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Numerical Analysis

An added-mass free semi-implicit coupling scheme for fluid–structure interaction

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Abstract

In this Note we propose a semi-implicit coupling scheme for the numerical simulation of fluid–structure interaction systems involving a viscous incompressible fluid. The scheme is stable irrespectively of the so-called added-mass effect and allows for conservative time-stepping within the structure. The efficiency of the scheme is based on the explicit splitting of the viscous effects and geometrical/convective non-linearities, through the use of the Chorin–Temam projection scheme within the fluid. Stability relies on the implicit treatment of the pressure stresses and on the Nitsche's treatment of the viscous coupling. ***To cite this article: M. Astorino et al., C. R. Acad. Sci. Paris, Ser. I 347 (2009).***

Résumé

Couplage semi-implicite en interaction fluide–structure insensible aux effets de masse-ajoutée. Dans cette Note, nous proposons un schéma de couplage semi-implicite pour la simulation numérique de phénomènes d'interaction fluide–structure dans lesquels le fluide est incompressible. Le schéma est stable indépendamment des effets de masse-ajoutée et permet l'utilisation de schémas conservatifs pour la structure. L'efficacité du schéma provient du traitement explicite des effets visqueux et des non-linéarités dues à la convection et à la géométrie, grâce à un schéma de projection de Chorin–Temam pour le fluide. D'autre part, la stabilité du schéma s'appuie sur le traitement implicite des efforts de pression et sur le couplage visqueux faible, effectué avec la méthode de Nitsche. ***Pour citer cet article : M. Astorino et al., C. R. Acad. Sci. Paris, Ser. I 347 (2009).***

Version française abrégée

Dans cette Note on s'intéresse à la résolution numérique des problèmes d'interaction fluide–structure, avec un fluide visqueux incompressible. Il est bien connu (voir [5], par exemple) que les schémas explicites, très peu coûteux en temps de calcul, sont instables pour des problèmes avec un effet de masse-ajoutée important. En général, ces instabilités sont évitées avec l'utilisation de schémas de couplage *implicites* (voir [7] pour une synthèse), *semi-implicites* [6,9] ou *explicites stabilisés* [3,4].

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Nous considérons ici le cas des schémas semi-implicites. Ces schémas possèdent des propriétés de stabilité supérieures à celles des schémas de couplage explicites, mais sont contraints par une condition de stabilité du type (1), encore dépendante des effets de masse-ajoutée. En outre, du point de vue théorique, la stabilité semble être difficile à établir lorsque des schémas conservatifs sont employés pour la structure (voir [6, Remarques 3 et 4]). Le schéma proposé ici (Algorithme 1, Section 3) pour le problème (2)–(3) s'appuie sur la méthode de projection de Chorin–Temam, utilisée pour la discrétisation en temps du fluide (voir [6]). La principale nouveauté présentée dans cette note consiste à imposer faiblement le couplage visqueux, grâce à la méthode de pénalisation de Nitsche (voir [2], par exemple). Ceci se traduit par d'excellentes propriétés de stabilité de l'algorithme. En effet, pour une version linéarisée du problème (2)–(3) et sous la condition (4), on peut montrer (Proposition 4.1), que l'Algorithme 1 est stable, indépendamment de l'effet de masse ajoutée et du caractère dissipatif de la discrétisation en temps du solide. Deux expériences numériques, sur des cas tests différents, confirment ces résultats théoriques.

1. Introduction

In this Note we address the numerical simulation of fluid–structure interaction problems involving a viscous incompressible fluid and an elastic structure. For this type of problems, it is well known that stability of the *explicit coupling* schemes (i.e. that only involve the solution of the fluid and the structure once, or just a few times, per time step) is dictated by the amount of *added-mass effect* (see e.g. [5]). In other words, a strong added-mass effect in the system (i.e. the fluid and solid densities are close or the domain is slender) gives rise to numerical instability of the explicit coupling, irrespectively of the discretization parameters. Examples in blood flows simulations are popular. Several strategies have been proposed in the literature in order to overcome these infamous numerical instabilities: *implicit coupling* (see e.g. [7]), *semi-implicit coupling* [6,9] or *stabilized explicit coupling* [3,4].

