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Group Theory

Staggered sheaves on partial flag varieties

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Abstract

Staggered t-structures are a class of t-structures on derived categories of equivariant coherent sheaves. In this Note, we show that the derived category of coherent sheaves on a partial flag variety, equivariant for a Borel subgroup, admits a staggered t-structure with the property that all objects in its heart have finite length. As a consequence, we obtain a basis for its equivariant K-theory consisting of simple staggered sheaves. ***To cite this article: P.N. Achar, D.S. Sage, C. R. Acad. Sci. Paris, Ser. I 347 (2009).***

Résumé

Faisceaux échelonnés sur les variétés de drapeaux partiels. Les *t-structures échelonnées* sont certaines t-structures sur des catégories dérivées des faisceaux cohérents équivariants. Nous montrons ici que la catégorie dérivée des faisceaux cohérents sur une variété de drapeaux partiels, équivariants sous un sous-groupe de Borel, admet une t-structure échelonnée telle que tout objet de son cœur soit de longueur finie. Par conséquent, l'ensemble des faisceaux échelonnés simples constitue une base pour sa K-théorie équivariante. ***Pour citer cet article : P.N. Achar, D.S. Sage, C. R. Acad. Sci. Paris, Ser. I 347 (2009).***

Let X be a variety over an algebraically closed field, and let G be a linear algebraic group acting on X with finitely many orbits. Let $\mathfrak{Coh}^G(X)$ be the category of G-equivariant coherent sheaves on X, and let $\mathcal{D}^G(X)$ denote its bounded derived category. Assume that $\mathfrak{Coh}^G(X)$ has enough locally free objects. *Staggered sheaves*, introduced in [1], are the objects in the heart of a certain t-structure on $\mathcal{D}^G(X)$, generalizing the perverse coherent t-structure [2]. The definition of this t-structure depends on the following data: (1) an *s-structure* on X (see below); (2) a choice of a Serre–Grothendieck dualizing complex $\omega_X \in \mathcal{D}^G(X)$ [4]; and (3) a *perversity*, which is an integer-valued function on the set of G-orbits, subject to certain constraints. When the perversity is "strictly monotone and comonotone," the category of staggered sheaves is particularly nice: every object has finite length, and every simple object arises by applying an intermediate-extension ("IC") functor to an irreducible vector bundle on a G-orbit.

An s-structure on X is a certain kind of increasing filtration of $\mathfrak{Coh}^G(X)$ by Serre subcategories $\{\mathfrak{Coh}^G(X)_{\leqslant n}\}_{n\in\mathbb{Z}}$, subject to various axioms (see Section 1). Philosophically, an s-structure plays a role analogous to that of weight

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filtrations in the theory of mixed constructible sheaves. Given an s-structure, let $\mathfrak{Coh}^G(X)_{\geqslant n} \subset \mathfrak{Coh}^G(X)$ denote the right-orthogonal to $\mathfrak{Coh}^G(X)_{\leqslant n-1}$. The *staggered codimension* of an orbit closure $i_C : \overline{C} \to X$, denoted $\operatorname{scod} \overline{C}$, is defined to be $\operatorname{codim} \overline{C} + n$, where n is the unique integer such that $i_C^! \omega_X|_C \in \mathcal{D}^G(C)$ is a shift of an object in $\mathfrak{Coh}^G(C)_{\leqslant n} \cap \mathfrak{Coh}^G(C)_{\geqslant n}$. (Staggered codimensions are, in general, sensitive to the choice of ω_X. In this paper, whenever X is smooth, ω_X will denote the canonical bundle of X.) Strictly monotone and comonotone perversities exist if and only if $\operatorname{scod} \overline{D} \geqslant \operatorname{scod} \overline{C} + 2$ whenever $\overline{D} \subsetneq \overline{C}$. (If this holds, one may take $\lfloor \frac{1}{2} \operatorname{scod} \overline{C} \rfloor$ as the perversity.)

The goal of this Note is to establish the existence of strictly monotone and comonotone perversities for suitable s-structures on partial flag varieties. As a consequence, we obtain a basis for the equivariant K-theory $K^B(G/P)$ consisting of simple staggered sheaves.

