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Optimal Control/Probability Theory

A Kalman-type condition for stochastic approximate controllability

Dan Goreac

Laboratoire de mathématiques, Unité CNRS UMR 6205, Université de Bretagne Occidentale, 6, avenue Victor-LeGorgeu, B.P. 809, 29285 Brest cedex, France

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Abstract

We are interested in the approximate controllability property for a linear stochastic differential equation. For deterministic control necessary and sufficient criterion exists and is called Kalman condition. In the stochastic framework criteria are already known either when the control fully acts on the noise coefficient or when there is no control acting on the noise. We propose a generalization of Kalman condition for the general case. ***To cite this article: D. Goreac, C. R. Acad. Sci. Paris, Ser. I 346 (2008).***

Résumé

Une condition de type Kalman pour la contrôlabilité stochastique approchée. On s'intéresse à la propriété de contrôlabilité approchée pour une équation différentielle stochastique linéaire. Pour le contrôle déterministe, il existe une condition nécessaire et suffisante appelée condition de Kalman. Pour le cas stochastique, des critères sont connus soit dans le cas où le contrôle agit pleinement sur le bruit, soit dans le cas où il n'y a aucun contrôle sur le bruit. Nous proposons une généralisation de la condition de Kalman pour le cas général. ***Pour citer cet article : D. Goreac, C. R. Acad. Sci. Paris, Ser. I 346 (2008).***

Version française abrégée

Nous étudions la contrôlabilité approchée pour l'équation différentielle stochastique linéaire

$$\begin{cases} \mathrm{d}y(t) = (Ay(t) + Bu(t))\,\mathrm{d}t + (Cy(t) + Du(t))\,\mathrm{d}W(t), \quad 0 \leqslant t \leqslant T, \\ y(0) = x \in \mathbb{R}^n, \end{cases} \tag{1}$$

gouvernée par un processus de contrôle $u(\cdot)$, où $A, C \in \mathcal{L}(\mathbb{R}^n, \mathbb{R}^n)$, et $B, D \in \mathcal{L}(\mathbb{R}^d, \mathbb{R}^n)$.

Pour le cas où le coefficient du processus de contrôle D est de rang plein, l'auteur de [7] (resp. [5] pour l'horizon infini) a montré que la propriété de contrôlabilité exacte pour (1) peut être caractérisée à l'aide de conditions algébriques de type Kalman. Si la matrice D n'est pas de rang plein, l'Éq. (1) n'est pas exactement contrôlable. Dans ce cas on étudie la contrôlabilité approchée. Cette propriété a été étudiée dans [4] pour le cas spécial $D = 0$. Les auteurs

E-mail address: Dan.Goreac@univ-brest.fr.

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généralisent la condition de Kalman pour obtenir un critère équivalent pour la contrôlabilité approchée de (1) à l'aide de la notion d'invariance stricte (cf. [9]).

Dans cette Note, nous proposons une extension de ces résultats au cas général où le contrôle peut agir également sur le bruit (i.e. rang $D \geqslant 0$) sans forcément avoir D de rang plein. Pour cela, il suffit de réduire l'étude à l'équation suivante, qui est équivalente à (1)

$$\mathrm{d}y(t) = \big(Ay(t) + B_1 u'(t) + B_2 u''(t)\big)\,\mathrm{d}t + \big(Cy(t) + D_1 u'(t)\big)\,\mathrm{d}W(t), \tag{2}$$

où $A, C \in \mathcal{L}(\mathbb{R}^n, \mathbb{R}^n)$, $B_1, D_1 \in \mathcal{L}(\mathbb{R}^r, \mathbb{R}^n)$, $B_2 \in \mathcal{L}(\mathbb{R}^{d-r}, \mathbb{R}^n)$ et $\operatorname{rang} D_1 = \operatorname{rang} D = r$. En utilisant $\operatorname{rang} D_1 = r$ on peut trouver $F \in \mathcal{L}(\mathbb{R}^n, \mathbb{R}^n)$ solution de $D_1^* F + B_1^* = 0$.