In this work we consider the semi-implicit coupling approach reported in [6]. Computational cost and numerical stability are then balanced by performing an explicit–implicit splitting, based on the use of Chorin–Temam' projection scheme within the fluid. At each time step, the projection sub-step is implicitly coupled with the structure, whereas the viscous sub-step, taking into account the convective–viscous effects and the geometrical non-linearities, is treated explicitly.

Although the theoretical and numerical results, reported in [6], showed that the resulting algorithm drastically improves the stability properties of explicit coupling and the efficiency of implicit coupling, the original semi-implicit coupling scheme has two limitations. On one hand, though much less sensitive to the added-mass effect than explicit coupling, numerical evidence (see Section 5) shows that the stability still depends on the fluid–solid density ratio. As a matter of fact, in the linear case, stability is obtained (see [6, Theorem 1]) under a condition of the type:

$$\rho_{\mathrm{s}}/\rho_{\mathrm{f}} \geqslant C\big[1+\mu\tau/\big(\rho_{\mathrm{f}}h^2\big)\big], \tag{1}$$

where ρ_{s}, ρ_{f} stand for the solid and fluid densities and h, τ for the space and time discretization parameters. On the other hand, from the theoretical point of view, a dissipative time-discretization is required within the structure in order to ensure stability (see [6, Remarks 3 and 4]).

In this Note, we propose a semi-implicit coupling scheme that circumvents the above mentioned inconveniences. The stability properties of the new scheme are independent of the added-mass effect and allow the use of a conservative time discretization within the structure. The key idea consists in treating the explicit part of the coupling in a weak sense, by using the Nitsche mortaring reported in [2] (see also [3]).

2. The coupled problem

We consider an ALE (Arbitrary Lagrangian Eulerian) formulation for the fluid and a total Lagrangian formulation for the solid. Let $\Omega = \Omega_{\mathrm{f}} \cup \Omega_{\mathrm{s}}$ be a reference configuration of the fluid–structure system, we denote by $\Sigma \stackrel{\mathrm{def}}{=} \partial\Omega_{\mathrm{s}} \cap \partial\Omega_{\mathrm{f}}$ the fluid–structure interface. The current configuration of the fluid domain, $\Omega^{\mathrm{f}}(t)$, is parametrized by the ALE map $\boldsymbol{\mathcal{A}} \stackrel{\mathrm{def}}{=} \boldsymbol{I}_{\Omega_{\mathrm{f}}} + \boldsymbol{\eta}_{\mathrm{f}}$ as $\Omega_{\mathrm{f}}(t) = \boldsymbol{\mathcal{A}}(\Omega_{\mathrm{f}}, t)$, where $\boldsymbol{\eta}_{\mathrm{f}} : \Omega_{\mathrm{f}} \times \mathbb{R}^+ \to \mathbb{R}^d$ stands for the displacement of the fluid domain. We denote by $\Sigma(t) \stackrel{\mathrm{def}}{=} \partial\Omega_{\mathrm{s}}(t) \cap \partial\Omega_{\mathrm{f}}(t)$ the current position of the fluid–structure interface. In practice, $\boldsymbol{\eta}_{\mathrm{f}} = \mathrm{Ext}(\boldsymbol{\eta}|_{\Sigma})$, where $\mathrm{Ext}(\cdot)$ denotes any reasonable lifting operator from the (reference) interface Σ into the (reference) fluid domain Ω_{f} (e.g., an harmonic lifting operator).

The non-linear fluid–structure problem under consideration reads as follows: Find the fluid velocity $\boldsymbol{u} : \Omega_{\mathrm{f}} \times \mathbb{R}^+ \to \mathbb{R}^d$, the pressure $p : \Omega_{\mathrm{f}} \times \mathbb{R}^+ \to \mathbb{R}$ and the solid displacement $\boldsymbol{\eta} : \Omega_{\mathrm{s}} \times \mathbb{R}^+ \to \mathbb{R}^d$ such that

