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C. R. Acad. Sci. Paris, Ser. I 346 (2008) 1073–1078

COMPTES RENDUS MATHEMATIQUE

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Geometry

Virtual fibering of certain cover of $\mathbb{S}^3$, branched over the figure eight knot

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Received 17 July 2008; accepted 15 September 2008

Available online 11 October 2008

Presented by Étienne Ghys

Abstract

In this Note we prove that there exists some integer $n_0 \geqslant 1$ such that if M is a closed, orientable 3-manifold which is a branched cover of $\mathbb{S}^3$, branched over the figure eight knot with all branching indices equal to a common even integer $n \geqslant n_0$, then M has a finite index cover which fibers over the circle. ***To cite this article: N. Bergeron, C. R. Acad. Sci. Paris, Ser. I 346 (2008).***

Résumé

Fibration virtuelle de certains revêtements de $\mathbb{S}^3$, ramifiés au-dessus du nœud de huit. Dans cette Note nous démontrons qu'il existe un entier $n_0 \geqslant 1$ tel que si M est une 3-variété compacte orientable qui est un revêtement ramifié de $\mathbb{S}^3$, ramifié au-dessus du nœud de huit et dont tous les indices de ramification sont égaux à un même entier pair $n \geqslant n_0$, alors M possède un revêtement fini qui fibre sur le cercle. ***Pour citer cet article : N. Bergeron, C. R. Acad. Sci. Paris, Ser. I 346 (2008).***

Version française abrégée

Jusqu'à l'été dernier le problème, posé par Thurston, de l'existence, au-dessus de toute 3-variété hyperbolique de volume fini, d'un revêtement fini qui fibre sur le cercle, était complètement mystérieux. Depuis, Ian Agol [1] a résolu ce problème par l'affirmative pour toute variété vérifiant la conjecture suivante :

Conjecture 0.1 *(Haglund–Wise). Le groupe fondamental de toute* 3*-variété hyperbolique compacte* (*ou de volume fini*) *se plonge virtuellement dans un groupe de Coxeter abstrait à angles droits.*

Rappelons qu'un **groupe de Coxeter abstrait à angles droits** est un groupe de type fini engendré par des éléments d'ordre deux et tel que les seules autres relations soient des relations de commutations entre certains générateurs. Dans [8], Haglund et Wise montrent que la Conjecture 0.1 est, de manière remarquable, essentiellement équivalente à

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doi:10.1016/j.crma.2008.09.014

deux conjectures bien connues, à savoir que les 3-variétés hyperboliques contiennent de nombreuses surfaces incompressibles (immergées) et que les sous-groupes quasi-convexes des groupes fondamentaux de 3-variétés hyperboliques sont séparables. À l'aide de ces résultats nous vérifions la Conjecture 0.1 pour les variétés arithmétiques standard compactes et les variétés non-arithmétiques de Gromov et Piatetski–Shapiro dans [4]. Dans cette Note nous étendons un peu ces méthodes pour démontrer le théorème suivant :

Théorème 0.1. *Il existe un entier $n_0 \in \mathbb{N}_{\geqslant 1}$ tel que si M est une 3-variété compacte orientable qui est un revêtement ramifié de $\mathbb{S}^3$, ramifié au-dessus du nœud de huit et dont tous les indices de ramification sont égaux à un même entier pair $n \geqslant n_0$, alors M possède un revêtement fini qui fibre sur le cercle.*

Remarquons que ce résultat doit être valable pour une classe bien plus grande de revêtements ramifiés au-dessus d'entrelacs hyperboliques arithmétiques. Néanmoins, le nœud de huit est un « petit bijou » qui a souvent servi à illustrer de nouvelles techniques ; sans compter qu'un énoncé général serait indigeste et que le développement rapide des travaux d'Haglund et Wise laisse espérer une démonstration de la Conjecture 0.1 pour toutes les variétés Haken dans un futur proche. Grâce au théorème d'Agol mentionné plus haut cela réduirait la solution affirmative du problème de la fibration virtuelle à la démonstration de la célèbre conjecture selon laquelle toute 3-variété hyperbolique compacte est virtuellement Haken (conjecture bien connue pour les revêtements ramifiés que nous considérons).

1. Introduction

Last summer Ian Agol [1] proved the answer to Thurston's virtual fibering question to be positive for any 3-manifold whose fundamental group virtually embeds in an abstract right-angled Coxeter group. This may be attacked using techniques of Haglund and Wise [8].

