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Partial Differential Equations

On the Γ-convergence of matrix fields related to the adjugate Jacobian

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Abstract

Adjugate Jacobians of mappings $f_j : \Omega \subset \mathbb{R}^2 \to \mathbb{R}^2$ can be represented in terms of Jacobian matrices: $\operatorname{adj} Df_j = \mathcal{A}_j(x) Df_j^t$, for $j = 1, 2, \ldots$, by mean of symmetric matrix fields $\mathcal{A}_j(x)$ with $\det \mathcal{A}_j(x) = 1$ a.e. Under suitable conditions, we prove that $Df_j \rightharpoonup Df$ weakly in $L^1_{\mathrm{loc}}(\Omega; \mathbb{R}^2)$ if and only if $\mathcal{A}_j(x)$ Γ-converges to a matrix $\mathcal{A}(x)$ with $\det \mathcal{A}(x) = 1$ satisfying $\operatorname{adj} Df = \mathcal{A}(x) Df^t$. ***To cite this article: C. Sbordone, C. R. Acad. Sci. Paris, Ser. I 337 (2003).***

Résumé

Sur la Γ-convergence de champs de matrices relatifs au jacobien adjugué. La transposée des cofacteurs de la matrice jacobienne de l'application $f_j : \Omega \subset \mathbb{R}^2 \to \mathbb{R}^2$ peut être exprimée en fonction de la matrice jacobienne : $\operatorname{adj} Df_j = \mathcal{A}_j(x) Df_j^t$, pour $j = 1, 2, \ldots$, en utilisant une matrice symétrique $\mathcal{A}_j(x)$ telle que $\det \mathcal{A}_j(x) = 1$ p.p. Sous des hypothèses appropriées, nous prouvons que Df_j converge faiblement vers Df dans $L^1_{\mathrm{loc}}(\Omega; \mathbb{R}^2)$ si et seulement si $\mathcal{A}_j(x)$ Γ-converge vers une matrice $\mathcal{A}(x)$ telle que $\det \mathcal{A}(x) = 1$ p.p. et vérifiant $\operatorname{adj} Df = \mathcal{A}(x) Df^t$. ***Pour citer cet article : C. Sbordone, C. R. Acad. Sci. Paris, Ser. I 337 (2003).***

Version française abrégée

Pour toute matrice $D \in \mathbb{R}^{2\times 2}$ on dénote par $\operatorname{adj} D$ la matrice transposée des cofacteurs. Si $K \geqslant 1$ et D est K-quasiconforme, c'est-à-dire $\|D\|^2 \leqslant K \det D$, alors il existe une unique matrice symétrique $\mathcal{A} \in \mathbb{R}^{2\times 2}$ verifiant :

$$\frac{|\xi|^2}{K} \leqslant \langle \mathcal{A}\xi, \xi \rangle \leqslant K|\xi|^2, \quad \text{et} \quad \det \mathcal{A} = 1$$

telle que $\operatorname{adj} D = \mathcal{A} D^t$.

Nous appelons $\mathcal{A}$ *le tenseur de distorsion* inverse de D.

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Pour une fonction measurable à valeurs matricielles $D = D(x)$, K-quasiconforme pour $x \in \Omega \subset \mathbb{R}^2$ p.p., nous prouvons que si $D_j \rightharpoonup D$ dans $L^1_{\mathrm{loc}}(\Omega, \mathbb{R}^2)$ et $\operatorname{Curl} D_j = 0$, alors les tenseurs inverses de distorsion $\mathcal{A}(x, D_j)$ de $D_j(x)$ satisfont $\mathcal{A}(x, D_j) \xrightarrow{G} \mathcal{A}(x, D)$ au sens de De Giorgi–Spagnolo (see [7,10]). En remplaçant $\operatorname{Curl} D_j = 0$ par $\operatorname{Div} D_j = 0$ nous trouvons :

$$\mathcal{A}(x, D_j)^{-1} \xrightarrow{G} \mathcal{A}(x, D)^{-1}.$$

Une généralisation aux matrices de distorsion $K(x)$ est présentée.

