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A locally asymptotically powerful test for nonlinear autoregressive models

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Abstract

We propose a locally asymptotically powerful test to simultaneously examine hypotheses relative to the parametric form of the conditional mean and the conditional variance functions in a class of nonlinear semi-parametric time series models without a specified error law. On the basis of a modified version of the Le Cam method of Hwang and Basawa (2001), we establish the local asymptotic normality relative to the model. The main result shows that the test statistic built by substituting consistent estimated residuals and parameters for the theoretical ones is asymptotically normal. Its asymptotic power is computed and the result is illustrated by some simulations. ***To cite this article: F. Chebana, N. Laïb, C. R. Acad. Sci. Paris, Ser. I 346 (2008).***

Résumé

Test localement asymptotiquemt puissant pour des modèles autoregressifs nonlinéaires. Dans cette Note, nous proposons un test localement asymptotiquement puissant pour traiter simultanément des hypothèses portant sur les fonctions moyenne et variance conditionnelles. Ceci est fait sous des conditions de stationnarité et d'érgodicité sur une classe de modèles semi-paramétriques non-linéaires que nous considérons et lorsque la loi des innovations n'est pas nécessairement spécifiée. Basé sur une version modifiée de la méthode de Le Cam, dûe à Hwang et Basawa (2001), nous établissons la normalité locale asymptotique relative aux modèles étudiés. Le résultat principal montre que la statistique du test, construite en substituant aux résidus et aux paramètres des estimateurs consistants, est asymptotiquement normale. La puissance asymptotique du test proposé est calculée et des simulations ont été effectuées pour évaluer sa performance. ***Pour citer cet article : F. Chebana, N. Laïb, C. R. Acad. Sci. Paris, Ser. I 346 (2008).***

Version française abrégée

De nombreux travaux ont été consacrés par le passé à l'étude des tests d'hypothèses simples ou composites en relation avec les fonctions moyenne et variance conditionnelles faisant partie d'une famille paramétrique (voir, par exemple, Diebolt et Laïb [1], McKeagne et Zhang [10], Laïb [8]). Des techniques non paramétriques sont utilisées

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et conduisent le plus souvent à des lois limites non tabulées lorsque les paramètres en présence sont estimés. Les tests ainsi obtenus ne sont pas directement opérationnels. L'hypothèse de linéarité dans les modèles autorégressifs constitue, dans ces travaux, l'objectif essentiel de ce type de procédure de test. Ces procédures de test sont établies souvent dans le cas réel, correspondant au modèle autorégressif d'ordre 1, et traitent séparément des hypothèses portant sur la moyenne conditionnelle et celles portant sur la variance conditionnelle. En outre, l'étude de la puissance locale des tests construits n'a pas été suffisamment étudiée (voir, par exemple, Ngatchou-Wandji et Laïb [12]).

Dans cette Note, nous construisons et étudions les propriétés asymptotiques d'un test traitant simultanément des hypothèses portant sur des fonctions moyennes et variances conditionnelles, sous des conditions de stationnarité et d'érgodicité du processus. A cet effet, nous envisageons une catégorie de suites d'alternatives locales tendant vers le vrai modèle avec une vitesse de l'ordre de $n^{-1/2}$. Cette catégorie de contre-hypothèses locale est additive approchant l'hypothèse nulle selon une direction bien spécifiée, dans ce cas les décalages « shift » sont fonctionnels plutôt que paramétriques. Adpotant une version modifiée de la méthode de Le Cam, dûe à Hwang et Basawa [7], nous établissons la normalité locale asymptotique (LAN) du modèle, supposé d'ordre $d \geqslant 1$. Le test ainsi construit sur la base de ce résultat de LAN est asymptotiquement efficace et sa puissance asymptotique a été explicitement déterminée. Le résultat principal prouve que la statistique modifiée, en substituant les résidus et les paramètres par leurs estimateurs consistants, reste asymptotiquement normale. Sa puissance asymptotique est également calculée. Des simulations ont été effectuées pour évaluer les performances du test proposé. Nos résultats sont applicables à une grande variété de modèles économétriques et financiers. Notre travail prolonge et généralise celui de Hwang et Basawa [7] à une classe plus générale de processus.