The purpose of this Note is to illustrate these new techniques on some particularly nice family of examples: some covers of $\mathbb{S}^3$ branched over the figure eight knot. We prove:

Theorem 1.1. *There exists some integer $n_0 \geqslant 1$ such that if M is a closed, orientable 3-manifold which is a branched cover of $\mathbb{S}^3$, branched over the figure eight knot with all branching indices equal to a common even integer $n \geqslant n_0$. Then M has a finite index cover which fibers over the circle.*

Note that these 3-manifolds are shown to be virtually Haken by Baker in [2].

2. An arithmetic knot

The figure eight knot is well known to be an arithmetic knot (it is even the only such knot by Reid). Its fundamental group Γ identifies with a subgroup of index 24 in the full group of symmetries of the tesselation $\mathcal{T}$ of the hyperbolic 3-space $\mathbb{H}^3$ by ideal tetrahedrons with angles $\pi/3$ between faces. The (Coxeter) group of symmetries of this tesselation is $\mathrm{O}(q,\mathbb{Z})$ factored by its center, where

$$q(x_1,x_2,x_3,x_4) = x_1^2 + x_2^2 + x_3^2 - 3x_4^2$$

and $\mathrm{O}(q,\mathbb{R}) \cong \mathrm{O}(3,1)$ is made to act on $\mathbb{H}^3$ through the projective model. The subgroup of $\mathrm{PSL}(2,\mathbb{C})$ (made to act on $\mathbb{H}^3$ through the upper half-space model) corresponding to this tesselation is the Bianchi group $\mathrm{PSL}(2,\mathbb{Z}[\omega])$ with $\omega = \mathrm{e}^{2\mathrm{i}\pi/3}$, it contains Γ as an index 12 subgroup. A meridian loop μ around the figure eight knot can be chosen so that μ is represented by $x = \left(\begin{smallmatrix}1 & 1\\ 0 & 1\end{smallmatrix}\right) \in \Gamma$. Given a positive integer n, we denote by $\Gamma(n)$ the finite index (congruence) subgroup of Γ which consists of the matrices in $\Gamma \subset \mathrm{O}(q,\mathbb{Z})$ which are congruent to the identity mod n in $\mathrm{GL}(4,\mathbb{Z})$.

A **hyperplane** of $\mathbb{H}^3$ is a codimension one totally geodesic subspace; note that a hyperplane is isometric to the hyperbolic plane $\mathbb{H}^2$. A Γ-**hyperplane** is a hyperplane H of $\mathbb{H}^3$ such that $\Gamma_H := \mathrm{Stab}_\Gamma(H)$ acts cocompactly on H. By reducing mod 4 it is easy to see that the rational sub-quadratic space of $(\mathbb{R}^4, q)$ generated by the three last vectors of the canonical basis is anisotropic over $\mathbb{Q}$. Let H be the corresponding hyperplane of $\mathbb{H}^3$. It follows from Godement's criterion that H is a Γ-hyperplane. We will retain from the arithmeticity of Γ that it contains plenty of such, namely all the translates $g \cdot H$ where $g \in \mathrm{O}(q,\mathbb{Q})$. These form a dense collection $\mathcal{H}$ of Γ-hyperplanes in the set of all hyperplanes of $\mathbb{H}^3$.

Let $\mathcal{F}$ be the set of hyperplanes supporting a face of the tesselation $\mathcal{T}$. The hyperplanes in $\mathcal{F}$ intersect any sufficiently small horosphere centered at some (ideal) vertex of $\mathcal{T}$ along a tiling by equilateral triangles. Hyperplanes in $\mathcal{F}$ are Γ-translates of the three hyperplanes supporting three adjacent faces of a tetrahedron of $\mathcal{T}$. This gives three types of hyperplanes in $\mathcal{F}$. We will refer to the first type as the one which contains a hyperplane fixed by x. Note that the hyperplanes in $\mathcal{F}$ are not Γ-hyperplanes, they nevertheless project onto finite volume submanifolds of $Y = \Gamma\backslash\mathbb{H}^3$. Hyperplanes of the first type intersect the boundary torus along a meridian.

3. A space with walls

Let $B \subsetneq B'$ be two open horoballs centered at some (ideal) vertex of $\mathcal{T}$ such that their Γ-translates are all disjoint. Let $X = \mathbb{H}^3 - \Gamma \cdot B$. The group Γ preserves X and acts cocompactly on it. Note that the hyperplanes in $\mathcal{F}$ yield cocompact submanifold of $\Gamma\backslash X$.