1. The G-convergence in the uniformly elliptic case

For any matrix $D \in \mathbb{R}^{2\times2}$, let $\operatorname{adj} D$ be its adjugate matrix, i.e., the transpose of its cofactors, defined by the identity

$$D \operatorname{adj} D = (\det D)\mathbf{I}, \tag{1}$$

where $\mathbf{I}$ is the unit matrix.

We note that if $\det D > 0$, then there exists a symmetric matrix $\mathcal{A} = \mathcal{A}(D)$ with $\det \mathcal{A} = 1$ such that

$$\operatorname{adj} D = \mathcal{A}(D) D^t, \tag{2}$$

namely

$$\mathcal{A}(D) = \left[\frac{D^t D}{\det D}\right]^{-1}. \tag{3}$$

In the following we supply $\mathbb{R}^{2\times2}$ with the operator norm $\|D\| = \max_{|\xi|=1} |D\xi|$ or, sometimes, with the Hilbert–Schmidt norm, $|D| = [\operatorname{Tr}(D^t D)]^{1/2}$ where $\operatorname{Tr}(C) = \sum_{i=1}^2 c_{ii}$ if $C = (c_{ij})$.

The natural ellipticity bounds we will consider on $\mathcal{A}$ are:

$$\frac{|\xi|^2}{K} \leqslant \langle \mathcal{A}\xi, \xi\rangle \leqslant K|\xi|^2 \tag{4}$$

for a $K \geqslant 1$, and we prove that (4) holds true for $\mathcal{A} = \mathcal{A}(D)$ if and only if D is a K-*quasiconformal* matrix, i.e.,

$$\|D\|^2 \leqslant K \det D. \tag{5}$$

Let us first introduce the following sets of matrices:

$$\mathcal{Q}_2(K) = \left\{D \in \mathbb{R}^{2\times2}\colon \|D\|^2 \leqslant K \det D\right\},$$
$$\mathcal{E}_2(K) = \left\{\mathcal{A} \in \mathbb{R}^{2\times2}\colon \mathcal{A}^t = \mathcal{A},\ \frac{\mathbf{I}}{K} \leqslant \mathcal{A} \leqslant K\mathbf{I},\ \det \mathcal{A} = 1\right\}.$$

The two sets are related via the mapping $D \to \mathcal{A}(D)$, according to the following:

Proposition 1.1. *If* $D \in \mathbb{R}^{2\times2}$, $\det D > 0$ *and* $\mathcal{A}(D)$ *is defined by* (3)*, then*:

$$D \in \mathcal{Q}_2(K) \quad \textit{if and only if} \quad \mathcal{A} \in \mathcal{E}_2(K). \tag{6}$$

Moreover $\mathcal{A}(D)$ *is the unique matrix in* $\mathcal{E}_2(K)$ *such that* $\operatorname{adj} D = \mathcal{A}(D)D^t$.

Proof. First of all it is easy to check that a matrix D belongs to $\mathcal{Q}_2(K)$ if and only if:

$$|D|^2 \leqslant \left(K + \frac{1}{K}\right) \det D. \tag{7}$$

Now for $D \in \mathcal{Q}_2(K)$ with $\det D > 0$, consider the inverse matrix of $\mathcal{A}$, $\mathcal{G} = \frac{D^t D}{\det D}$. Then, obviously $\det \mathcal{G} = 1$ and the distortion inequality (7) is equivalent to $\operatorname{Tr}(\mathcal{G}) \leqslant K + \frac{1}{K}$. Let λ and $\frac{1}{\lambda}$ be the eigenvalues of $\mathcal{G}$. Then the last inequality means that $\lambda + \frac{1}{\lambda} \leqslant K + \frac{1}{K}$; hence $\frac{1}{K} \leqslant \lambda \leqslant K$ and so $\mathcal{A}(D)$ belongs to $\mathcal{E}_2(K)$.