1. Introduction

The present work is concerned with the construction of a locally asymptotically powerful test in a class of nonlinear time series models of higher-order d. To describe the model, let $(\theta, \rho) \in \Theta := \Theta_1 \times \Theta_2$, where Θ_1 and Θ_2 are open subsets of $\mathbb{R}^q$, and let (X_i) be a sequence of random variables (r.v.) and $\mathbf{X}_{i-1} = (X_{i-1}, \ldots, X_{i-d})^\top$, $i \geqslant d$. The model that we deal with is

$$X_i = m_{\theta,a}(\mathbf{X}_{i-1}) + \sigma_{\rho,a}(\mathbf{X}_{i-1})\epsilon_i, \quad i \geqslant d, \tag{1}$$

where $m_{\theta,a}(\mathbf{x}) = m_\theta(\mathbf{x}) + aG(\mathbf{x})$ and $\sigma_{\rho,a}(\mathbf{x}) = \sigma_\rho(\mathbf{x}) + aS(\mathbf{x})$ are the mean and the volatility functions such that $\sigma_{\rho,a}(\cdot) > 0$. Here $G(\mathbf{x})$ and $S(\mathbf{x})$ are specified functions and a is a real constant which allows us to distinguish the null hypothesis ($a = 0$) from the local alternative ($a = 1/\sqrt{n}$), and it is not a parameter to be estimated. The model is assumed to be identifiable and the sequence $(X_i)_{i \geqslant d}$ is assumed to be stationary and ergodic with finite second order moment. The ϵ_i's are iid r.v. with continuous positive twice differentiable density function f with mean 0 and variance 1. Moreover, we assume that the ϵ_i's are independent of $\mathbf{X}_{i-1}$ for any $i \geqslant d$.

Model (1) has been considered with specific assumptions on f in several research areas such as econometrics and control theory. Several papers have been devoted to the problem of testing hypotheses on the parametric form of the conditional mean and conditional variance functions, but none of them have considered both of these functions together. Two major kinds of test statistics are presented in the literature: (i) the first kind is based on the nonparametric estimation of the conditional mean and conditional variance functions, it is studied for instance by Härdle and Mammen [6], Diebolt and Laïb [1], McKeague and Zhang [10]; (ii) the second one is based on the cumulative residual empirical process, see e.g., Diebolt et al. [2] and Laïb [8], for testing hypotheses on the conditional mean function and Laïb [9] for testing hypotheses on the conditional variance function. However, most of these tests are usually derived for testing first order autoregressive models. Furthermore, the study of the local power has not been developed before except in Stute's [13] paper in i.i.d. regression setting. The same problem (but with different techniques) is considered by Ngatchou-Wandji and Laïb [12] in the setting of autoregression models with ARCH errors.

The problem of testing hypotheses simultaneously on the conditional mean and conditional variance functions pertaining to model (1) has attracted less attention. It has been studied recently by Gao and King [4] and Ngatchou-Wandji [11]. The simultaneous approach considers the correlation between the two functions. By contrast the usual approach uses two separate tests for the mean and variance functions and this leads to increase the error since each test already has its own first kind error.

The hypotheses we deal with can be formulated as

$$H_0\colon\ m_{\theta,a} = m_{\theta_0} \text{ and } \sigma_{\rho,a} = \sigma_{\rho_0} \tag{2}$$

and the local alternatives are

$$H_1^n\colon\ m_{\theta,a} = m_{\theta_0} + n^{-1/2}G \text{ and } \sigma_{\rho,a} = \sigma_{\rho_0} + n^{-1/2}S. \tag{3}$$

The shifts in the alternatives are functions instead of parameters. Note that Hwang and Basawa [7] have considered efficient test against local alternatives for testing a linear autoregressive models. Here we extend their work to a more general class of processes defined by (1). The results obtained here are applicable to a large variety of econometric and financial models.

Based on an adapted version of the Le Cam procedure due to Hwang and Basawa [7], we show in Section 2 that model (1) is locally asymptotically normal (LAN). Consequently, an efficient and asymptotically optimal test to test simultaneously hypotheses upon the conditional mean and conditional variance functions in autoregressive models of order d with ARCH errors (AR(d)-ARCH(d)) is derived. Its asymptotic power is explicitly obtained. We also give the limiting distribution of the test statistic when substituting the theoretical values with their consistent estimators. Note however, that the estimator of the autoregression parameters severely affect the behavior of the limiting law and therefore the test constructed in this case does not have an optimal power since the Neyman–Person Lemma can not be directly invoked in the present context. Simulations are drawn in Section 3 to illustrate our results.