$$\begin{cases} \rho_{\mathrm{f}} \partial_t \boldsymbol{u}|_{\boldsymbol{\mathcal{A}}} + \rho_{\mathrm{f}}(\boldsymbol{u} - \boldsymbol{w}) \cdot \boldsymbol{\nabla} \boldsymbol{u} - \boldsymbol{\nabla} \cdot \boldsymbol{\sigma}(\boldsymbol{u}, p) = \boldsymbol{0} & \text{in } \Omega_{\mathrm{f}}(t), \\ \boldsymbol{\nabla} \cdot \boldsymbol{u} = 0 & \text{in } \Omega_{\mathrm{f}}(t), \\ \rho_{\mathrm{s}} \partial_{tt} \boldsymbol{\eta} - \boldsymbol{\nabla} \cdot \boldsymbol{\Pi}(\boldsymbol{\eta}) = 0 & \text{in } \Omega_{\mathrm{s}}, \end{cases} \tag{2}$$

with the interface coupling conditions

$$\begin{cases} \boldsymbol{\eta}_{\mathrm{f}} = \mathrm{Ext}(\boldsymbol{\eta}|_{\Sigma}), \quad \boldsymbol{w} = \partial_t \boldsymbol{\eta}_{\mathrm{f}} \quad \text{in } \Omega_{\mathrm{f}}, \quad \Omega_{\mathrm{f}}(t) = (\boldsymbol{I}_{\Omega_{\mathrm{f}}} + \boldsymbol{\eta}_{\mathrm{f}})(\Omega_{\mathrm{f}}), \\ \boldsymbol{u} = \partial_t \boldsymbol{\eta} \quad \text{on } \Sigma(t), \\ \boldsymbol{\Pi}(\boldsymbol{\eta}) \boldsymbol{n}_{\mathrm{s}} = -J_{\mathrm{f}} \boldsymbol{\sigma}(\boldsymbol{u}, p) \boldsymbol{F}_{\mathrm{f}}^{-T} \boldsymbol{n} \quad \text{on } \Sigma, \end{cases} \tag{3}$$

where $\partial_t|_{\boldsymbol{\mathcal{A}}}$ represents the ALE time derivative, $\boldsymbol{\sigma}(\boldsymbol{u}, p) \overset{\text{def}}{=} -p\boldsymbol{I} + 2\mu \boldsymbol{\epsilon}(\boldsymbol{u})$ the fluid Cauchy stress tensor, μ the fluid dynamic viscosity, $\boldsymbol{\epsilon}(\boldsymbol{u}) \overset{\text{def}}{=} \frac{1}{2}(\boldsymbol{\nabla} \boldsymbol{u} + \boldsymbol{\nabla} \boldsymbol{u}^{\mathrm{T}})$ the strain rate tensor, $\boldsymbol{\Pi}(\boldsymbol{\eta})$ the first Piola–Kirchhoff stress tensor of the structure, $\boldsymbol{F}_{\mathrm{f}} \overset{\text{def}}{=} \boldsymbol{\nabla} \boldsymbol{\mathcal{A}}$ the fluid domain gradient of deformation and $J_{\mathrm{f}} \overset{\text{def}}{=} \det \boldsymbol{F}_{\mathrm{f}}$ the Jacobian. Note that a field defined in the reference fluid domain, Ω_{f}, is evaluated in the current fluid domain, $\Omega_{\mathrm{f}}(t)$, by composition with $\boldsymbol{\mathcal{A}}^{-1}(\cdot, t)$. In order to simplify the exposition, in this Note we have deliberately neglected initial and boundary conditions for (3).

3. A Nitsche-based semi-implicit coupling scheme

We now propose to semi-discretize in time the non-linear coupled problem (2)–(3) using the framework of the semi-implicit coupling scheme introduced in [6]. The fluid equation $(2)_{1,2}$ are discretized in time using a projection Chorin–Temam's scheme, whereas for the structure $(2)_3$ we consider a conservative mid-point scheme. The time discretization of (3) is semi-implicit, that is, the fluid-domain geometry $(3)_1$, the viscous kinematic condition $(3)_2$ and the viscous stresses in $(3)_3$ are treated explicitly whereas the pressure stresses are implicitly treated.

The main contribution of this note concerns the discretization in space, particularly, how condition $(3)_2$ is enforced at the discrete level. Indeed, in [6], this condition is enforced in a strong fashion. In this note we consider a different point of view, in order to enhance stability. We propose to treat weakly the *viscous coupling* by using Nitsche's penalty method (see e.g. [2,3]).