Notice that, if $\mathcal{A} \in \mathcal{E}_2(K)$ and $D \in \mathbb{R}^{2\times 2}$ with positive determinant are related by the identity (2) then we deduce:

$$\operatorname{Tr}(D^t D) = \operatorname{Tr}\big(\mathcal{A}^{-1}(\operatorname{adj} D) D\big) = \operatorname{Tr}\big(\mathcal{A}^{-1}(\det D)\mathbf{I}\big) = \operatorname{Tr}\big(\mathcal{A}^{-1}\big) \det D \leqslant \left(K + \frac{1}{K}\right) \det D,$$

since $\operatorname{Tr}(\mathcal{A}^{-1}) = \operatorname{Tr}(\mathcal{A}) \leqslant (K + \frac{1}{K})$.

To prove the last statement, observe that $\mathcal{A} D^t = (\det D) D^{-1} [D^t]^{-1} D^t = (\det D) D^{-1} = \operatorname{adj} D$. □

Now we are interested in variable matrices $D = D(x) \in \mathcal{Q}_2(K)$, for a.e. $x \in \Omega$ where $\Omega \subset \mathbb{R}^2$ is a simply connected bounded domain. If $D(x)$ is measurable then the pointwise distortion tensor $\mathcal{A}(x) = \mathcal{A}(x, D(x))$, associated to $\operatorname{adj} D(x)$, i.e., satisfying

$$\operatorname{adj} D(x) = \mathcal{A}(x) D(x)^t \tag{8}$$

is a measurable matrix field which is uniformly elliptic with $\det \mathcal{A}(x) = 1$ a.e. An important point here is that a converse statement is also true. By the so-called measurable Riemann mapping theorem, given any measurable symmetric matrix field $\mathcal{A}(x)$ in $\Omega \subset \mathbb{R}^2$ such that $\mathcal{A}(x) \in \mathcal{E}_2(K)$ a.e. $x \in \Omega$ we can find $D \in L^2(\Omega, \mathbb{R}^{2\times 2})$ such that $D(x) \in \mathcal{Q}_2(K)$ a.e., $\operatorname{Curl} D(x) = 0$, for which (8) holds. A natural question is to see how does the *pointwise inverse distortion tensor* $\mathcal{A} = \mathcal{A}(x, D)$ change with $D(x)$.

We are particularly concerned with the continuity properties of the operator, $D \in L^2(\Omega, \mathbb{R}^2) \to \mathcal{A}(x, D) \in L^\infty(\Omega, \mathbb{R}^2)$, when we supply $L^2(\Omega, \mathbb{R}^2)$ with the weak topology. If D_j converges weakly to D, this does not guarantee convergence of matrices $\mathcal{A}(x, D_j)$ to $\mathcal{A}(x, D)$ in any familiar sense. Note that the condition $\det \mathcal{A}(x, D_j) = 1$ is not necessarily preserved under the weak$*$ convergence of $\mathcal{A}(x, D_j)$. The relevant concept to be considered here is that of G-convergence, at least in the case $\operatorname{Curl} D_j = 0$ a.e. in Ω (see also related ideas in [1–3,10]) . Let $A_j = A_j(x)$ be a sequence of measurable matrix valued functions $A_j : \Omega \to \mathbb{R}^{2\times 2}$ satisfying the ellipticity condition:

$$\frac{|\xi|^2}{K} \leqslant \big\langle A_j(x)\xi, \xi\big\rangle \leqslant K|\xi|^2 \tag{9}$$

for a.e. $x \in \Omega$ and $\forall \xi \in \mathbb{R}^2$, with $K \geqslant 1$. Assume that

$$\det A_j(x) = 1 \quad \text{a.e. } x \in \Omega. \tag{10}$$

We are ready for the definition of G-convergence of A_j to a matrix valued function $A = A(x)$ satisfying (9) and (10).