2. Results

We first introduce some notations. Set $\|\cdot\|_q$ the Euclidian norm in $\mathbb{R}^q$. For a given function $u(\mathbf{x},\mathbf{s})$, denote by $\dot{u}(\mathbf{x},\mathbf{s})$ and $u'(\mathbf{x},\mathbf{s})$ its gradients with respect to the parameter $\mathbf{s}$ and to the variable $\mathbf{x}$ respectively. A similar notation is employed for higher derivatives. Let us also define the functions $\ell(x) = \log f(x)$ and $\varphi_x(a,b) = b^{-1}f((x-a)b^{-1})$, and the quantity $I_k := E[\ell'(\epsilon_d)^2\epsilon_d^k]$ for $k = 0, 1, 2$.

The following regularity conditions are required to state our first result:

(A1) There exit a positive measurable function M satisfying $E|M(\epsilon_d)|^{1+\gamma} < \infty$ for some $\gamma > 0$, and some $\delta > 0$ such that for $|a| < \delta$ and $|b-1| < \delta$ we have

$$\left|\frac{1}{f(x)}\frac{\partial^2}{\partial a^j\partial b^k}\varphi_x(a,b)\right| \leqslant M(x) \quad \text{for positive integers } j \text{ and } k \text{ with } j+k=2.$$

(A2) There exists $\gamma' > 0$ such that $E|\frac{G(X_d)}{\sigma_{\rho_0}(X_d)}|^{2+\gamma'} < \infty$ and $E|\frac{S(X_d)}{\sigma_{\rho_0}(X_d)}|^{2+\gamma'} < \infty$.

(A3) $E|\ell'(\epsilon_d)\epsilon_d^k|^{2+\gamma''} < \infty$ for $k = 0, 1$ and some positive γ''.

(A4)

$$\text{A4-1 } E\big[\ell'(\epsilon_d)\epsilon_d\big] = -1, \quad \text{A4-2 } E\big[\big(\ell''(\epsilon_d) + \ell'(\epsilon_d)^2\big)\epsilon_d^2\big] = 2,$$

$$\text{A4-3 } E\big[\ell'(\epsilon_d)\big] = 0, \quad \text{A4-4 } E\big[\ell''(\epsilon_d) + \ell'(\epsilon_d)^2\big] = 0, \quad \text{A4-5 } E\big[\big(\ell''(\epsilon_d) + \ell'^2(\epsilon_d)\big)\epsilon_d\big] = 0.$$

Condition (A4) is satisfied whenever $\lim_{|x|\to\infty} x^{l-1}f^{(k)}(x) = 0$ for k and l taking values 1 or 2. Various distributions including standard centered normal distribution and t-distribution with a degree of freedom greater than 3 satisfy conditions (A1), (A3) and (A4). The condition (A2) is comparable to the statements (4.4) and (4.5) in Hwang and Basawa [7] when σ is constant.

Let $\Lambda_n = \sum_{i=1}^n \log g_{ni}$ be the log-likelihood ratio, where g_{ni} is the nonnegative ratio of the densities corresponding to hypotheses H_1^n and H_0 respectively. Simple computation shows that

$$g_{ni} - 1 = f^{-1}(\epsilon_i)\big[\varphi_{\epsilon_i}(\alpha_{ni},\beta_{ni}) - f(\epsilon_i)\big], \tag{4}$$

where $\alpha_{ni} := n^{-1/2}G(\mathbf{X}_{i-1})\sigma_{\rho_0}^{-1}(\mathbf{X}_{i-1})$ and $\beta_{ni} := 1 + n^{-1/2}S(\mathbf{X}_{i-1})\sigma_{\rho_0}^{-1}(\mathbf{X}_{i-1})$. Let

$$V_n = -n^{-1/2}\sum_{i=1}^n\big\{\ell'(\epsilon_i)G(\mathbf{X}_{i-1})\sigma_{\rho_0}^{-1}(\mathbf{X}_{i-1}) + \big(\ell'(\epsilon_i)\epsilon_i + 1\big)S(\mathbf{X}_{i-1})\sigma_{\rho_0}^{-1}(\mathbf{X}_{i-1})\big\}.$$

The following theorem states the local asymptotic normality of model (1):

Theorem 2.1. *Suppose that* (A1)–(A4) *are fulfilled. If H_0 is true, then*

$$\Lambda_n = V_n - \frac{\tau_0^2}{2} + o_P(1) \quad \textit{and} \quad V_n \xrightarrow{\mathcal{D}} \mathcal{N}\big(0, \tau_0^2\big), \tag{5}$$

where $\tau_0^2 := (I_2 - 1)E[S^2(\mathbf{X}_d)\sigma_{\rho_0}^{-2}(\mathbf{X}_d)] + 2I_1 E[S(\mathbf{X}_d)G(\mathbf{X}_d)\sigma_{\rho_0}^{-2}(\mathbf{X}_d)] + I_0 E[G^2(\mathbf{X}_{d-1})\sigma_{\rho_0}^{-2}(\mathbf{X}_{d-1})]$.

