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Probability Theory

Approximation of the occupation measure of Lévy processes

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Abstract

Consider a real-valued Lévy process with non-zero Gaussian component and jumps with locally finite variation. We obtain an invariance principle theorem for the speed of approximation of its occupation measure by means of functionals defined on regularizations of the paths. This theorem is a first extension to processes with jumps of previous results for semimartingales with continuous paths. ***To cite this article: E. Mordecki, M. Wschebor, C. R. Acad. Sci. Paris, Ser. I 340 (2005).***

Résumé

Approximation de la mesure d'occupation d'un processus de Lévy. On considère un processus de Lévy à valeurs réelles, dont la partie gaussienne ne s'annule pas et dont la variation totale des sauts est localement finie. Nous donnons un théorème central limite pour la vitesse d'approximation de sa mesure d'occupation, moyennant des fonctionnelles définies sur des régularisations des trajectoires. Ce théorème est une première extension à des processus avec sauts, de résultats précédents obtenus pour des semi-martingales à trajectoires continues. ***Pour citer cet article : E. Mordecki, M. Wschebor, C. R. Acad. Sci. Paris, Ser. I 340 (2005).***

Version française abrégée

Soit $X = \{X_t\colon\ t \geqslant 0\}$ un processus de Lévy, défini sur un espace de probabilité $(\Omega, \mathcal{F}, P)$, représenté par $X_t = mt + \sigma W_t + S_t$. $\{W_t\colon\ t \geqslant 0\}$ est un processus de Wiener standard, $m \in \mathbb{R}$, et nous allons supposer que $\sigma > 0$ et que p.s. la partie de sauts purs $\{S_t\colon\ t \geqslant 0\}$ est càdlàg et localement à variation finie. On observe des régularisations des trajectoires de X par convolution de la forme

$$X_t^\varepsilon = (\psi_\varepsilon * X)_t = \int_{\mathbb{R}} \psi_\varepsilon(t-s) X_s \,\mathrm{d}s, \tag{1}$$

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avec $\psi_\varepsilon(t) = (1/\varepsilon)\psi(t/\varepsilon)$ où le noyau ψ est non-négatif, de classe C^1, avec un support contenu dans $[-1, 1]$, et $\int_{\mathbb{R}} \psi(x)\,\mathrm{d}x = 1$.

Le but des deux théorèmes suivants est de donner des approximations de la mesure d'occupation du processus X sur l'intervalle $[0, T]$ à partir de renormalisations de $N_u^{X^\varepsilon}[0, T]$, le nombre de racines de l'equation $X_t^\varepsilon = u$ appartenant à $[0, T]$. Dans le Théorème 0.1 il s'agit de la convergence des distributions fini-dimensionnelles et dans le Théorème 0.2, de convergence faible, après modification de la forme du terme de biais.

Théorème 0.1. *Soit X un processus de Lévy du type précédent. Alors, pour toute fonction $f\colon \mathbb{R} \to \mathbb{R}$ de classe C^2 et dérivée seconde bornée, nous avons*

$$\frac{1}{\sqrt{\varepsilon}}\left[C\int_{\mathbb{R}} f(u)\sqrt{\varepsilon}\,N_u^{X^\varepsilon}[0,t]\,\mathrm{d}u - \sigma\int_0^t f(X_s)\,\mathrm{d}s\right] \xrightarrow{(c)} \delta\int_0^t f(X_s)\,\mathrm{d}B_s + C\sum_{0<s\leqslant t} L(f,s)|\Delta X_s| \tag{2}$$

lorsque $\varepsilon \to 0$, où : $B = \{B_t\colon t \geqslant 0\}$ est un processus de Wiener indépendant de X ; $C = \frac{1}{\|\psi\|}\sqrt{\frac{\pi}{2}}$; la constante $\delta > 0$ est donnée par $\delta^2 = 2\sigma^2\int_0^2 (r(t)\operatorname{Arsin} r(t) + \sqrt{1 - r^2(t)} - 1)\,\mathrm{d}t$, où $r(t)$ est la covariance définie par $r(t) = \frac{1}{\|\psi\|^2}\int_{\mathbb{R}} \psi(t-u)\psi(-u)\,\mathrm{d}u$. La variable aléatoire $L(f, t)$ est donnée par

$$L(f,t) = \int_{-1}^{1} \psi(z) f\left(X_{t-}\int_z^1 \psi(w)\,\mathrm{d}w + X_t\int_{-1}^{z} \psi(w)dw\right)\mathrm{d}z, \tag{3}$$

$\xrightarrow{(c)}$ signifie la convergence faible des distributions fini-dimensionnelles.