Definition 1.2. The sequence A_j G-converges to A if and only if, for $D_j \in L^2_{\mathrm{loc}}(\Omega, \mathbb{R}^{2\times 2})$ satisfying:

$$\operatorname{Div}\big(A_j(x) D_j^t(x)\big) = 0, \quad \text{and} \quad \operatorname{Curl} D_j(x) = 0,$$

the conditions, (i) $D_j(x) \rightharpoonup D(x)$; and (ii) $A_j(x) D_j^t(x) \rightharpoonup A(x) D^t(x)$, are equivalent each other.

Here, the Div operator is defined as

$$\big(\operatorname{Div} M(x)\big)_i = \sum_{k=1}^{2} \frac{\partial M_{ki}(x)}{\partial x_k}, \quad i = 1, 2,\ M \in L^2_{\mathrm{loc}}\big(\Omega, \mathbb{R}^{2\times 2}\big).$$

We have the following:

Theorem 1.3. *Let Ω be a simply connected bounded open set in $\mathbb{R}^2$. Let D_j belong to $L^2(\Omega,\mathbb{R}^2)$ and $D_j(x)\in\mathcal{Q}_2(K)$ a.e. ($K\geqslant 1$). Assume*

$$D_j \rightharpoonup D \neq 0 \quad \text{weakly in } L^2\big(\Omega,\mathbb{R}^{2\times 2}\big). \tag{11}$$

Then (i) *and* (ii) *holds true*:

(i) *if* $\operatorname{Curl} D_j = 0$, *then* $D(x)\in\mathcal{Q}_2(K)$ *a.e. and* $\mathcal{A}(x,D_j)\xrightarrow{G}\mathcal{A}(x,D)$;
(ii) *if* $\operatorname{Div} D_j = 0$, *then* $D(x)\in\mathcal{Q}_2(K)$ *a.e. and* $\mathcal{A}(x,D_j)^{-1}\xrightarrow{G}\mathcal{A}(x,D)^{-1}$.

Remark 1. Roughly speaking, in case (i) Theorem 1.3 has the following meaning: if $D_j = Df_j$ then, by a well known property of the adjugate Jacobian, $\operatorname{Div}(\operatorname{adj} Df_j)=0$, and so the mappings f_j solve their own second order elliptic system $\operatorname{Div}(\mathcal{A}(x,Df_j)Df_j^t)=0$. Actually, the mappings f_j behave as “principal solutions” to such a system, their convergence governing the convergence of all other sequences g_j of solutions.

Proof of Theorem 1.3. (i) Let $f_j,\ f\in W^{1,2}(\Omega,\mathbb{R}^2)$ satisfy $Df_j = D_j$, $Df = D$. By our assumption:

$$Df_j \rightharpoonup Df \tag{12}$$

we obtain, via a classical result of Reshetnyak [8,4]:

$$\det Df_j \rightharpoonup \det Df \quad \text{weakly in } L^1_{\mathrm{loc}}(\Omega)$$

and so $Df(x)\in\mathcal{Q}_2(K)$, a.e. in Ω in virtue of the lower semicontinuity of the norm. By the G-compactness theorem [10] (see also [5] for more general cases of degenerate elliptic equations) we may assume $\mathcal{A}(x,Df_j)\xrightarrow{G}A_0(x)$. Since $\operatorname{Div}(\mathcal{A}(x,Df_j)Df_j^t)=\operatorname{Div}(\operatorname{adj} Df_j)=0$ by definition of G-convergence, we have $\mathcal{A}(x,Df_j)Df_j^t\rightharpoonup A_0(x)Df^t$. But (12) and the definition of $\mathcal{A}(x,Df_j)$ imply $\mathcal{A}(x,Df_j)Df_j^t\rightharpoonup\mathcal{A}(x,Df)Df^t$ and so $\mathcal{A}(x,Df)Df^t = A_0(x)Df^t$. Since $Df^t\neq 0$ a.e., we deduce $\mathcal{A}(x,Df)=A_0(x)$.