1. A gluing theorem for s-structures

If X happens to be a G-homogeneous space (i.e., of the form $X = G/H$ for some closed subgroup $H \subset G$), the axioms for an s-structure are equivalent to the following: (1) If $\mathcal{F} \in \mathfrak{Coh}^G(X)_{\leqslant n}$ and $\mathcal{G} \in \mathfrak{Coh}^G(X)_{\leqslant m}$, then $\mathcal{F} \otimes \mathcal{G} \in \mathfrak{Coh}^G(X)_{\leqslant n+m}$. (2) Each $\mathfrak{Coh}^G(X)_{\geqslant n}$ is a Serre subcategory of $\mathfrak{Coh}^G(X)$. (3) If $\mathcal{F} \in \mathfrak{Coh}^G(X)_{\geqslant n}$ and $\mathcal{G} \in \mathfrak{Coh}^G(X)_{\geqslant m}$, then $\mathcal{F} \otimes \mathcal{G} \in \mathfrak{Coh}^G(X)_{\geqslant n+m}$. If X consists of many G-orbits, the last two axioms must be replaced by a collection of "local" conditions on all G-stable closed subschemes (see [1] for details), and specifying an s-structure on X directly can become quite arduous. The following "gluing theorem" lets us instead specify an s-structure on X by specifying one on each G-orbit:

Theorem 1.1. *For each orbit $C \subset X$, let $\mathcal{I}_C \subset \mathcal{O}_X$ denote the ideal sheaf corresponding to the closed subscheme $i_C : \overline{C} \hookrightarrow X$. Suppose each orbit C is endowed with an s-structure, and that $i_C^* \mathcal{I}_C|_C \in \mathfrak{Coh}^G(C)_{\leqslant -1}$. There is a unique s-structure on X whose restriction to each orbit is the given s-structure.*

Proof. This statement is nearly identical to [1, Theorem 10.2]. In that result, the requirement that $i_C^* \mathcal{I}_C|_C \in \mathfrak{Coh}^G(C)_{\leqslant -1}$ is replaced by the following two assumptions: (F1) For each orbit C, $i_C^* \mathcal{I}_C|_C \in \mathfrak{Coh}^G(C)_{\leqslant 0}$. (F2) Each $\mathcal{F} \in \mathfrak{Coh}^G(C)_{\leqslant w}$ admits an extension $\mathcal{F}_1 \in \mathfrak{Coh}^G(\overline{C})$ whose restriction to any smaller orbit $C' \subset \overline{C}$ is in $\mathfrak{Coh}^G(C')_{\leqslant w}$. Condition (F1) is trivially implied by the stronger assumption that $i_C^* \mathcal{I}_C|_C \in \mathfrak{Coh}^G(C)_{\leqslant -1}$. It suffices, then, to show that (F2) is implied by it as well. Given $\mathcal{F} \in \mathfrak{Coh}^G(C)_{\leqslant w}$, let $\mathcal{G} \in \mathfrak{Coh}^G(\overline{C})$ be some sheaf such that $\mathcal{G}|_C \simeq \mathcal{F}$. Let $C' \subset \overline{C} \smallsetminus C$ be a maximal orbit (with respect to the closure partial order) such that $i_{C'}^* \mathcal{G}|_{C'} \notin \mathfrak{Coh}^G(C')_{\leqslant w}$. (If there is no such C', then $\mathcal{G}$ is the desired extension of $\mathcal{F}$, and there is nothing to prove.) Let $v \in \mathbb{Z}$ be such that $i_{C'}^* \mathcal{G}|_{C'} \in \mathfrak{Coh}^G(C')_{\leqslant v}$. By assumption, we have $v > w$. Let $\mathcal{G}' = \mathcal{G} \otimes \mathcal{I}_{C'}^{\otimes v-w}$. Since $\mathcal{I}_{C'}|_{X \smallsetminus \overline{C'}}$ is isomorphic to the structure sheaf of $X \setminus \overline{C'}$, we see that $\mathcal{G}'|_{\overline{C} \setminus \overline{C'}} \simeq \mathcal{G}|_{\overline{C} \setminus \overline{C'}}$. On the other hand, according to axiom (1) above, the fact that $i_{C'}^* \mathcal{I}_{C'}|_{C'} \in \mathfrak{Coh}^G(C')_{\leqslant -1}$ implies that $i_{C'}^* \mathcal{G}'|_{C'} \simeq i_{C'}^* \mathcal{G}|_{C'} \otimes (i_{C'}^* \mathcal{I}_{C'}|_{C'})^{\otimes v-w} \in \mathfrak{Coh}^G(C')_{\leqslant w}$. Thus, $\mathcal{G}'$ is a new extension of $\mathcal{F}$ such that the number of orbits in $\overline{C} \setminus C$ where (F2) fails is fewer than for $\mathcal{G}$. Since the total number of orbits is finite, this construction can be repeated until an extension $\mathcal{F}_1$ satisfying (F2) is obtained. □

2. s-structures on Bruhat cells

Let G be a reductive algebraic group over an algebraically closed field, and let $T \subset B \subset P$ be a maximal torus, a Borel subgroup, and a parabolic subgroup, respectively, and let L be the Levi subgroup of P containing T. The Lie algebras of G, P, and B are denoted $\mathfrak{g}$, $\mathfrak{b}$, and $\mathfrak{p}$.