Note that if $\sigma_\rho(\cdot)$ is a constant, we obtain the same result as in Hwang and Basawa [7]. Based on Theorem 2.1 it is possible to treat nonlinear and heteroscedasticity testing problems in an asymptotic manner as this is shown below.

According to the Neyman–Person Lemma, for any fixed $\alpha \in (0,1)$, the best α-level test, for testing the null hypothesis H_0 against the sequence of alternatives H_1^n specified by (3), has the critical region $\{T_n \geqslant c_\alpha\}$, where $T_n = V_n/\tau_0$. Using Theorem 2.1, one can see that the test based on T_n consists in rejecting H_0 whenever $T_n \geqslant c_\alpha$, where c_α is the $(1-\alpha)$-quantile of a standard normal distribution Φ. It can be also deduced from Theorem 2.1 that H_0 and H_1^n are contiguous. It follows then from Le Cam's third Lemma (see Hall and Mathiason, [5]) that under H_1^n, V_n converges in distribution to $\mathcal{N}(\tau_0^2, \tau_0^2)$. The proof of Theorem 2.2 below is similar to that of Theorem 3 of Hwang and Basawa [7].

Theorem 2.2. *Under the assumptions of Theorem* 2.1, *the asymptotic power of T_n under H_1^n is* $1 - \Phi(c_\alpha - \tau_0)$. *Furthermore, T_n is asymptotically efficient in the sense that the limiting power of any other limiting size α test does not exceed that of T_n.*

For practical use, the unknown parameters in T_n should be estimated. To this end, let $\hat{\nu}_n = (\hat{\theta}_n, \hat{\rho}_n)$ be a $\sqrt{n}$-consistent estimator of $\nu_0 = (\theta_0, \rho_0)$ under H_0. An estimator $\hat{V}_n$ of V_n can be obtained with substituting the parameters by their estimators. Let $\hat{\tau}_n^2$ and $\hat{I}_{k,n}$ be the empirical versions of τ_0^2 and I_k respectively.

In what follows we investigate the limiting distribution of $\hat{V}_n$ and the consistency of the quantities appearing in its limiting variance. The following additional assumptions are required:
(A5) For every fixed x, the function $\theta \mapsto m_\theta(x)$ (resp. $\rho \mapsto \sigma_\rho(x)$) has continuous derivatives, and for all θ (resp. ρ) the functions $x \mapsto m_\theta(x)$ and $\dot{m}_\theta(x)$ (resp. $x \mapsto \sigma_\rho(x)$ and $\dot{\sigma}_\rho(x)$) are continuous.
(A6) There exist closed balls $\overline{B}_0(\theta_0, r_0) \times \overline{B}_1(\rho_0, r_1) \subset \mathrm{int}(\Theta_1 \times \Theta_2)$ and positive functions M_0 and M_1 satisfying $E[M_0(X_d)]^{2+\gamma} < \infty$ and $E[M_1(X_d)]^{2+\gamma} < \infty$ such that for all $x \in \mathbb{R}^d$

$$\sup_{\theta \in \overline{B}_0} \big\|\dot{m}_\theta(x)\sigma_\rho^{-1}(x)\big\|_q \leqslant M_0(x) \quad \text{and} \quad \sup_{\rho \in \overline{B}_1} \big\|\dot{\sigma}_\rho(x)\sigma_\rho^{-1}(x)\big\|_q \leqslant M_1(x).$$

(A7) The estimators $(\hat{\theta}_n, \hat{\rho}_n)$ of the parameters (θ, ρ) are such that

$$n^{1/2}(\hat{\theta}_n - \theta_0) = n^{-1/2}\sum_{i=d}^{n} \varphi_1(\mathbf{X}_{i-1}, \theta_0, \rho_0)\epsilon_i + o_P(1),$$

$$n^{1/2}(\hat{\rho}_n - \rho_0) = n^{-1/2}\sum_{i=d}^{n} \varphi_2(\mathbf{X}_{i-1}, \rho_0)\big(\epsilon_i^2 - 1\big) + o_P(1)$$