One may find a finite set of hyperplanes in $\mathcal{H}$ such that their Γ-translates of the H_i separate B from the boundary of B'. This and the proof of [4, Proposition 2.1] imply the following proposition:

Proposition 3.1. *There exists a finite set of hyperplanes $H_1, \ldots, H_m \in \mathcal{H}$ and a positive integer n_1 such that for all $n \geqslant n_1$, the projection of the Γ-translates of the H_i induce a decomposition of $Y(n) = \Gamma(n)\backslash\mathbb{H}^3$ as a finite union of compact contractible sets and of products* $[0, +\infty) \times T_i$ *for each boundary torus T_i of $\Gamma(n)\backslash\mathbb{H}^3$.*

Adding the hyperplanes in $\mathcal{F}$ cuts the tori into triangles. Let $\mathcal{W}$ be the reunion of the Γ-translates of the H_i and of the hyperplanes in $\mathcal{F}$.

We now examine the situation from a metric viewpoint. Each cusp of $Y(n)$ has metrically the structure of a warped product $C = [0, +\infty) \times_g T$ with the warping function $g : [0, +\infty) \to \mathbb{R}$, $g(t) = \mathrm{e}^{-t}$, i.e. the metric is given by $\mathrm{d}t^2 + g^2(t)\,\mathrm{d}s^2$ where $\mathrm{d}s^2$ is the standard Euclidean metric on the corresponding boundary torus $T = \mathbb{R}^2/L$, where L is the lattice of $\mathbb{R}^2$ generated by $(n, 0)$ and $(0, n)$. On the other hand, let c be a geodesic in $\mathbb{H}^3$ and consider the distance tube $T_r(c)$. For $r \to +\infty$, the boundary $\partial T_r(c)$ can be identified with $\mathbb{R}^2/2\pi\mathrm{e}^r\mathbb{Z}$ where $2\pi\mathrm{e}^r\mathbb{Z}$ is the set of vectors $(2\pi\mathrm{e}^r k, 0) \in \mathbb{R}^2$ with $k \in \mathbb{Z}$; it converges metrically to a horosphere. Choose r such that $2\pi\mathrm{e}^r = n$. Now let β be the isometry of $T_r(c)$ which operates on $\partial T_r(c) = \mathbb{R}^2/2\pi\mathrm{e}^r\mathbb{Z}$ as the translation $(0, n)$. For n sufficiently large, say $n \geqslant n_2$, $\partial(T_r(c)/\mathbb{Z})$ is metrically close to ∂C. Thus one can glue the tube onto each boundary torus. with a “small” metric singularity. After smoothing we obtain a metric with curvature close to -1. We denote by $V(n)$ the manifolds thus obtained, topologically they are obtained by $(1, 0)$-Dehn filling on each cusp of $Y(n)$. (Note that Thurston has actually shown the existence of constant curvature metrics on these manifolds.)

Each hyperplane in $\mathcal{W}$ yields an immersed surface in $Y(n)$; it is compact if the hyperplane is a translate of some H_i and it is of finite volume if the hyperplane is in $\mathcal{F}$. In the last case we distinguish between hyperplanes of the first type and the others. The $(1, 0)$-Dehn fillings of the cusps of $Y(n)$ induce a filling of the surfaces corresponding to the hyperplanes of the first type. These surfaces cut the solid tori into solid cylinder. The two other types of hyperplanes in $\mathcal{F}$ intersect the boundary tori along **traces**, simple closed curves, that we distinguish into the corresponding two types. Note that traces of a same type on a same torus are homotopic. Consider a boundary torus T_i; we may assume that it corresponds to the cusp at infinity. The smallest positive integer p such that $x^p \in \Gamma(n)$ is n. From now on we will assume that n is even. The element $x^{n/2}$ then induces a permutation of order 2 on the set of traces on T_i of one given type. After $(1, 0)$-Dehn filling, one may close the corresponding surfaces by gluing them by pairs by an annulus in the interior of the solid torus T_i with boundary the two corresponding traces associated by the order 2 permutation. The cut of the filling by a meridian disk is represented in Fig. 1. In any case the hyperplanes in $\mathcal{W}$ yield immersed quasi-convex surfaces in $V(n)$ whose principle curvatures are close to zero when n is big. These surfaces cut $V(n)$ in compact contractible sets. The set of hyperplanes $\mathcal{W}$ thus induces a decomposition of $V(n)$ as a finite union of compact contractible sets. This yields a decomposition of the universal cover $\widetilde{V(n)}$ of $V(n)$ into a union of compact contractible sets. Any geodesic starting from a point in such a contractible will have to hit after some time (depending only on n but not of the starting point) a hyperplane in $\mathcal{W}$ at an angle bigger than some uniform (in n) positive constant. Assuming n_2 sufficiently big (and hence the principle curvatures of the hyperplanes in $\mathcal{W}$ sufficiently close to zero) this forces the geodesic to never cross back the hyperplane. Using this, it is now a standard task to prove that $\mathcal{W}$ induces a **wallspace structure** (see Haglund and Paulin [7] for a definition and [4] for a related construction)