On rappelle la définition de la contrôlabilité approchée et de la 0-contrôlabilité approchée :

Définition 0.1. L'équation (1) a la propriété de contrôlabilité approchée si, pour tout $x \in \mathbb{R}^n$, tout $T > 0$, tout $\eta \in L^2(\Omega; \mathcal{F}_T; P; \mathbb{R}^n)$, tout $\varepsilon > 0$, il existe un contrôle admissible u tel que

$$E\big[\big|y(T, x, u) - \eta\big|^2\big] \leqslant \varepsilon.$$

L'équation (1) a la propriété de 0-contrôlabilité approchée si la condition ci-dessus a lieu pour $\eta = 0$.

Nous utilisons la définition suivante :

Définition 0.2. Etant donnés les opérateurs linéaires $L, M, N \in \mathcal{L}(\mathbb{R}^n, \mathbb{R}^n)$ et les sous-espaces linéaires $V \subset \mathbb{R}^n$ et $U \subset \mathbb{R}^n$, on dit que V est $(L; M)$-strictement invariant conditionnellement à (N, U) si pour tout $v \in V$ il existe $w \in V$ tel que

$$w - Nv \in U \text{ et } Lv + Mw \in V.$$

Le résultat principal de cette Note est :

Théorème 0.3. *Les trois assertions suivantes sont équivalentes* :

1. *L'équation* (2) *a la propriété de contrôlabilité approchée.*
2. *L'équation* (2) *a la propriété de* 0*-contrôlabilité approchée.*
3. *Le plus grand sous-espace linéaire de* $\operatorname{Ker} B_2^*$ *qui est* $(A^*; C^*)$*-strictement invariant conditionnellement à* $(F, \operatorname{Ker} D_1^*)$ *est le sous-espace* $\{0\}$.

Remarque 1. Étant donnés les sous-espaces linéaires $V, U \subset \mathbb{R}^n$, le plus grand sous-espace de V qui est $(L; M)$-strictement invariant conditionnellement à (N, U) peut être obtenu en considérant le schéma itératif suivant :

$$V_0 = V; \quad V_{i+1} = \big\{v \in V_i\colon\ M\big((U + Nv) \cap V_i\big) \cap (V_i - Lv) \neq \phi\big\}, \quad i \in \mathbb{N}.$$

Il en découle que la condition 3. du résultat principal est calculable.

1. Introduction

Given $(W(t), t \geqslant 0)$ a standard Brownian motion on the complete probability space $(\Omega, \mathcal{F}, P)$, we consider the natural complete filtration $(\mathcal{F}_t)_{t\geqslant 0}$ generated by W. We let $\mathcal{A} = \mathcal{A}(\Omega, \mathcal{F}, P; W)$ be the set of all $(\mathcal{F}_t)$-progressively measurable $\mathbb{R}^d$-valued processes $v(\cdot)$ such that $E[\int_0^T |v(s)|^2\,\mathrm{d}s] < \infty$ for all $T > 0$. A process $v(\cdot) \in \mathcal{A}$ is called an admissible control process.

We consider the following linear stochastic differential equation

$$\begin{cases} \mathrm{d}y(t) = (Ay(t) + Bu(t))\,\mathrm{d}t + (Cy(t) + Du(t))\,\mathrm{d}W(t), & 0 \leqslant t \leqslant T, \\ y(0) = x \in \mathbb{R}^n, \end{cases} \tag{1}$$

governed by the control process $u(\cdot) \in \mathcal{A}$, where $A, C \in \mathcal{L}(\mathbb{R}^n, \mathbb{R}^n)$, and $B, D \in \mathcal{L}(\mathbb{R}^d, \mathbb{R}^n)$.

In the case when the control coefficient D is of full rank, it has been shown in [7] (resp. [5] for the infinite horizon setting) that exact controllability for (1) can be characterized by some algebraic conditions of Kalman type. If the matrix D is not of full rank, one has to study approximate controllability. This has been done in [4] for the special case $D = 0$. The authors generalize the Kalman condition to some equivalent criterion for approximate controllability of (1). In this Note we give an extension of the above results to the general case where the control is allowed to act on the noise (i.e. rank $D \geqslant 0$) without necessarily having D of full rank.