We consider a finite element discretization in space. Let $V_{\mathrm{f},h}$, Q_h, and $V_{\mathrm{s},h}$ the corresponding discrete spaces for the velocity, pressure and displacement, respectively. We shall also make use of $V_{\mathrm{f},h}^0 \overset{\text{def}}{=} V_h \cap H^1_{\Sigma}(\Omega_{\mathrm{f}})$, the space of discrete test functions vanishing on Σ. In what follows, Ext_h stands for the discrete counterpart of Ext and $D_\tau X^{n+1} = (X^{n+1} - X^n)/\tau$ for the time first order backward difference.

Our projection semi-implicit coupling scheme, applied to the non-linear coupled problem (2)–(3), is then given by the following iterative procedure:

Algorithm 1. Nitsche-based semi-implicit coupling scheme

(i) Update fluid domain (mesh):

$$\boldsymbol{\eta}_{\mathrm{f},h}^{n+1} = \mathrm{Ext}_h\big(\boldsymbol{\eta}_h^n|_{\Sigma}\big), \quad \boldsymbol{w}_h^{n+1} = D_\tau \boldsymbol{\eta}_{\mathrm{f},h}^{n+1} \quad \text{in } \Omega_{\mathrm{f}}, \quad \Omega_{\mathrm{f}}^{n+1} = \big(\boldsymbol{I}_{\Omega_{\mathrm{f}}} + \boldsymbol{\eta}_{\mathrm{f},h}^{n+1}\big)(\Omega_{\mathrm{f}}).$$

(ii) Implicit part:

(a) Solid: Find $\boldsymbol{\eta}_h^{n+1} \in V_{\mathrm{s},h}$ such that

$$A_{\mathrm{s},\tau}\big(\boldsymbol{\eta}_h^{n+1}, \boldsymbol{w}_h\big) + \gamma \frac{\mu}{h} \int_{\Sigma^n} D_\tau \boldsymbol{\eta}_h^{n+1} \cdot \boldsymbol{w}_h = \gamma \frac{\mu}{h} \int_{\Sigma^n} \tilde{\boldsymbol{u}}_h^n \cdot \boldsymbol{w}_h - \int_{\Sigma^n} \boldsymbol{\sigma}\big(\tilde{\boldsymbol{u}}_h^n, p_h^{n+1}\big)\boldsymbol{n} \cdot \boldsymbol{w}_h$$

for all $\boldsymbol{w}_h \in V_{\mathrm{s},h}$.

(b) Fluid projection sub-step: Find $(\boldsymbol{u}_h^{n+1}, p_h^{n+1}) \in V_{\mathrm{f},h} \times Q_h$ such that

$$\boldsymbol{u}_h^{n+1} = D_\tau \boldsymbol{\eta}_h^{n+1} \quad \text{on } \Sigma^n,$$

$$\frac{\rho_{\mathrm{f}}}{\tau}\Bigg(\int\limits_{\Omega_{\mathrm{f}}^n}\boldsymbol{u}_h^{n+1}\cdot\boldsymbol{v}_h-\int\limits_{\Omega_{\mathrm{f}}^n}\tilde{\boldsymbol{u}}_h^{n}\cdot\boldsymbol{v}_h\Bigg)-\int\limits_{\Omega_{\mathrm{f}}^n}p_h^{n+1}\nabla\cdot\boldsymbol{v}_h+\int\limits_{\Omega_{\mathrm{f}}^n}\nabla\cdot\boldsymbol{u}_h^{n+1}q_h=0$$

for all $(\boldsymbol{v}_h,q_h)\in V_{\mathrm{f},h}^0\times Q_h$.