where $\varphi_k = (\varphi_{k,1}, \ldots, \varphi_{k,q})^\top$ denotes a measurable function such that $E\|\varphi_k(\mathbf{X}_{d-1})\|_q^\beta < \infty$ for some $\beta > 2$, and the symmetric matrix $\Gamma_k = E\varphi_k(\mathbf{X}_{d-1})\varphi_k^\top(\mathbf{X}_{d-1})$ is positive-definite $(k = 1, 2)$.
(A8) $E|\epsilon_d|^{4+\delta} < \infty$ for some $\delta > 0$.
(A9) There exists an integrable function $L(\cdot)$ such that $\sigma_\rho(x) \geqslant L(x)$ for any $x \in \mathbb{R}^d$ and for any ρ in a neighborhood of ρ_0.

Condition (A7) assumes $\sqrt{n}$-convergence of the estimators $\hat{\theta}_n$ and $\hat{\rho}_n$, which leads to obtain the asymptotic distribution of $\hat{V}_n$. It is satisfied by most estimators. For instance, under the condition **B** of McKeague and Zhong [10], the first condition in (A7) holds true for the conditional least squares estimator, whereas the second condition is fulfilled whenever conditions of Lemma 2 in Laïb [9] are satisfied.

Let us now define the following quantities which are finite in view of (A2), (A4) and (A6):

$$\xi_1 = -E\left[\frac{\dot{m}_{\theta_0}(\mathbf{X}_{d-1})}{\sigma_{\rho_0}^2(\mathbf{X}_{d-1})}\big\{G(\mathbf{X}_{d-1})I_0 + S(\mathbf{X}_{d-1})I_1\big\}\right] \quad \text{and}$$

$$\xi_2 = -E\left[\frac{\dot{\sigma}_{\rho_0}(\mathbf{X}_{d-1})}{\sigma_{\rho_0}^2(\mathbf{X}_{d-1})}\big\{G(\mathbf{X}_{d-1})I_1 + S(\mathbf{X}_{d-1})(I_2 - 1)\big\}\right]. \tag{6}$$

Proposition 2.3. *Under* H_0, (A5)–(A9) *and the conditions of Theorem* 2.1*, we have*

(i) $\hat{V}_n = V_n + Z_n + o_P(1)$*, where* $Z_n = \frac{1}{\sqrt{n}} \sum_{i=d}^{n} [\varphi_1^\top(\mathbf{X}_{i-1}, \theta_0, \rho_0)\epsilon_i \xi_1 + \varphi_2^\top(\mathbf{X}_{i-1}, \rho_0)(\epsilon_i^2 - 1)\xi_2]$.

(ii) $(\hat{\gamma}_{n11} + 2\hat{\gamma}_{n12} + \hat{\gamma}_{n22})^{-1/2}\hat{V}_n \xrightarrow{\mathcal{D}} \mathcal{N}(0,1)$*, where* $\hat{\gamma}_{n11}, \hat{\gamma}_{n12}$ *and* $\hat{\gamma}_{n22}$ *denote the empirical estimators of* γ_{11}, γ_{12} *and* γ_{22} *respectively, with* $\gamma_{11} = \lim_{n\to\infty} \operatorname{Var}(V_n)$, $\gamma_{12} = \lim_{n\to\infty} \operatorname{Cov}(V_n, Z_n)$ *and* $\gamma_{22} = \lim_{n\to\infty} \operatorname{Var}(Z_n)$.

Based on the result of Proposition 2.3 'an estimate' of the test statistic T_n can be defined as follows:

$$\hat{T}_n := (\hat{\gamma}_{n11} + 2\hat{\gamma}_{n12} + \hat{\gamma}_{n22})^{-1/2}\hat{V}_n. \tag{7}$$

Since H_0 and H_1^n are contiguous, according to the third Lemma of Le Cam, one gets the limiting distribution of $\hat{V}_n$ under H_1^n, namely

$$\hat{V}_n \xrightarrow{D} \mathcal{N}(\gamma_{11} + \gamma_{12}, \gamma_{11} + 2\gamma_{12} + \gamma_{22}).$$

Thus, the asymptotic power of the test based on $\hat{T}_n$ is given by

$$1 - \Phi\big(c_\alpha - (\gamma_{11} + 2\gamma_{12} + \gamma_{22})^{-1/2}(\gamma_{11} + \gamma_{12})\big). \tag{8}$$

3. Simulations

The aim of these simulations is to evaluate the performances of the test based on $\hat{T}_n$ for short and moderate sample sizes. The observations X_i are generated according to the model:

$$X_i = \theta_1 X_{i-1} + \theta_2 X_{i-1} e^{-\varsigma X_{i-1}^2} + abX_{i-1} + \Big(\sqrt{\rho_1^2 + \rho_2^2 X_{i-1}^2 e^{-\eta X_{i-1}^2}} + abX_{i-1}\Big)\epsilon_i, \quad i \geqslant 1, \tag{9}$$

where ς and η are positive constants, $\rho_1 > 0$ and $|\theta_1| + |\theta_2| + \rho_2 < 1$.