Il faut observer que dans l'énoncé du Théorème 0.1, si la partie sauts ne s'anulle pas, on ne peut pas avoir la convergence faible dans l'espace de Skorokhod, sauf pour des fonctions f triviales. En effet, raisonnons par l'absurde, si la convergence a lieu au sens faible, la limite est donneé par le second membre de (2) et est continue. Ce qui génerè une contradiction puisque le terme $\sum_{0<s\leqslant t} L(f,s)\,|\Delta X(s)|$ a des sauts qui en général ne s'annulent pas.

D'autre part, on peut obtenir la convergence faible, en modifiant le terme de biais comme l'indique l'énoncé suivant.

Théorème 0.2. *Soient X, X^ε et f vérifiant les conditions du Théorème 0.1. Alors,*

$$\frac{1}{\sqrt{\varepsilon}}\left[C\int_{\mathbb{R}} f(u)\sqrt{\varepsilon}N_u^{X^\varepsilon}[0,t]\,\mathrm{d}u - \sigma\int_0^t f(X_s)\,\mathrm{d}s\right] - C\int_0^t f(X_s^\varepsilon)|\dot{S}_s^\varepsilon|\,\mathrm{d}s \xrightarrow{\mathcal{L}} \delta\int_0^t f(X_s)\,\mathrm{d}B_s$$

lorsque $\varepsilon \to 0$, où $B = \{B_t\colon t \geqslant 0\}$ est un processus de Wiener indépendant de X, $\dot{S}_s^\varepsilon$ est la dérivée de la fonction $s \rightsquigarrow S_s^\varepsilon$, les constantes C et δ sont les mêmes que celles du Théorème 0.1, et $\xrightarrow{\mathcal{L}}$ note la convergence faible dans l'espace $\mathcal{C} = \mathcal{C}([0, +\infty), \mathbb{R})$ des fonctions continues.

En plus, pour chaque $t \geqslant 0$, presque sûrement on a

$$\int_0^t f(X_s^\varepsilon)|\dot{S}_s^\varepsilon|\,\mathrm{d}s \to \sum_{0<s\leqslant t} L(f,s)|\Delta X_s|$$

lorsque $\varepsilon \to 0$, $L(f, t)$ étant donnée par (3).

1. Introduction

Let $X=\{X_t\colon t\geqslant 0\}$ be a real-valued Lévy process, defined on a probability space $(\Omega,\mathcal{F},P)$, that we represent by $X_t=mt+\sigma W_t+S_t$. Here $W=\{W_t\colon t\geqslant 0\}$ is a standard Wiener process, $S=\{S_t\colon t\geqslant 0\}$ a pure jump process with càdlàg paths, m and σ are real constants, and we assume that the Gaussian part does not vanish, i.e. $\sigma>0$.

Furthermore, assume **(FV)**: the jump part of the process has locally finite variation, i.e. $\int(1\wedge|y|)\Pi(\mathrm{d}y)<\infty$ where Π is the Lévy–Khinchine measure of the process, so that for each positive t, $\sum_{0<s\leqslant t}|\Delta S_s|$ is almost surely finite. For general references on Lévy processes see Skorokhod [15], Bertoin [3] or Sato [16].

We now describe the regularization of the trajectories, that, in our context, can be interpreted as a partial observation of the process through a physical device. Let $\psi\colon\mathbb{R}\to\mathbb{R}^+$ be a C^1 function with compact support, say $\operatorname{supp}(\psi)\subset[-1,1]$, such that $\int_{-1}^{1}\psi(t)\,\mathrm{d}t=1$ and, for $\varepsilon>0$, define the approximation of unity $\psi_\varepsilon(t)=(1/\varepsilon)\psi(t/\varepsilon)$. We denote by $\|\psi\|=(\int_{-1}^{1}\psi^2(t)\,\mathrm{d}t)^{1/2}$ the norm of ψ in $L^2(\mathbb{R},\mathrm{d}t)$. The regularization $X^\varepsilon=\{X_t^\varepsilon\colon t\geqslant 0\}$ of the process is obtained by convolution with ψ_ε in the following way:

$$X_t^\varepsilon=(\psi_\varepsilon*X)_t=\int_{\mathbb{R}}\psi_\varepsilon(t-s)X_s\,\mathrm{d}s=\int_{-1}^{1}\psi(-w)X_{t+w\varepsilon}\,\mathrm{d}w,\tag{4}$$

where we set $X_s=W_s=S_s=0$ if $s<0$. In the same way, we define $W^\varepsilon=\{W_t^\varepsilon\colon t\geqslant 0\}$ and $S^\varepsilon=\{S_t^\varepsilon\colon t\geqslant 0\}$, and obtain that $X_t^\varepsilon=mt-\varepsilon m\alpha+\sigma W_t^\varepsilon+S_t^\varepsilon$ where $\alpha=\int_{\mathbb{R}}w\psi(w)\,\mathrm{d}w$.

Observe that the regularized process inherits the regularity properties of ψ, so that X^ε has C^1 paths. Its time-derivative is denoted with a dot: $\dot{X}_t^\varepsilon=\frac{\mathrm{d}}{\mathrm{d}t}(X^\varepsilon)_t$.

If $F\colon\mathbb{R}^+\to\mathbb{R}$ is a C^1 function, we denote the *number of crossings* of the level u by the function F on an interval $I=[s,t]$, by $N_u^F[s,t]=\operatorname{card}\{r\colon F_r=u, r\in I\}$, that is, the number of roots of the equation $F_t=u$ belonging to I. For a given continuous function $f\colon\mathbb{R}\to\mathbb{R}$, we use the fact that

$$\int_{-\infty}^{\infty}f(u)N_u^F[0,T]\,\mathrm{d}u=\int_0^T f(F_t)|\dot{F}_t|\,\mathrm{d}t.\tag{5}$$

This formula is immediate for $F(t)=at+b$ and extends easily to piecewise linear functions. For C^1 functions F, formula (5) follows using approximations by piecewise linear functions.

2. Main results

The aim of the results below is to approximate the occupation measure of the process X on the interval $[0,T]$ by a re-normalization of the number of crossings of the process $X^\varepsilon=\{X_t^\varepsilon\colon t\geqslant 0\}$ with horizontal levels, on the same time interval. Theorem 2.1 is about weak convergence of finite dimensional distributions. In order to obtain a stronger result including tightness, one has to modify the bias term, as it is done in Theorem 2.2.

Theorem 2.1. *Consider X and X^ε as in Section 1. Then, for each C^2-function $f\colon\mathbb{R}\to\mathbb{R}$ with bounded second derivative, we have*

$$\frac{1}{\sqrt{\varepsilon}}\left[C\int_{\mathbb{R}}f(u)\sqrt{\varepsilon}\,N_u^{X^\varepsilon}[0,t]\,\mathrm{d}u-\sigma\int_0^t f(X_s)\,\mathrm{d}s\right]\xrightarrow{(c)}\delta\int_0^t f(X_s)\,\mathrm{d}B_s+C\sum_{0<s\leqslant t}L(f,s)|\Delta X_s|\tag{6}$$

as $\varepsilon\to 0$, where: $B=\{B_t\colon t\geqslant 0\}$ is a Wiener process independent of X; $C=\frac{1}{\|\psi\|}\sqrt{\frac{\pi}{2}}$. The second constant $\delta>0$ satisfies

$$\delta^2 = 2\sigma^2 \int_0^2 \big(r(t)\,\mathrm{Arsin}\, r(t) + \sqrt{1-r^2(t)} - 1\big)\,\mathrm{d}t,$$

where $r(t)$ is a covariance function defined by $r(t) = \frac{1}{\|\psi\|^2}\int_{\mathbb{R}} \psi(t-u)\psi(-u)\,\mathrm{d}u$. The random variable $L(f,t)$ in (6) *is given by*

$$L(f,t) = \int_{-1}^{1} \psi(z) f\left(X_{t-}\int_z^1 \psi(w)\,\mathrm{d}w + X_t \int_{-1}^{z} \psi(w)\,\mathrm{d}w\right)\mathrm{d}z. \tag{7}$$

$\xrightarrow{(c)}$ *denotes weak convergence of finite dimensional distributions, i.e. convergence of cylindrical projections.*