Let W be the Weyl group of G (with respect to T), and let Φ be its root system. Let Φ^+ be the set of positive roots corresponding to B. Let $W_L \subset W$ and $\Phi_L \subset \Phi$ be the Weyl group and root system of L, and let $\Phi_P = \Phi_L \cup \Phi^+$. Let $W^L \subset W$ be the set of minimal-length right coset representatives for W_L. For each $w \in W^L$, we fix once and for all a representative in G, also denoted w. We put $B_w = wBw^{-1}$ and $P_w = wPw^{-1}$, and we write $\mathfrak{b}_w$ and $\mathfrak{p}_w$ for their Lie algebras. Let X_w° denote the Bruhat cell BwP/P, let X_w denote its closure (a Schubert variety), and let $i_w : X_w \to G/P$ be the inclusion. Let $\mathcal{I}_w$ denote the ideal sheaf on G/P corresponding to X_w.

Let Λ denote the weight lattice of T, and let $\rho = \frac{1}{2} \sum \Phi^+$. (For a set $\Psi \subset \Phi$, we write "$\sum \Psi$" for $\sum_{\alpha \in \Psi} \alpha$.) For any $w \in W$, we define various subsets of Φ^+ and elements of Λ as follows:

$$\Pi(w) = \Phi^+ \cap w(\Phi^+), \quad \pi(w) = \sum \Pi(w), \quad \Pi_L(w) = \Phi^+ \cap w(\Phi^+ \setminus \Phi_L), \quad \pi_L(w) = \sum \Pi_L(w),$$
$$\Theta(w) = \Phi^+ \cap w(\Phi^-), \quad \theta(w) = \sum \Theta(w), \quad \Theta_L(w) = \Phi^+ \cap w(\Phi^- \setminus \Phi_L), \quad \theta_L(w) = \sum \Theta_L(w).$$

Let $\langle \cdot,\cdot \rangle$ denote a W-invariant positive-definite bilinear form on Λ such that $\langle 2\rho, \lambda \rangle \in \mathbb{Z}$ for all $\lambda \in \Lambda$. Now, for $w \in W^L$, the category $\mathfrak{Coh}^B(X_w^\circ)$ is equivalent to the category $\mathfrak{Rep}(B_w \cap B)$ of representations of the isotropy group $P_w \cap B = B_w \cap B$. We define an s-structure on X_w° via this equivalence as follows: $\mathfrak{Coh}^B(X_w^\circ)_{\leqslant n} \simeq \{V \in \mathfrak{Rep}(B_w \cap B) \mid \langle \lambda, -2w\rho \rangle \leqslant n$ for all weights λ occurring in $V\}$. It follows that $\mathfrak{Coh}^B(X_w^\circ)_{\geqslant n} \simeq \{V \in \mathfrak{Rep}(B_w \cap B) \mid \langle \lambda, -2w\rho \rangle \geqslant n$ for all weights λ occurring in $V\}$. Below, we regard $\omega_{X_w^\circ}$ and $i_w^* \mathcal{I}_w$ as objects of $\mathfrak{Rep}(B_w \cap B)$:

Lemma 2.1. *The T-weight on $\omega_{X_w^\circ}$ is $-\theta_L(w)$, and the set of T-weights on $i_w^* \mathcal{I}_w$ is $\Pi_L(w)$.*

Proof. For any weight $\psi \in \Lambda$, let V_ψ denote the 1-dimensional T-representation of weight ψ, and for any subset $\Psi \subset \Lambda$, let $V(\Psi) = \bigoplus_{\psi \in \Psi} V_\psi$. The tangent space to G/P at the point wP/P is $\mathfrak{g}/\mathfrak{p}_w$. As a T-representation, this is isomorphic to $V(w(\Phi \setminus \Phi_P)) \simeq V(w(\Phi^- \setminus \Phi_L))$. The tangent space to the B-orbit through that point is the subspace $\mathfrak{b}/\mathfrak{b} \cap \mathfrak{p}_w \simeq V(\Phi^+ \cap w(\Phi^- \setminus \Phi_L)) \simeq V(\Theta_L(w))$, and the normal space is the quotient $\mathfrak{g}/(\mathfrak{b} + \mathfrak{p}_w) \simeq V(\Phi^- \cap w(\Phi^- \setminus \Phi_L)) \simeq V(-\Pi_L(w))$. Since the canonical bundle $\omega_{X_w^\circ}$ is the top exterior power of the cotangent bundle, and $i_w^* \mathcal{I}_w$ is the conormal bundle, the result follows. □