(iii) Fluid viscous sub-step: Find $\tilde{\boldsymbol{u}}_h^{n+1}\in V_{\mathrm{f},h}$ such that

$$\tilde{A}_{\mathrm{f},\tau}\big(\tilde{\boldsymbol{u}}_h^{n+1},\tilde{\boldsymbol{v}}_h\big)-\int\limits_{\Sigma^{n+1}}2\mu\boldsymbol{\epsilon}\big(\tilde{\boldsymbol{u}}_h^{n}\big)\boldsymbol{n}\cdot\tilde{\boldsymbol{v}}_h-\int\limits_{\Sigma^{n+1}}2\mu\big(\tilde{\boldsymbol{u}}_h^{n+1}-D_\tau\boldsymbol{\eta}_h^{n+1}\big)\cdot\boldsymbol{\epsilon}(\tilde{\boldsymbol{v}}_h)\boldsymbol{n}$$
$$+\gamma\frac{\mu}{h}\int\limits_{\Sigma^{n+1}}\big(\tilde{\boldsymbol{u}}_h^{n+1}-D_\tau\boldsymbol{\eta}_h^{n+1}\big)\cdot\tilde{\boldsymbol{v}}_h=0\quad\forall\tilde{\boldsymbol{v}}_h\in V_{\mathrm{f},h}.$$

Here, the term $A_{\mathrm{s},\tau}(\boldsymbol{\eta}_h^{n+1},\boldsymbol{w}_h)$ represents the standard fully discrete mass and stiffness contributions of the structure, while $\tilde{A}_{\mathrm{f},\tau}(\tilde{\boldsymbol{u}}_h^{n+1},\tilde{\boldsymbol{v}}_h)$ is given by

$$\begin{aligned}\tilde{A}_{\mathrm{f},\tau}\big(\tilde{\boldsymbol{u}}_h^{n+1},\tilde{\boldsymbol{v}}_h\big)\overset{\mathrm{def}}{=}\frac{\rho_{\mathrm{f}}}{\tau}\Bigg(\int\limits_{\Omega_{\mathrm{f}}^{n+1}}\tilde{\boldsymbol{u}}_h^{n+1}\cdot\tilde{\boldsymbol{v}}_h-\int\limits_{\Omega_{\mathrm{f}}^{n}}\boldsymbol{u}_h^{n+1}\cdot\tilde{\boldsymbol{v}}_h\Bigg)+\frac{\rho_{\mathrm{f}}}{2}\int\limits_{\Omega_{\mathrm{f}}^{n+1}}\big(\nabla\cdot\tilde{\boldsymbol{u}}_h^{n}\big)\tilde{\boldsymbol{u}}_h^{n+1}\cdot\tilde{\boldsymbol{v}}_h\\ -\rho_{\mathrm{f}}\int\limits_{\Omega_{\mathrm{f}}^{n+1}}\big(\nabla\cdot\boldsymbol{w}_h^{n+1}\big)\tilde{\boldsymbol{u}}_h^{n+1}\cdot\tilde{\boldsymbol{v}}_h+\rho_{\mathrm{f}}\int\limits_{\Omega_{\mathrm{f}}^{n+1}}\big(\tilde{\boldsymbol{u}}_h^{n}-\boldsymbol{w}_h^{n+1}\big)\cdot\nabla\tilde{\boldsymbol{u}}_h^{n+1}\cdot\tilde{\boldsymbol{v}}_h\\ +\int\limits_{\Omega_{\mathrm{f}}^{n+1}}2\mu\boldsymbol{\epsilon}\big(\tilde{\boldsymbol{u}}_h^{n+1}\big):\boldsymbol{\epsilon}(\tilde{\boldsymbol{v}}_h),\end{aligned}$$

and $\gamma>0$ is the Nitsche's (dimensionless) penalty parameter.

Remark 1. As usual, the projection step (ii)(b) can be reformulated as a pressure-Poisson problem as follows: find $p_h^{n+1}\in Q_h$ such that

$$\frac{\tau}{\rho_{\mathrm{f}}}\int\limits_{\Omega_{\mathrm{f}}^n}\nabla p_h^{n+1}\cdot\nabla q_h=-\int\limits_{\Omega_{\mathrm{f}}^n}\nabla\cdot\tilde{\boldsymbol{u}}_h^{n}q_h-\int\limits_{\Sigma^n}\big(D_\tau\boldsymbol{\eta}_h^{n+1}-\tilde{\boldsymbol{u}}_h^{n}\big)\cdot\boldsymbol{n}q_h\quad\forall q_h\in Q_h.$$

Note that, as in [6], steps (ii)(a) and (ii)(b) are fully coupled, whereas (ii)(a) is explicitly treated with respect to (i) and (iii). Moreover, the kinematic condition in step (ii)(b) is strongly imposed. However, now the coupling between steps (ii)(a) and (iii) is enforced weakly. Note the additional interface integrals in steps (ii)(a) and (iii), these are Nitsche's interface coupling terms.