Observe that one cannot expect weak convergence in the Skorokhod space in the statement of Theorem 2.1 if the jump part of the process does not vanish, excepting for trivial functions f. If this convergence would hold, the limit should be the right-hand member of (6) and at the same time should be continuos. However, if the jump part of X does not vanish, for a given f the discontinuous term $\sum_{0<s\leqslant t} L(f,s)|\Delta X(s)|$ in the right-hand member of (6) has non-vanishing jumps with positive probability. This generates a contradiction. On the other hand, one can obtain weak convergence replacing the bias term by a certain continuous approximation of this discontinuous bias, as stated in the following result.

Theorem 2.2. *Consider X, X^ε and f as in Theorem* 2.1. *Then*

$$\frac{1}{\sqrt{\varepsilon}}\left[C\int_{\mathbb{R}} f(u)\sqrt{\varepsilon}\,N_u^{X^\varepsilon}[0,t]\,\mathrm{d}u - \sigma\int_0^t f(X_s)\,\mathrm{d}s\right] - C\int_0^t f(X_s^\varepsilon)|\dot{S}_s^\varepsilon|\,\mathrm{d}s \xrightarrow{\mathcal{L}} \delta\int_0^t f(X_s)\,\mathrm{d}B_s \tag{8}$$

as $\varepsilon \to 0$, where $B = \{B_t\colon t \geqslant 0\}$ is a Wiener process independent of X, the constants C and δ are as in Theorem 2.1 *and $\xrightarrow{\mathcal{L}}$ denotes weak convergence in the space $\mathcal{C} = \mathcal{C}([0,+\infty),\mathbb{R})$ of continous functions.*

Furthermore, for each $t \geqslant 0$,

$$\int_0^t f(X_s^\varepsilon)|\dot{S}_s^\varepsilon|\,\mathrm{d}s \to \sum_{0<s\leqslant t} L(f,s)|\Delta X_s| \quad a.s. \tag{9}$$

as $\varepsilon \to 0$, where $L(f,t)$ is given in (7).

Remark 1. A simple consequence of Theorem 2.1 is that for each $t > 0$, one has

$$C\int_{\mathbb{R}} f(u)\sqrt{\varepsilon}\,N_u^{X^\varepsilon}[0,t]\,\mathrm{d}u \to \sigma\int_0^t f(X_s)\,\mathrm{d}s \quad \text{in probability} \tag{10}$$

as $\varepsilon \to 0$. This result can be used to estimate σ from the observation of the smoothed path X^ε. Results of type (10) are well-known for semimartingales having continuous paths (Azaïs and Wschebor, [2]) and also other classes of processes (Azaïs and Wschebor, [1]), where almost sure convergence is proved.

Remark 2. Theorem 2.1 contains the speed of convergence in (10). This allows to make inference on σ from the observation of X^ε. Analogous results for processes with continuous paths are in Berzin and León [4] for Brownian motion and in Perera and Wschebor [13,14] for certain classes of continuous semimartingales having Itô-integrals as martingale part. There exist also some related results for Brownian motion and general diffusions, where the approximation X^ε of the actual path X is replaced by polygonal approximation and the smooth function f by a Dirac-delta function, or considering functionals defined on random walks. See for example, Dacunha-Castelle and Florens-Zmirou [6], Florens-Zmirou [7], Génon-Catalot and Jacod [8], Borodin and Ibragimov [5], and Jacod [9,10]. In this context, if ε is the size of the discretization in time, then the speed of convergence turns out to be of the order $\varepsilon^{1/4}$.

3. Brief sketch of the proofs

A detailed proof of Theorem 2.2 is given in Mordecki and Wschebor [12]. It is also proved that Theorem 2.1 is a consequence of Theorem 2.2. Let us outline the proof of Theorem 2.2. In view of (5) we first observe:

$$\int_{\mathbb{R}} f(u) N_u^{X^\varepsilon}[0,t]\,\mathrm{d}u = \int_0^t f(X_s^\varepsilon)|\dot{X}_s^\varepsilon|\,\mathrm{d}s, \quad \text{a.s.}$$