Since $\langle \alpha, -2w\rho \rangle = \langle w^{-1}\alpha, -2\rho \rangle < 0$ for all $\alpha \in \Pi_L(w)$, we see from Lemma 2.1 that $i_w^* \mathcal{I}_w|_{X_w^\circ} \in \mathfrak{Coh}^B(X_w^\circ)_{\leqslant -1}$, and then Theorem 1.1 gives us an s-structure on G/P. Separately, Lemma 2.1 also tells us that $\operatorname{scod} X_w = \operatorname{codim} X_w + \langle -\theta_L(w), -2w\rho \rangle$. Recall that because $w \in W^L$, we have $\operatorname{codim} X_w = |\Phi^+ \setminus \Phi_L| - \ell(w)$ and $\theta_L(w) = \theta(w)$. (See [3, Chap. 2].) Moreover, $\langle -\theta(w), -2w\rho \rangle = \langle w^{-1}\theta(w), 2\rho \rangle = \langle -\theta(w^{-1}), 2\rho \rangle$. Combining these observations gives us the following theorem:

Theorem 2.2. *There is a unique s-structure on G/P compatible with those on the various X_w°. For $w \in W^L$, the staggered codimension of X_w, with respect to $\omega_{G/P}$, is given by* $\operatorname{scod} X_w = |\Phi^+ \setminus \Phi_L| - \ell(w) - \langle \theta(w^{-1}), 2\rho \rangle$.

3. Main result

Theorem 3.1. *With respect to the s-structure and dualizing complex of Theorem 2.2, $\mathcal{D}^B(G/P)$ admits a strictly monotone and comonotone perversity function. For any such perversity, all objects in the heart of the corresponding staggered t-structure have finite length. In particular, the set of simple staggered sheaves $\{\mathcal{IC}(X_w, \mathcal{O}_{X_w^\circ}(\lambda))\}$, where $\lambda \in \Lambda$ and $w \in W^L$, forms a basis for $K^B(G/P)$.*

By the remarks in the introduction, this theorem follows from Proposition 3.6 below. Throughout this section, the notation "$u \cdot v$" for the product of $u, v \in W$ will be used to indicate that $\ell(uv) = \ell(u) + \ell(v)$. Note that if s is a simple reflection corresponding to a simple root α, $\ell(sw) > \ell(w)$ if and only if $\alpha \in \Pi(w)$.

Lemma 3.2. *Let s be a simple reflection, and let α be the corresponding simple root. If $\ell(sw) > \ell(w)$, then $\pi(sw) = s\pi(w) + \alpha$ and $\theta(sw) = s\theta(w) + \alpha$.*

Proof. Since $\Pi(s) = \Phi^+ \setminus \{\alpha\}$, it is easy to see that if $\alpha \in \Pi(w)$, then $\Pi(sw) = s(\Pi(w) \setminus \{\alpha\})$, and hence that $\pi(sw) = s(\pi(w) - \alpha) = s\pi(w) + \alpha$. The proof of the second formula is similar. □

Lemma 3.3. *For any $w \in W$, we have $\langle \pi(w), \theta(w) \rangle = 0$.*

Proof. Note that $\Pi(w) \cup \Theta(w) = \Phi^+$. Also, since $-\Theta(w) = \Phi^- \cap w(\Phi^+)$, we have $\Pi(w) \cup -\Theta(w) = w(\Phi^+)$. Thus, $\pi(w) + \theta(w) = 2\rho$, and $\pi(w) - \theta(w) = w(2\rho)$. Then $4\langle \pi(w), \theta(w) \rangle = \langle \pi(w) + \theta(w), \pi(w) + \theta(w) \rangle - \langle \pi(w) - \theta(w), \pi(w) - \theta(w) \rangle = \langle 2\rho, 2\rho \rangle - \langle w(2\rho), w(2\rho) \rangle = 0$. □

Proposition 3.4. *If $\alpha \in \Pi(w)$ is a simple root, then $\langle \alpha, \theta(w) \rangle \leqslant 0$.*