The ϵ_i's are iid and ϵ_1 and X_0 have standard normal distribution. The alternative H_1^n can be expressed by taking $a = n^{-1/2}$ and $G(x) = S(x) = bx$. Here we have taken the same real constant b for simplicity. Using Theorem 3.2.11 in Taniguchi and Kakizawa [14, page 86] and Doukhan [3, pages 106–107] it can be easily seen that model (9) is stationary and ergodic, under the alternative, whenever $|\theta_1| + |\theta_2| + \rho_2 + 2n^{-1/2}b < 1$.

In our simulations, the true value of the parameters $(\theta_1, \theta_2, \rho_1, \rho_2)$ are fixed at $(0.1, 0.3, 0.5, 0.4)$ and the sample sizes are $n = 50, 100, 200$ and 500. The quantities I_k, $k = 0, 1, 2$ take the values $I_0 = E\epsilon_1^2 = 1$, $I_1 = E\epsilon_1^3 = 0$, $I_2 = E\epsilon_1^4 = 3$, and the statistic $\hat{V}_n$ takes the form

$$\hat{V}_n = -bn^{-1/2} \sum_{i=d}^{n} \mathbf{X}_{i-1} \sigma_{\hat{\rho}_n}^{-1}(\mathbf{X}_{i-1}) \big[(1 - \hat{\epsilon}_i^2) - \hat{\epsilon}_i\big].$$

$\hat{T}_n$ has been computed when the parameters θ_1, θ_2, ρ_1, ρ_2 are substituted by their conditional least square estimators. To evaluate the power of the test, a number $N = 1000$ of generated samples is considered and the test statistic $\hat{T}_n$ is computed for each sample. For fixed $\alpha = 5\%$, the empirical power of the test is computed.

Fig. 1 shows that the empirical first kind error increases with respect to n to reach 5%. One can also observe that when the alternatives move away from the null hypothesis, the power increases until it almost reaches 100%. However, it begins to decrease when b increases until it stabilizes around 50%. When n increases, the power remains at its maximum value for a large interval of b.

The expected power behavior would usually be increasing with respect to b since generally the power takes higher values when the alternative is further from the null hypothesis.

Two elements may explain the unusual power behavior in the present context: (1) the effect of the estimation of the parameters, since when b increases the limiting covariance γ_{12} between the sequences V_n and Z_n becomes negative which leads to reducing the asymptotic power; (2) the other explanation is related to the condition $\max_{1\leqslant i\leqslant n} n^{-1/2}|G(X_{i-1})| = o_P(1)$, which is implied by (A2). This condition prevents b from taking high values in order to keep the local aspect of alternatives.

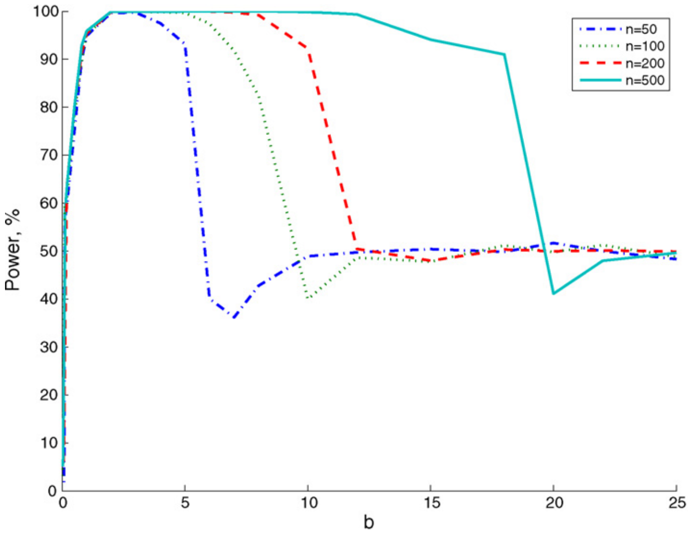

Fig. 1. $\hat{T}_n$ power with $\eta = 3$ and $\varsigma = 3$.

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