Let us write the left-hand side of (8) as the sum of three terms:

$$\begin{aligned} &C\int_0^t f(X_s^\varepsilon)|\dot{X}_s^\varepsilon|\,\mathrm{d}s - \frac{\sigma}{\sqrt{\varepsilon}}\int_0^t f(X_s)\,\mathrm{d}s - C\int_0^t f(X_s^\varepsilon)|\dot{S}_s^\varepsilon|\,\mathrm{d}s \\ &\quad = C\int_0^t f(X_s^\varepsilon)\big(|\dot{X}_s^\varepsilon| - |\sigma\dot{W}_s^\varepsilon + m| - |\dot{S}_s^\varepsilon|\big)\,\mathrm{d}s + C\int_0^t f(X_s^\varepsilon)|\sigma\dot{W}_s^\varepsilon + m|\,\mathrm{d}s \\ &\qquad - C\int_0^t f(X_s)|\sigma\dot{W}_{s-\varepsilon}^\varepsilon + m|\,\mathrm{d}s + \frac{1}{\sqrt{\varepsilon}}\int_0^t \big(C\sqrt{\varepsilon}|\sigma\dot{W}_{s-\varepsilon}^\varepsilon + m| - \sigma\big) f(X_s)\,\mathrm{d}s. \end{aligned}$$

For $a > 0$ consider the processes $S_a = \{S_{a,t}\colon t \geqslant 0\}$ and $X_a = \{X_{a,t}\colon t \geqslant 0\}$ defined by

$$S_{a,t} = \sum_{0<s\leqslant t} \Delta X_s \mathbf{1}_{\{|\Delta X_s| \geqslant a\}},$$

that is the (a.s. finite) sum of jumps of the process greater or equal than a, and $X_{a,t} = mt + \sigma W_t + S_{a,t}$, respectively. Given an arbitrary $\delta \in (0,1)$ there exists $a > 0$ such that there is no jump with absolute value greater than a, with probability greater that $1-\delta$. The given Lévy process can be written as $X_t = (X_t - S_{a,t}) + S_{a,t}$, where the random processes $S_a = \{S_{a,t}\colon t \geqslant 0\}$ and $\{X_t - S_{a,t}\colon t \geqslant 0\}$ are independent. A standard argument shows that it is enough to prove the result for the process $\{X_t - S_{a,t}\colon 0 \leqslant t \leqslant T\}$, so that we may assume that the support of Π is contained in the interval $[-a,a]$. Under this additional hypothesis, it is easy to see that for each $t \geqslant 0$ the random variable X_t has finite moments of all orders.

In what follows, the parameter of the various processes we will consider vary in a fixed interval $[0,T]$.

The proof is divided into three steps:

$$Z_t^{1,\varepsilon} = \int_0^t f(X_s^\varepsilon)\big(|\dot{X}_s^\varepsilon| - |\sigma\dot{W}_s^\varepsilon + m| - |\dot{S}_s^\varepsilon|\big)\,\mathrm{d}s \xrightarrow{\mathcal{L}} 0. \tag{11}$$

$$Z_t^{2,\varepsilon} = \int_0^t f(X_s^\varepsilon)|\sigma\dot{W}_s^\varepsilon + m|\,\mathrm{d}s - \int_0^t f(X_s)|\sigma\dot{W}_{s-\varepsilon}^\varepsilon + m|\,\mathrm{d}s \xrightarrow{\mathcal{L}} 0 \tag{12}$$

$$Z_t^{3,\varepsilon} = \frac{1}{\sqrt{\varepsilon}}\int_0^t \big(C\sqrt{\varepsilon}|\sigma\dot{W}_{s-\varepsilon}^\varepsilon + m| - \sigma\big) f(X_s)\,\mathrm{d}s \xrightarrow{\mathcal{L}} \delta\int_0^t f(X_t)\,\mathrm{d}B_s. \tag{13}$$

In the proof of (11), one can separate the behaviour of the small jumps, which is easy to bound and consider processes X with jumps greatear or equal than $a > 0$. In the latter case, the number of jumps N_T in the bounded interval $[0, T]$ is finite, and the main point in the computation is that if they occur at the points $\tau_1, \ldots, \tau_N$ the derivative $\dot{S}_t^\varepsilon$ vanishes for t outside $\bigcup_{n=1}^N (\tau_n - \varepsilon, \tau_n + \varepsilon)$.