Let us now recall the notions of approximate controllability and null approximate controllability:

Definition 1. We say that Eq. (1) is approximately controllable if, for all $x \in \mathbb{R}^n$, all $T > 0$, all $\eta \in L^2(\Omega; \mathcal{F}_T; P; \mathbb{R}^n)$, and all $\varepsilon > 0$, there exists an admissible control u such that

$$E\big[\big|y(T,x,u) - \eta\big|^2\big] \leqslant \varepsilon.$$

Moreover, we say that Eq. (1) is approximately null controllable if the above condition holds for the particular case $\eta = 0$. For further results on stochastic control, the reader is referred to [1–3] or [8].

2. Main result

In order to study approximate controllability for (1), we notice that (1) is equivalent to he following

$$\mathrm{d}y(t) = \big(Ay(t) + B_1u'(t) + B_2u''(t)\big)\,\mathrm{d}t + \big(Cy(t) + D_1u'(t)\big)\,\mathrm{d}W(t), \tag{2}$$

where $A, C \in \mathcal{L}(\mathbb{R}^n, \mathbb{R}^n)$, $B_1, D_1 \in \mathcal{L}(\mathbb{R}^r, \mathbb{R}^n)$, $B_2 \in \mathcal{L}(\mathbb{R}^{d-r}, \mathbb{R}^n)$ and rank $D_1 = \operatorname{rank} D = r$. Since rank $D_1 = r$ we establish the existence of some $F \in \mathcal{L}(\mathbb{R}^n, \mathbb{R}^n)$ solution of $D_1^*F + B_1^* = 0$.

We introduce the following:

Definition 2. Given the linear operators $L, M, N \in \mathcal{L}(\mathbb{R}^n, \mathbb{R}^n)$ and two linear subspaces $V \subset \mathbb{R}^n$ and $U \subset \mathbb{R}^n$, we say that V is $(L; M)$-strictly invariant conditioned to (N, U) if for all $v \in V$ there exists $w \in V$ such that

$$w - Nv \in U \quad \text{and} \quad Lv + Mw \in V.$$

The main result of our Note is:

Theorem 3. *The following statements are equivalent*:

1. *Eq.* (2) *enjoys the approximate controllability property.*
2. *Eq.* (2) *enjoys the approximate null-controllability property.*
3. *The largest linear subspace of* Ker B_2^* *which is* $(A^*; C^*)$*-strictly invariant conditioned to* $(F, \operatorname{Ker} D_1^*)$ *is the trivial space* $\{0\}$.

Remark 4. For arbitrary linear subspaces $V, U \subset \mathbb{R}^n$, the largest subspace of V which is $(L; M)$-strictly invariant conditioned to (N, U) can be obtained in at most n iterations by considering the following schema

$$V_0 = V; \quad V_{i+1} = \big\{v \in V_i\colon\ M\big((U + Nv) \cap V_i\big) \cap (V_i - Lv) \neq \phi\big\}, \quad i \in \mathbb{N}.$$

This implies that condition 3. in our main result is computable.

3. Sketch of the proof of main result

To prove our main result, we first establish a connection between approximate controllability for (2) and some observability condition on the dual equation

$$\begin{cases} \mathrm{d}p(t) = [-(A^* + C^*F)p(t) - C^*q(t)]\,\mathrm{d}t + (Fp(t) + q(t))\,\mathrm{d}W(t), \\ p(T) = \eta. \end{cases} \tag{3}$$

The existence and uniqueness results for the solution of (3) are classic (cf. Pardoux, Peng [6]). We prove

Proposition 5. *Eq.* (2) *is approximately-controllable if and only if, for all* $T > 0$, *every solution of* (3) *such that* $B_2^* p(s) = 0$ *and* $D_1^* q(s) = 0$, *P-a.s., for all* $s \in [0, T]$ *is trivially reduced to* 0.

Moreover, Eq. (2) *is approximately null controllable if and only if, for all* $T > 0$, *every solution of* (3) *such that* $B_2^* p(s) = 0$ *and* $D_1^* q(s) = 0$, *P-a.s., for all* $s \in [0, T]$ *satisfies* $p(0) = 0$.