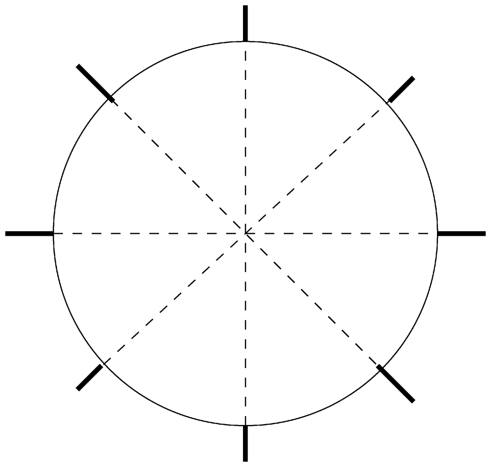

Fig. 1. The cut of the filling by a meridian disk.

$\mathcal{W}(n)$ on $\widetilde{V(n)}$. The fundamental group $\Lambda(n) = \pi_1(V(n))$ acts properly on $\widetilde{V(n)}$ and preserves the wallspace structure $\mathcal{W}(n)$.

Remark 1. The walls in $\mathcal{W}(n)$ are in one-to-one correspondence with the equivalence classes of hyperplanes in $\mathcal{W}$ under:

- the action of the (peripheral) subgroups of $\Gamma(n)$ generated by the conjugates of x^n on the hyperplanes of $\mathcal{F}$ of the first type, and
- the action of the (peripheral) subgroups of $\Gamma(n)$ generated by the conjugates of $x^{n/2}$ on the other hyperplanes of $\mathcal{F}$.

Two walls in $\mathcal{W}(n)$ intersect if and only if they may be represented by two intersecting hyperplanes in $\mathcal{W}$.

4. Cubulation

The cubulation of a space with walls embeds it in a CAT(0) cube complex (see [5] for a definition); it is an abstract version of a construction due to Sageev [11]. In [10] Nica shows that any space with walls admits such a cubulation, see [6] for an alternate construction. As a consequence he obtains that a proper group action on a space with walls extends naturally to a proper group action on a CAT(0) cube complex. Using the Gromov-hyperbolicity of $\Lambda(n)$ and a general lemma of Hruska ans Wise [9] (see also [4, Lemma 3.3]) we obtain the following proposition:

Proposition 4.1. *Let $n \geqslant n_1, n_2$. The group $\Lambda(n)$ acts properly and cocompactly on a* CAT(0) *cube complex $\mathcal{C}(n)$. Moreover, the hyperplanes of $\mathcal{C}(n)$ are in one-to-one correspondence with the walls of $\mathcal{W}(n)$, with equal stabilizers in $\Lambda(n)$.*

The quotient $\Lambda(n)\backslash\mathcal{C}(n)$ is a non-positively curved (npc) cube complex. In [8] Haglund and Wise introduce and study a particular class of npc cube complex. A npc cube complex is **special** if its immersed hyperplanes avoid certain pathologies: each hyperplane embeds, no hyperplane self-osculates and no two hyperplane interosculates. See [8] for definitions.

