has been found that there are infinitely many stable nodes along the vertical axis. Observing the fact that all the trajectories on the phase portrait are directed towards the line $x = 273.556$ and the situation being stable it may be interpreted that the daily TO over Mumbai has the tendency to be in the vicinity of 273 DU. It is further observed that all of contours along the phase portrait are converging towards the point (273, 310). This leads us to conclude that the concentration of daily total ozone is increasing as well as the concentration at time point lagged by 1 is also increasing. In the vicinity of (273, 315), both variables get confined and thus it may be interpreted that the concentration of the daily total ozone concentration is unlikely to go beyond 315 DU (approximately) and it is almost stable. Based on the time series under consideration we have constructed a phase portrait and obtained a stable node based on the autonomous system. The trajectories are tending towards the said point and hence it is an approximate stable node. If longer time series are considered then more complexity would be explored and we propose to study this in future.

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