Proof. Let us fix $T \geqslant 0$. We denote by $L^2_{\mathcal{P}}([0, T], \mathbb{R}^d)$ the space of all predictable processes $u : \Omega \times [0, \infty[\to \mathbb{R}^d$ satisfying $E[\int_0^T |u(s)|^2 \, \mathrm{d}s] < \infty$ and consider the linear operator M_T given by

$$M_T : L^2_{\mathcal{P}}\big([0, T], \mathbb{R}^d\big) \longrightarrow L^2\big(\Omega, \mathcal{F}_T, P; \mathbb{R}^n\big), \quad M_T u = y(T, 0, u). \tag{4}$$

It is straightforward that (2) is approximately-controllable if and only if, for all $T > 0$, $M_T(L^2_{\mathcal{P}}([0, T], \mathbb{R}^d))$ is dense in $L^2(\Omega, \mathcal{F}_T, P; \mathbb{R}^n)$. Itô's formula applied to $\langle p(T), y(T)\rangle$ implies $M_T^* \eta = \binom{D_1^* q}{B_2^* p}$. We use the fact that the image of a linear operator is dense if and only if the kernel of its adjoint is trivial and the uniqueness and the continuity of the solution of (3) to get $\eta = 0$ if and only if $p(s) = 0$, P-a.s., for all $s \in [0, T]$ and $q(s) = 0 \, \mathrm{d}s \, \mathrm{d}P$-almost everywhere on $[0, T] \times \Omega$.

In order to prove the second assertion in our statement, we introduce the linear operator

$$L_T : \mathbb{R}^n \longrightarrow L^2\big(\Omega, \mathcal{F}_T, P; \mathbb{R}^n\big), \quad L_T x = y(T, x, 0). \tag{5}$$

It is obvious that approximate null controllability is equivalent to the fact that for all $T > 0$, $L_T[\mathbb{R}^n] \subset \overline{M_T[L^2_{\mathcal{P}}([0, T], \mathbb{R}^d)]}$ (or $\mathrm{Ker}(M_T^*) \subset \mathrm{Ker}(L_T^*)$). To conclude the proof of this second part, we use Itô's formula applied to $\langle p(T), y(T)\rangle$ for $u = 0$ and obtain $L_T^* \eta = p(0)$. The conclusion follows. □

At this point, we interpret the dual equation (3) as a controlled forward equation

$$\begin{cases} \mathrm{d}p(t, q, \theta) = [-(A^* + C^* F) p(t, q, \theta) - C^* q(t)] \, \mathrm{d}t + (F p(t, q, \theta) + q(t)) \, \mathrm{d}W(t); \\ p(0, q, \theta) = \theta \in \mathbb{R}^n. \end{cases} \tag{6}$$

Thus, Proposition 5 can be restated as:

Proposition 6. *Eq.* (2) *is approximately-controllable if and only if, for all* $T > 0$, *all* $\theta \in \mathbb{R}^n$, *and all* $q \in L^2_{\mathcal{P}}([0, T], \mathrm{Ker}\, D_1^*)$ *such that* $B_2^* p(s, q, \theta) = 0$, *P-a.s., for all* $s \in [0, T]$, *it holds* $q(s) = 0$, $\mathrm{d}s \, \mathrm{d}P$*-almost everywhere on* $[0, T] \times \Omega$ *and* $\theta = 0$.

Eq. (2) *is approximately null controllable if and only if, for all* $T > 0$, *all* $\theta \in \mathbb{R}^n$ *and all* $q \in L^2_{\mathcal{P}}([0, T], \mathrm{Ker}\, D_1^*)$ *such that* $B_2^* p(s, q, \theta) = 0$, *P-a.s., for all* $s \in [0, T]$, *it holds* $\theta = 0$.

This justifies our interest in the following notion:

Definition 7. Let $U, V \subset \mathbb{R}^n$ be two linear subspaces of $\mathbb{R}^n$. The family of all $\theta \in V$ for which there exists a $T > 0$ and $q \in L^2_{\mathcal{P}}([0, T], U)$ such that $p(s, q, \theta) \in V$, P-a.s., for all $s \in [0, T]$ is called the viability kernel of V conditioned to U with respect to (6) (we denote this set by $\mathrm{Viab}(V/U)$). Moreover, we say that V is local in time viable conditioned to U with respect to (6) if $\mathrm{Viab}(V/U) = V$.