The walls in $\mathcal{W}(n)$ project onto a finite number of (immersed) compact connected submanifolds in $V(n)$. They moreover induce a decomposition of $\widetilde{V(n)}$ as a locally finite union of compact connected contractible sets: the closure of the connected components of the complementary region $\widetilde{V(n)} - \bigcup_{W \in \mathcal{W}(n)} W$. Note that each such connected component C is a manifold with corners with boundary a finite union of connected components of $C \cap W$, open in $W \in \mathcal{W}(n)$. We call such an intersection a **face** of C and say that the wall W **supports** this face. In analogy with the terminology of [8] we say that two walls in $\mathcal{W}(n)$ **osculates** if they are parallel and are note separated by another wall. It follows from Proposition 4.1 that the cube complex $\Lambda(n)\backslash\mathcal{C}(n)$ will be special if the walls in $\mathcal{W}(n)$ avoid certain pathologies:

(i) Each wall in $\mathcal{W}(n)$ projects onto an embedded submanifold in $V(n)$.
(ii) Two osculating walls in $\mathcal{W}(n)$ project on two different submanifold.
(iii) Two osculating walls in $\mathcal{W}(n)$ project on two non-intersecting submanifold.

Note that it follows from Remark 1 that this will be the case as soon as the projection of the hyperplanes $H \in \mathcal{W}$ into $\Gamma(n)\backslash\mathbb{H}^3$ already avoid the corresponding pathologies. In the next paragraph we prove that there exists some positive integer $n_0 \geqslant n_1, n_2$ such that for any even integer $n \geqslant n_0$ the projection of the hyperplanes $H \in \mathcal{W}$ into $\Gamma(n)\backslash\mathbb{H}^3$ avoid these pathologies. This implies that $\Lambda(n)\backslash\mathcal{C}(n)$ is special. The following proposition then follows from [8, Proposition 3.10 and Theorem 4.2]:

Proposition 4.2. *For each even integer $n \geqslant n_0$, the group $\Lambda(n)$ virtually embeds in an abstract right-angled Coxeter group.*

5. Separation

Using finite unions of closures of connected components of $X - \bigcup_{H\in\mathcal{W}} H$ we may cover X by a family of compact subsets $D \subset X$ of uniformly bounded size such that two osculating walls must intersect some D. It then follows from Remark 1 that the projection of the hyperplanes $H \in \mathcal{W}$ into $\Gamma(n)\backslash\mathbb{H}^3$ will avoid the pathologies mentioned in the preceding paragraph if for every $\gamma \in \Gamma(m)$ ($m = n/2$) we have:

(i) For each hyperplane $H \in \mathcal{W}$, if $\gamma \cdot H \cap H \neq \emptyset$ then $\gamma \in \Gamma(m)_H$.
(ii) For each hyperplane $H \in \mathcal{W}$ intersecting some D, if $\gamma \cdot D \cap H \neq \emptyset$ then $\gamma \in \Gamma(m)_H$.
(iii) For each pair of parallel hyperplanes $H_1, H_2 \in \mathcal{W}$ intersecting some D, then $\gamma \cdot H_1 \cap H_2 = \emptyset$.

For each hyperplane $H \in \mathcal{W}$ we let $N(H)$ be the reunion of all the D that intersect H. To avoid the pathologies it is thus sufficient to show that for m sufficiently large, the sets:

$$\mathrm{Bad}\big(\Gamma(m), H\big) := \big\{\gamma \in \Gamma(m)\colon\ \gamma \cdot N(H) \cap N(H) \neq \emptyset\big\} - \Gamma(m)_H$$

and

$$\mathrm{Bad}\big(\Gamma(m), H, H'\big) := \big\{\gamma \in \Gamma(m)\colon\ \gamma \cdot H \cap H' \neq \emptyset\big\}$$

are both empty for every $H, H' \in \mathcal{W}$ disjoint and intersecting some D.

First note that there are only a finite number of hyperplanes $H_1, \ldots, H_k \in \mathcal{W}$ (resp. of couples of hyperplanes $(H_1, H_1'), \ldots, (H_k, H_k')$ in $\mathcal{W}$) such that each hyperplane of $\mathcal{W}$ (resp. each pair of disjoint hyperplanes intersecting some D) is in the Γ-orbit of one of the H_i's (resp. (H_i, H_i')'s). It is thus sufficient to show that for every $H, H' \in \mathcal{W}$ disjoint and intersecting some D, there exists some integer n_0 such that for every $n \geqslant n_0$, $\mathrm{Bad}(\Gamma(n), H) = \emptyset$ and $\mathrm{Bad}(\Gamma(n), H, H') = \emptyset$. But as in the proofs of [4, Lemmas 5.2 and 5.3] this follows from the facts that both Γ_H and $\Gamma \cap \mathrm{Stab}_{\mathrm{SO}(q)}(H)\mathrm{Stab}_{\mathrm{SO}(q)}(H')$ are closed subsets of Γ for the topology generated by the basis consisting of cosets of finite index subgroups containing some $\Gamma(n)$ with $n \in \mathbb{N}_{\geqslant 1}$. This in turn follows from the next lemma and the fact that both $\mathrm{Stab}_{\mathrm{SO}(q)}(H)$ and $\mathrm{Stab}_{\mathrm{SO}(q)}(H)\mathrm{Stab}_{\mathrm{SO}(q)}(H')$ are rational algebraic subsets of $\mathrm{SO}(q)$. This is obvious in the first case and is a consequence of the proof of [3, Proposition 10] in the last case.