(ii) Taking into account that Ω is a simply connected open set in $\mathbb{R}^2$, the condition $\operatorname{Div} D_j = 0$ implies $D_j = \operatorname{adj} Dg_j$ for some $g_j\in W^{1,2}(\Omega;\mathbb{R}^2)$. Hence, by the definition of $\mathcal{A}(x,Dg_j)$, $D_j=\mathcal{A}(x,Dg_j)D^t g_j$ which, of course can be rewritten as: $Dg_j=\mathcal{A}(x,Dg_j)^{-1}D_j^t$. Now, the hypothesis $D_j\rightharpoonup D$ in $L^2(\Omega,\mathbb{R}^{2\times 2})$ is equivalent to $Dg_j\rightharpoonup Dg$ and so, by part (i) we have $D(x)\in \mathrm{Q}_2(K)$ a.e. in Ω, and $\mathcal{A}(x,Dg_j)\xrightarrow{G}\mathcal{A}(x,Dg)$. Note that

$$\mathcal{A}(x,Dg_j)=\mathcal{A}(x,D_j)^{-1}. \tag{13}$$

Actually (13) is a consequence of the equivalence

$$C=\operatorname{adj} D \quad\Leftrightarrow\quad D=\operatorname{adj} C$$

which holds for all 2×2 matrices $C,D\in\mathbb{R}^{2\times 2}$. □

2. The case of mappings with unbounded distortion

For the purpose of this section we adopt the following variant of De Giorgi’s notion of Γ-convergence. Let $A_j,\ A$ be symmetric matrix fields satisfying:

$$0\leqslant\big\langle A_j(x)\xi,\xi\big\rangle\leqslant K_j(x)|\xi|^2, \tag{14}$$

$$0\leqslant\big\langle A(x)\xi,\xi\big\rangle\leqslant K(x)|\xi|^2, \tag{15}$$

where $K_j, K\in L^1(\Omega)$.

Definition 2.1. We say that A_j Γ-converges to A, ($A_j \xrightarrow{\Gamma} A$), if the following two conditions are verified:

(i) the inequality

$$\int_\Omega \langle A(x)\nabla u, \nabla u\rangle \,\mathrm{d}x \leqslant \liminf_{j\to\infty} \int_\Omega \langle A_j(x)\nabla u_j, \nabla u_j\rangle \,\mathrm{d}x$$

holds whenever $u_j, u \in \mathrm{Lip}(\Omega)$ and $u_j \to u$ in $L^1(\Omega)$.

(ii) For every $v \in \mathrm{Lip}(\Omega)$ there exist a sequence $v_j \in \mathrm{Lip}(\Omega)$ converging to v in $L^1(\Omega)$ such that $v_j - v \in C_0^0(\Omega)$ and

$$\int_\Omega \langle A(x)\nabla v, \nabla v\rangle \,\mathrm{d}x = \lim_{j\to\infty} \int_\Omega \langle A_j(x)\nabla v_j, \nabla v_j\rangle \,\mathrm{d}x.$$

Note that by general properties of Γ-convergence, (2.1)(i) and (2.1)(ii) remain true if we replace Ω by any of its open subsets. It is worth pointing out here that if $\mathcal{A}_j$ satisfy the ellipticity conditions (9) the two definitions of Γ and G convergence agree (see, e.g., [6]).

We report here a compactness result concerning Γ-convergence [6].

Theorem 2.2. *Let A_j be a sequence of symmetric 2×2 matrices satisfying* (14) *with $K_j \rightharpoonup K$ weakly in $L^1(\Omega)$. Then, there exists a subsequence A_{j_r} Γ-converging to a symmetric matrix A satisfying* (15).