4. Stability analysis

The next proposition (proved in [1]) states the energy based stability of the numerical solution provided by Algorithm 1, when applied to a linearized version of problem (2)–(3): Stokes/linear elasticity coupling.

Proposition 4.1. *The linearized version of Algorithm* 1 *is stable, in the energy norm, under the condition*

$$\gamma\geqslant C_{\mathrm{TI}},\quad\gamma\tau=O(h),\tag{4}$$

where C_{TI} *is a constant depending only on the local mesh geometry and polynomial order.*

Note that, Proposition 4.1 provides the unconditional stability of the Nitsche-based semi-implicit coupling scheme with respect to the added-mass effect. Indeed, contrary to (1), the stability condition (4) is independent of the fluid–solid density ratio and the geometry of the domain. The stability result is also independent of the dissipative character

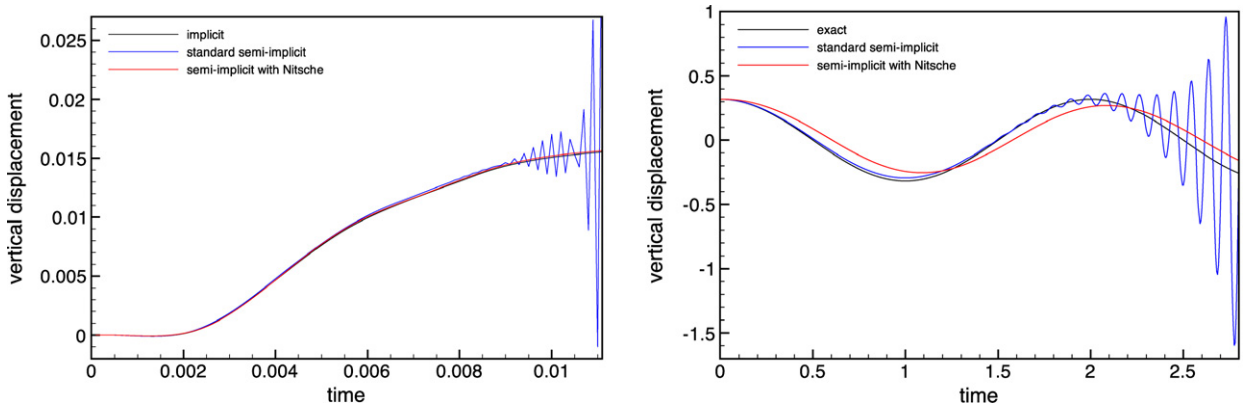

Fig. 1. Comparison of the semi-implicit and semi-implicit with Nitsche coupling schemes: interface mid-point vertical displacement.

of the time-marching scheme for the structure, which is not the case for the original semi-implicit coupling scheme (see [6, Remarks 3 and 4]). Finally, it is worth noticing that the stability condition (4) arises also in the stability analysis of the stabilized explicit coupling schemes reported in [3,4].

5. Numerical experiments

In order to illustrate the above results, two different numerical tests involving a Stokes-linear elasticity coupling in 2D have been realized. The solid equations have been discretized in time using the neutrally stable mid-point scheme. All the computations have been performed with FreeFem++ [8].

First we consider the 2D test-case used in [4, Section 6.1], with identical physical and geometrical parameters, except for the fluid viscosity $\mu = 10$ poise and the solid density $\rho_s = 1.2 \times 10^{-2}$ g/cm^3. Note that these values (high viscosity and small solid density) have been chosen so that the stability condition (1), for the original semi-implicit coupling scheme, is expected not to be satisfied. The Taylor–Hood finite element was used for the fluid and a standard $\mathbb{P}_1$-continuous discretization for the structure, with meshes size $h = 0.1$. The time step was fixed to $\tau = 10^{-4}$ s and the Nitsche penalty parameter to $\gamma = 10$.