Lemma 5.1. *Let V be a rational algebraic subset of* $\mathrm{GL}(4,\mathbb{R})$. *Let $\gamma \in \Gamma$ such that $\gamma \in V\Gamma(n)$ for every $n \geqslant 1$ then $\gamma \in V$.*

Proof. Assume by contradiction that $\gamma \notin \Gamma \cap V$. Then there exists a polynomial P with $\mathbb{Z}$-coefficients on the 4×4 matrices such that $P(\gamma) \neq 0$. Choosing n sufficiently big so that $P(\gamma) \neq 0 \pmod n$ we conclude that $\gamma \notin V\Gamma(n)$. □

6. Conclusion

Let $r : M \to \mathbb{S}^3$ be a finite cover, branched over the figure eight knot K with all branching indices equal to a common even integer $n \geqslant n_0$ and let $W \to Y$ be the associated unbranched cover obtained by removing an open tubular neighborhood of K. Let $\Gamma' \subset \Gamma$ be the finite index subgroup corresponding to this cover. Consider the regular cover $W(n) \to W$ corresponding to the normal subgroup $\Gamma'(n) = \Gamma' \cap \Gamma(n)$. It follows from [2, Lemma 2] that the cover $W(n) \to W$ extends to a regular (unbranched) cover $M(n) \to M$ by performing $(1, 0)$-Dehn filling on each cusp. Note now that $M(n)$ is also a finite regular cover of $V(n)$. It follows from Proposition 4.2, that $\pi_1(V(n))$ virtually embeds in an abstract right-angled Coxeter group. Thus M satisfies Conjecture 0.1 and Agol's theorem finally implies that M has a finite index cover which fibers over the circle.

Acknowledgements

I would like to thank Frédéric Haglund and Ian Agol for their careful reading. Ian Agol, especially, indicated and corrected a gap in an earlier version.

References

[1] I. Agol, Criteria for virtual fibering, preprint, arXiv: 0707.4522v1.
[2] M.D. Baker, The virtual $\mathbb{Z}$-representability of certain 3-manifold groups, Proc. Amer. Math. Soc. 103 (1988) 996–998.
[3] N. Bergeron, Cycles géodésiques transverses dans les variétés hyperboliques, Geom. Funct. Anal. 12 (3) (2002) 437–463.
[4] N. Bergeron, F. Haglund, D.T. Wise, Hyperplane sections in arithmetic hyperbolic manifolds, submitted for publication.
[5] M.R. Bridson, A. Haefliger, Metric Spaces of Non-positive Curvature, Grundlehren der Mathematischen Wissenschaften (Fundamental Principles of Mathematical Sciences), vol. 319, Springer-Verlag, Berlin, 1999.
[6] I. Chatterji, G. Niblo, From wall spaces to CAT(0) cube complexes, Internat. J. Algebra Comput. 15 (5–6) (2005) 875–885.
[7] F. Haglund, F. Paulin, Simplicité de groupes d'automorphismes d'espaces à courbure négative, in: The Epstein Birthday Schrift, in: Geom. Topol. Monogr., vol. 1, Geom. Topol. Publ., Coventry, 1998, pp. 181–248 (electronic).
[8] F. Haglund, D.T. Wise, Special cube complexes, Geom. Funct. Anal. 17 (5) (2008) 1551–1620.
[9] G.C. Hruska, D.T. Wise, Finiteness properties of groups acting on CAT(0)-cube complex, in preparation.
[10] B. Nica, Cubulating spaces with walls, Algebr. Geom. Topol. 4 (2004) 297–309 (electronic).
[11] M. Sageev, Ends of group pairs and non-positively curved cube complexes, Proc. London Math. Soc. (3) 71 (3) (1995) 585–617.