Let us now consider a sequence $f_j = (f_j^1, f_j^2) \in W^{1,1}(\Omega, \mathbb{R}^2)$ of non constant mappings of *finite distortion*, i.e., satisfying the *distortion inequality*:

$$\|Df_j(x)\|^2 \leqslant K_j(x) J(x, f_j) \quad \text{a.e. } x \in \Omega \tag{16}$$

under the following assumptions:

$$\text{there exist } \lambda_0 \geqslant 1, c_0 > 0 \text{ such that } \int_\Omega \mathrm{e}^{\lambda_0 K_j(x)} \,\mathrm{d}x \leqslant c_0 \quad \text{for } j = 1, 2, \ldots, \tag{17}$$

$$K_j \rightharpoonup K \quad \text{weakly in } L^1(\Omega), \tag{18}$$

$$f_j \rightharpoonup f = (f^1, f^2) \quad \text{weakly in } W^{1,1}(\Omega, \mathbb{R}^2), \tag{19}$$

$$\text{there exists } c_1 > 0 \text{ such that } \int_\Omega J(x, f_j) \,\mathrm{d}x \leqslant c_1 \quad \text{for } j = 1, 2, \ldots. \tag{20}$$

Note that we do not assume $f_j \in W^{1,2}(\Omega, \mathbb{R}^2)$. Actually (16), (17) and (20) imply, via Hölder inequality, only $Df_j \in L^2 \lg^{-1} L(\Omega)$.

Set now $\mathcal{A}_j(x) = \mathcal{A}(x, Df_j)$ then, by our previous results, $\mathcal{A}_j$ enjoy the ellipticity bounds,

$$\frac{|\xi|^2}{K_j(x)} \leqslant \langle \mathcal{A}_j(x)\xi, \xi\rangle \leqslant K_j(x)|\xi|^2, \tag{21}$$

for almost every $x \in \Omega$ and all $\xi \in \mathbb{R}^2$.

Theorem 2.3. *Under the above assumptions* (16)–(20)*, the limit mapping f is either constant or, if not, has finite distortion $K(x)$ and*

$$\int_\Omega \langle \mathcal{A}_j(x)\nabla v, \nabla v\rangle \,\mathrm{d}x \xrightarrow{\Gamma} \int_\Omega \langle \mathcal{A}(x)\nabla v, \nabla v\rangle \,\mathrm{d}x, \tag{22}$$

where $\mathcal{A}(x) = \mathcal{A}(x, Df)$.

We emphasize that, under our general assumptions (17), (21) of degenerate ellipticity on A_j, no compactness theorem is avaible with respect to G-convergence.

This is why we are invoking Γ-convergence as a tool, since it enjoys the above mentioned compactness property (Theorem 2.2).

We conclude our paper with the following:

Corollary 1. *Under the assumptions of Theorem* 2.3*, we have*: $\mathcal{A}_j = \mathcal{A}(x, Df_j) \xrightarrow{G} \mathcal{A} = \mathcal{A}(x, Df)$ *in the following sense*: *if* $v_j \in W^{1,1}_{\mathrm{loc}}(\Omega_1)$ *are finite energy solutions to the equations*

$$\operatorname{div}\big(\mathcal{A}_j(x)\nabla v_j\big) = 0 \quad \text{in } \Omega_1 \subset \Omega,$$

$(\int_{\Omega_1} \langle A_j(x)\nabla v_j, \nabla v_j\rangle \leqslant c_1,\ j = 1, 2, \ldots)$ *and*

$$\nabla v_j \rightharpoonup \nabla v \quad \text{weakly in } L^1\big(\Omega_1, \mathbb{R}^2\big),$$

then

$$\mathcal{A}_j \nabla v_j \rightharpoonup \mathcal{A}\nabla v \quad \text{weakly in } L^1\big(\Omega_1, \mathbb{R}^2\big)$$

and v *is a finite energy solution in* Ω_1 *to the equation*

$$\operatorname{div}\big(\mathcal{A}(x)\nabla v\big) = 0.$$

The proof of Theorem 2.3 will appear in [9].

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