In order to give a description of the conditional viability kernel, we introduce the following Riccati equation

$$\begin{cases} P_N'(s) = -P_N(s)(A^* + C^* F) - (A + F^* C) P_N(s) + F^* P_N(s) F \\ \qquad - (F^* P_N(s) - P_N(s) C^*)(I + N \Pi_{U^\perp} + P_N(s))^{-1}(P_N(s) F - C P_N(s)) + N \Pi_{V^\perp}, \\ P_N(0) = 0. \end{cases} \tag{7}$$

Proposition 8. *The viability kernel of V conditioned to U with respect to* (6) *has the following representation*:

$$\mathrm{Viab}(V|U) = \Big\{\theta \in V\colon \exists T > 0 \text{ s.t. } \lim_{N \to \infty} \big\langle P_N(T)\theta, \theta\big\rangle < \infty\Big\}.$$

Proof. Let us consider $\theta \in \mathrm{Viab}(V|U)$. Then, there exist $T > 0$ and $q \in L^2_{\mathcal{P}}([0,T],U)$ such that $p(s,q,\theta) \in V$, P-a.s., for all $s \in [0,T]$ and Itô's formula yields $E[\langle P_N(T)\theta,\theta\rangle] \leqslant E\int_0^T |q(s)|^2\,\mathrm{d}s$.

For the converse, we consider, for all N, the following linear quadratic optimal control problem

$$\inf_{q \in L^2_{\mathcal{P}}([0,T];\mathbb{R}^{n\times d})} \left\{ E\left[\int_0^T N\langle \Pi_{V^\perp} p(t,q,\theta), p(t,q,\theta)\rangle\,\mathrm{d}t\right] + E\left[\int_0^T \langle (N\Pi_{V^\perp}+I)q(t), q(t)\rangle\,\mathrm{d}t\right]\right\}. \tag{SLQ}$$

We may choose the optimal control process $\bar{q}_N \in L^2_{\mathcal{P}}([0,T],\mathbb{R}^n)$ (see Yong and Zhou [10], Th. 6.6.1) and we have

$$K > E[\langle P_N(T)\theta,\theta\rangle] = E\left[\int_0^T \left(N\left(\left|\Pi_{V^\perp} p(s,\bar{q}_N,\theta)\right|^2 + \left|\Pi_{U^\perp}\bar{q}_N(s)\right|^2\right) + \left|\bar{q}_N(s)\right|^2\right)\mathrm{d}s\right], \tag{8}$$

for some constant K independent of N. Standard estimates show that $(p(\cdot,\bar{q}_N,\theta),\bar{q}_N)_N$ is bounded in $L^2_{\mathcal{P}}([0,T];\mathbb{R}^n) \times L^2_{\mathcal{P}}([0,T];\mathbb{R}^{n\times d})$. Therefore, there exists a subsequence (still denoted by $(p(\cdot,\bar{q}_N,\theta),\bar{q}_N)_N$ which converges in the weak topology to some $(\bar{p},\bar{q})$. We can identify, in $L^2_{\mathcal{P}}([0,T];\mathbb{R}^n)$, $\bar{p}(\cdot) = p(\cdot,\bar{q},\theta)$. The inequality (8) yields

$$\lim_{N\to\infty} E\left[\int_0^T \left|\Pi_{V^\perp} p(s,\bar{q}_N,\theta)\right|^2\mathrm{d}s\right] = 0, \quad \text{and} \quad \lim_{N\to\infty} E\left[\int_0^T \left|\Pi_{U^\perp}\bar{q}_N(s)\right|^2\mathrm{d}s\right] = 0.$$

Using the fact that the $L^2_{\mathcal{P}}([0,T];\mathbb{R}^n)$-norm (respectively the $L^2_{\mathcal{P}}([0,T];\mathbb{R}^{n\times d})$-norm) are lower semicontinuous for the weak topology on $L^2_{\mathcal{P}}([0,T];\mathbb{R}^n)$ (respectively on $L^2_{\mathcal{P}}([0,T];\mathbb{R}^{n\times d})$), we get

$$E\left[\int_0^T \left|\Pi_{V^\perp} p(t,\bar{q},\theta)\right|^2\mathrm{d}t\right] = 0, \quad \text{and} \quad E\left[\int_0^T \left|\Pi_{U^\perp}\bar{q}(s)\right|^2\mathrm{d}s\right] = 0.$$

Therefore, we have $\Pi_{V^\perp} p(s,\bar{q},\theta) = 0$ P-a.s., for all $s \in [0,T]$ and $\bar{q} \in L^2_{\mathcal{P}}([0,T],U)$. □