A comparison between our Nitsche-based semi-implicit coupling (in its pressure-Darcy version), an implicit coupling and the original semi-implicit coupling [6] is given in Fig. 1 (left). Strong numerical instabilities are observed for the latter strategy. However, Algorithm 1 provides a stable numerical solution which accurately predicts the results of the fully implicit coupling.

As second numerical test we approximate an analytical solution of the Stokes-linear elasticity coupling:

$$\begin{cases} p = [-2L_2\cos(\pi t)/\pi - 2\mu\sin(\pi t)]\sin(x)\sin(y), \quad u_x = -\sin(\pi t)\cos(x)\sin(y), \\ u_y = \sin(\pi t)\sin(x)\cos(y), \quad \eta_x = \cos(\pi t)\cos(x)\sin(y)/\pi, \quad \eta_y = -\cos(\pi t)\sin(x)\cos(y)/\pi, \end{cases}$$

where L_2 stands for the second Lamé constant of solid. The fluid and solid domains are given by $\Omega_f = [0,\pi]\times[0,\pi]$ and $\Omega_s = [0,\pi]\times[\pi, 1.25\pi]$. The physical parameters are $\rho_f = 1.0$ g/cm^3, $\mu = 4$ poise, $\rho_s = 4$ g/cm^3, the elastic modulus $E = 1$ dyne/cm^2, and the Poisson's ratio $\nu = 0.3$.

The pressure-Poisson version of Algorithm 1 and the original semi-implicit coupling are tested using $\mathbb{P}_1$ finite elements. Here, $h = \pi/20$, $\tau = 5\times 10^{-3}$, and the value of γ is the same as in previous test case. Once again, strong instabilities are observed for the original semi-implicit coupling scheme, Fig. 1 (right). On the contrary, our semi-implicit coupling scheme is stable and predicts the behavior of the exact solution.

6. Conclusion

In this Note, we have proposed a semi-implicit coupling scheme whose stability properties are independent of the added-mass effect in the system (fluid–solid density ratio and geometry of the domain) and of the numerical dissipation of the solid time-discretization. As a result, it allows for conservative time-marching on the structure

without compromising stability. The main idea consists in treating the explicit part of the coupling with a Nitsche-based mortaring. Numerical tests confirm the theoretical results.

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References

[1] M. Astorino, F. Chouly, M.A. Fernández, An added-mass free semi-implicit coupling scheme in fluid–structure interaction: numerical analysis and applications, in preparation.
[2] R. Becker, P. Hansbo, R. Stenberg, A finite element method for domain decomposition with non-matching grids, M2AN Math. Model. Numer. Anal. 37 (2) (2003) 209–225.
[3] E. Burman, M.A. Fernández, Stabilized explicit coupling for fluid–structure interaction using Nitsche's method, C. R. Math. Acad. Sci. Paris 345 (8) (2007) 467–472.
[4] E. Burman, M.A. Fernández, Stabilization of explicit coupling in fluid–structure interaction involving fluid incompressibility, INRIA Research Report RR-6445, 2008, Comput. Methods Appl. Mech. Engrg., doi:10.1016/j.cma.2008.10.012, http://dx.doi.org/10.1016/j.cma.2008.10.012.
[5] P. Causin, J.-F. Gerbeau, F. Nobile, Added-mass effect in the design of partitioned algorithms for fluid–structure problems, Comput. Methods Appl. Mech. Engrg. 194 (42–44) (2005) 4506–4527.
[6] M.A. Fernández, J.F. Gerbeau, C. Grandmont, A projection semi-implicit scheme for the coupling of an elastic structure with an incompressible fluid, Int. J. Numer. Meth. Engrg. 69 (4) (2007) 794–821.
[7] L. Formaggia, A. Quarteroni, A. Veneziani (Eds.), Cardiovascular Mathematics, vol. 1, Springer-Verlag, 2009, Chapter 9: Algorithms for fluid–structure interaction problems. Modeling, Simulation and Applications.
[8] F. Hecht, O. Pironneau, A. Le Hyaric, K. Ohtsuka, FreeFem++ v. 2.11. User's Manual, University of Paris 6.
[9] A. Quaini, A. Quarteroni, A semi-implicit approach for fluid–structure interaction based on an algebraic fractional step method, Math. Models Methods Appl. Sci. 17 (6) (2007) 957–983.