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Problèmes mathématiques de la mécanique/*Mathematical Problem in Mechanics*

Erratum

to the Note by Paolo Bisegna, Frédéric Lebon, Franco Maceri

entitled: *D-PANA: a convergent block-relaxation solution method for the discretized dual formulation of the Signorini–Coulomb contact problem*
published in Série I, t. 333, n° 11, pp. 1053–1058.

The Note: *D-PANA: a convergent block-relaxation solution method for the discretized dual formulation of the Signorini–Coulomb contact problem* was published in Tome 333, number 11, pp. 1053–1058. The authors' corrections were omitted; we publish here the corrected Subsection 3.3.

3.3. Well-posedness and convergence result

THEOREM 3.2. – *Under the hypothesis stated in Section* 3.1, *there exists a positive constant* $\mathcal{M}$ *such that for* $0 \leqslant \mu < \mathcal{M}$ *the transformation* $f : \mathcal{H} \to \mathcal{H}$ *defined by Eq.* (6) *is a contraction. As a consequence, for* $0 \leqslant \mu < \mathcal{M}$ *the discrete dual condensed formulation* (4) *of the Signorini–Coulomb contact problem has a unique solution* $(\overline{\sigma}, \overline{\tau}) \in \mathcal{H} \times \mathcal{K}_{\overline{\sigma}}$ *and the D-PANA algorithm converges to this solution for any initial vector* $\sigma_0 \in \mathcal{H}$. *Moreover, a constant* $0 < \beta < 1$ *exists such that the error estimates*

$$\|\sigma_k - \overline{\sigma}\| \leqslant \frac{\|\sigma_0 - \sigma_1\|\beta^k}{1-\beta}, \quad \|\tau_k - \overline{\tau}\| \leqslant \frac{\|\sigma_0 - \sigma_1\|\beta^k}{(1-\beta)\|C\|}, \quad k \in N,$$

hold.

Proof of Theorem 3.2. – For any $\sigma_1, \sigma_2 \in \mathcal{H}$, by using Lemma 3.1, the following estimate is obtained:

$$\begin{aligned}\big\|f(\sigma_1) - f(\sigma_2)\big\| &\leqslant \big\|C^t\big(p(d_\tau - C\sigma_1, -\mu Q_\sigma \sigma_1) - p(d_\tau - C\sigma_2, -\mu Q_\sigma \sigma_2)\big)\big\| \\ &\leqslant \|C\|\big(\|-C\sigma_1 + C\sigma_2\| + \big\|Q_\tau^{-1}\big\|\|-\mu Q_\sigma \sigma_1 + \mu Q_\sigma \sigma_2\|\big) \leqslant \|C\|\big(\|C\| + \mu\|Q_\sigma\|\big\|Q_\tau^{-1}\big\|\big)\|\sigma_1 - \sigma_2\|.\end{aligned}$$

It follows that f is Lipschitz continuous with Lipschitz coefficient $\beta = \|C\|(\|C\| + \mu\|Q_\sigma\|\|Q_\tau^{-1}\|)$. From the positive definiteness of the compliance matrix in the diagonalized formulation, it easily follows that $\|C\| < 1$. Therefore, setting $\mathcal{M} = (1 - \|C\|^2)/(\|C\|\|Q_\sigma\|\|Q_\tau^{-1}\|)$, for every $0 \leqslant \mu < \mathcal{M}$ it turns out that $0 < \beta < 1$. Hence, the assert follows from the Banach–Caccioppoli contraction principle, by observing that Eqs. (5) can be summarized by the equation $\sigma_k = f(\sigma_{k-1})$. The error estimates follow from the contraction principle and Lemma 3.1. It is worth observing that the constant $\mathcal{M}$ depends on quantities having a significant mechanical interest: they are $\|C\|$, which is a measure of the energetic coupling between σ and τ, and $\|Q_\sigma\|\|Q_\tau^{-1}\|$, related to the direct complementary energies associated to σ and τ.

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