Using this characterization we get the following useful result:

Corollary 9. *The viability kernel of the linear subspace $V \subset \mathbb{R}^n$ conditioned to the linear subspace $U \subset \mathbb{R}^n$ with respect to* (6) *is conditional locally in time viable. In particular, the conditional viability kernel is locally in time viable.*

Corollary 10. *Eq.* (2) *is approximately controllable if and only if the viability kernel of* $\mathrm{Ker}\,B_2^*$ *conditioned to* $\mathrm{Ker}\,D_1^*$ *is trivial.*

Proof. We only have to prove the 'if' part. Let us suppose that $\mathrm{Viab}(\mathrm{Ker}\,B_2^*|\,\mathrm{Ker}\,D_1^*)$ is trivial and let $q \in L^2_{\mathcal{P}}([0,T],\mathrm{Ker}\,D_1^*)$ such that $p(s,q,\theta) \in \mathrm{Ker}\,B_2^*$, P-a.s., for all $s \in [0,T]$. We get $\theta = 0$, and $p(s,q,\theta) \in \mathrm{Viab}(\mathrm{Ker}\,B_2^*|\,\mathrm{Ker}\,D_1^*)$. Therefore, $p(s,q,\theta) = 0$, P-a.s., for all $s \in [0,T]$. Recall that $p(\cdot,q,\theta)$ is the solution of (6) to conclude $q(s) = 0$, $\mathrm{d}s\,\mathrm{d}P$-almost everywhere on $[0,T]\times\Omega$. □

The following result links the conditional viability property and the conditional invariance:

Proposition 11. *The linear subspace $V \subset \mathbb{R}^n$ is local in time viable conditioned to the linear subspace $U \subset \mathbb{R}^n$ with respect to* (6) *if and only if V is $(A^*;C^*)$-strictly invariant conditioned to (F,U).*

Proof. If V is $(A^*;C^*)$-strictly invariant conditioned to (F,U), then there exists a linear operator $K \in \mathcal{L}(\mathbb{R}^n,\mathbb{R}^n)$ such that $KV \subset V$, $(K-F)V \subset U$ and $(A^*+C^*K)V \subset V$. For any $\theta \in V$, we consider the linear stochastic differential equation

$$\mathrm{d}\bar{p}(t) = -(A^*+C^*K)p(t)\,\mathrm{d}t + Kp(t)\,\mathrm{d}W(t), \quad \bar{p}(0) = \theta.$$

Obviously, the solution of this equation is in V. Moreover, if we set $q(t) = (K - F)\bar{p}(t) \in U$, we notice that $p(t, q(t), \theta) = \bar{p}(t) \in V$ for all $t > 0$.

For the converse,we fix $\theta \in V$ and suppose that $p(s, q, \theta) \in V$, P-a.s., for all $s \in [0, T]$, for some $T > 0$ and $q \in L^2_{\mathcal{P}}([0, T], U)$. Then the quadratic variation of $(I - \Pi_V)p$ is zero. Moreover, we have $(I - \Pi_V)[-(A^* + C^*F)p(t) - C^*q(t)] = 0$, $\mathrm{d}t\,\mathrm{d}P$-almost everywhere on $[0, T] \times \Omega$ (the left-hand term being the drift coefficient in the differential expression of $(I - \Pi_V)p$).

At this point, we consider the linear subspace $W = \{\theta \in V \text{ s.t. } \exists \alpha \in V\colon \alpha - F\theta \in U,\ A^*\theta + C^*\alpha \in V\}$ and notice that $p(t, q, \theta) \in W$, $\mathrm{d}t\,\mathrm{d}P$-almost everywhere on $[0, T] \times \Omega$. We use the continuity of the trajectories of p to finally get $\theta \in W$. This implies that $V = W$ (i.e. V is $(A^*; C^*)$-strictly invariant conditioned to (F, U)). □

We deduce that $\mathrm{Viab}(\mathrm{Ker}\, B_2^* \,|\, \mathrm{Ker}\, D_1^*)$ is the largest space which is $(A^*; C^*)$-strictly invariant conditioned to $(F, \mathrm{Ker}\, D_1^*)$ and combine this with Corollary 10 to conclude the proof of the main theorem.

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