For (12), one can first replace $Z_t^{2,\varepsilon}$ by $\widehat{Z}_t^{2,\varepsilon} = \int_0^t [f(X_s^\varepsilon) - f(X_{s+\varepsilon})]|\sigma \dot{W}_s^\varepsilon + m| \, \mathrm{d}s$. $\widehat{Z}_t^{2,\varepsilon} \xrightarrow{\mathcal{L}} 0$ follows from the moment estimations:

$$E\big[(\widehat{Z}_t^{2,\varepsilon} - \widehat{Z}_s^{2,\varepsilon})^2\big] \leqslant (const)(t-s)^2 \quad \text{for } s, t \in [0, T],\ 0 < \varepsilon \leqslant 1$$

and $E[(\widehat{Z}_t^{2,\varepsilon})^2] \leqslant (const)\varepsilon$ where in both inequalities $(const)$ denotes a constant that may depend only on T.

For (13), we put

$$Y_t^\varepsilon = \frac{1}{\sqrt{\varepsilon}} \int_0^t \big(C\sqrt{\varepsilon}|\sigma \dot{W}_{s-\varepsilon}^\varepsilon + m| - \sigma\big) \, \mathrm{d}s,\ t \geqslant 0$$

and prove that $(Y^\varepsilon, W) \xrightarrow{\mathcal{L}} (\delta B, W)$ as $\varepsilon \to 0$. Tightness follows using fourth moments and the value of δ using second moments. In order to complete the proof of (13) one can use a theorem of convergence of stochastic integrals, as in Kurtz and Protter [11], Remark 2.5.

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References

[1] J.-M. Azaïs, M. Wschebor, Almost sure oscillation of certain random processes, Bernoulli 2 (3) (1996) 257–270.
[2] J.-M. Azaïs, M. Wschebor, Oscillation presque sûre de martingales continues, in: Séminaire de Probabilités XXXI, in: Lecture Notes in Math., vol. 1655, Springer, 1997, pp. 69–76.
[3] J. Bertoin, Lévy Processes, Cambridge University Press, Cambridge, 1966.
[4] C. Berzin, J.R. León, Weak convergence of the integrated number of level crossings to the local time of the Wiener process, C. R. Acad. Sci. Paris, Sér. I 319 (1994) 1311–1316.
[5] A.N. Borodin, I.A. Ibragimov, Limit Theorems for Functionals of Random Walks, Proc. Steklov Instit. Math., vol. 195, Nauka, Sankt-Peterburg, 1994 (in Russian).
[6] D. Dacunha-Castelle, D. Florens-Zmirou, Estimation of the coefficient of a diffusion from discrete observations, Stochastics 19 (1986) 263–284.
[7] D. Florens-Zmirou, On estimating the difussion coefficient from discrete observations, J. Appl. Probab. 30 (1993) 790–804.
[8] V. Génon-Catalot, J. Jacod, On the estimation of the diffusion coefficient for multidimensional diffusion processes, Ann. Inst. H. Poincaré Probab. Statist. 29 (1993) 119–151.
[9] J. Jacod, Rates of convergence to the local time of a diffusion, Ann. Inst. H. Poincaré Probab. Statist. 34 (1998) 505–544.
[10] J. Jacod, Non-parametric kernel estimation of the diffusion coefficient of a diffusion, Scand. J. Statist. 27 (1) (2000) 83–96.
[11] T.G. Kurtz, P. Protter, Weak limit theorems for stochastic integrals and stochastic differential equations, Ann. Probab. 19 (3) (1991) 1035–1070.
[12] E. Mordecki, M. Wschebor, Smoothing the paths and weak approximation of the occupation measure for Lévy processes, Pre-Mat 2003/74, Publ. Mat. Uruguay, in press.
[13] G. Perera, M. Wschebor, Crossings and occupation measures for a class of semimartingales, Ann. Probab. 26 (1) (1998) 253–266.
[14] G. Perera, M. Wschebor, Inference on the variance and smoothing of the paths of diffusions, Ann. Inst. H. Poincaré Probab. Statist. 38 (6) (2002) 1009–1022.
[15] A.V. Skorokhod, Random Processes with Independent Increments, Kluwer Academic, Dordrecht, 1991.
[16] K.-I. Sato, Lévy Processes and Infinitely Divisible Distributions, Cambridge Stud. Adv. Math., vol. 68, 1999.