Proof. Let s denote the simple reflection corresponding to α, and let $\gamma \in \Theta(w)$. Of course, if $s\gamma = \gamma$, then $\langle \alpha, \gamma \rangle = 0$. If $s\gamma \neq \gamma$ but $s\gamma \in \Theta(w)$ as well, then $\langle \alpha, \gamma + s\gamma \rangle = 0$. It remains to show that if $s\gamma \notin \Theta(w)$, then $\langle \alpha, \gamma \rangle \leqslant 0$. Suppose instead that $\langle \alpha, \gamma \rangle > 0$, and consider $\langle s\gamma, w(2\rho) \rangle = \langle \gamma, w(2\rho) \rangle - \langle \alpha^\vee, \gamma \rangle \langle \alpha, w(2\rho) \rangle$. Since $\gamma \in w(\Phi^-)$ and $\alpha \in w(\Phi^+)$ by assumption, we have $\langle \gamma, w(2\rho) \rangle < 0$ and $\langle \alpha, w(2\rho) \rangle > 0$. Also, $\langle \alpha^\vee, \gamma \rangle > 0$ since $\langle \alpha, \gamma \rangle > 0$, so the calculation above shows that $\langle s\gamma, w(2\rho) \rangle < 0$, and hence that $s\gamma \in w(\Phi^-)$. But clearly $s\gamma \in \Phi^+$ as well (since $\gamma \neq \alpha$), so we find that $s\gamma \in \Theta(w)$, a contradiction. □

Proposition 3.5. *Let s be a simple reflection, corresponding to the simple root α. Let $v, w \in W$ be such that $\ell(vsw) = \ell(v) + 1 + \ell(w)$. Then* $\langle \pi(vw), 2\rho \rangle - \langle \pi(vsw), 2\rho \rangle = (1 - \langle \alpha^\vee, \theta(v^{-1}) \rangle) \langle w^{-1}\alpha, 2\rho \rangle > 0$.

Proof. We proceed by induction on $\ell(v)$. First, suppose that $v = 1$. Note that $\theta(v^{-1}) = 0$. Since $2\rho = \pi(w) + \theta(w)$, Lemma 3.3 implies that $\langle \pi(w), 2\rho \rangle = \langle \pi(w), \pi(w) \rangle$. Similarly,

$$\begin{aligned}\langle \pi(sw), 2\rho \rangle &= \langle \pi(sw), \pi(sw) \rangle = \langle s\pi(w) + \alpha, s\pi(w) + \alpha \rangle \\ &= \langle s\pi(w), s\pi(w) \rangle + 2\langle s\pi(w), \alpha \rangle + \langle \alpha, \alpha \rangle = \langle \pi(w), \pi(w) \rangle + 2\langle \pi(w), s\alpha \rangle + \langle 2\rho, \alpha \rangle \\ &= \langle \pi(w), 2\rho \rangle - 2\langle \pi(w), \alpha \rangle + \langle \pi(w) + \theta(w), \alpha \rangle = \langle \pi(w), 2\rho \rangle - \langle \pi(w) - \theta(w), \alpha \rangle.\end{aligned}$$

It is easy to see that $\pi(w) - \theta(w) = w(2\rho)$, whence it follows that $\langle \pi(w), 2\rho \rangle - \langle \pi(sw), 2\rho \rangle = \langle w^{-1}\alpha, 2\rho \rangle$. Finally, the fact that $\ell(sw) > \ell(w)$ implies that $w^{-1}\alpha \in \Phi^+$, so $\langle w^{-1}\alpha, 2\rho \rangle > 0$.

Now, suppose $\ell(v) \geqslant 1$, and write $v = t \cdot x$, where t is a simple reflection with simple root β. Using the special case of the proposition that is already established, we find $\langle \pi(xsw), 2\rho \rangle - \langle \pi(txsw), 2\rho \rangle = \langle w^{-1}sx^{-1}\beta, 2\rho \rangle$ and $\langle \pi(xw), 2\rho \rangle - \langle \pi(txw), 2\rho \rangle = \langle w^{-1}x^{-1}\beta, 2\rho \rangle$. Using the fact that $sx^{-1}\beta = x^{-1}\beta - \langle \alpha^\vee, x^{-1}\beta \rangle \alpha$, we find

$$\begin{aligned}\langle \pi(txw), 2\rho \rangle - \langle \pi(txsw), 2\rho \rangle &= \big(\langle \pi(xw), 2\rho \rangle - \langle \pi(xsw), 2\rho \rangle\big) + \big(\langle w^{-1}sx^{-1}\beta, 2\rho \rangle - \langle w^{-1}x^{-1}\beta, 2\rho \rangle\big) \\ &= \big(1 - \langle \alpha^\vee, \theta(x^{-1}) \rangle\big)\langle w^{-1}\alpha, 2\rho \rangle - \langle \alpha^\vee, x^{-1}\beta \rangle \langle w^{-1}\alpha, 2\rho \rangle = \big(1 - \langle \alpha^\vee, \theta(x^{-1}) + x^{-1}\beta \rangle\big)\langle w^{-1}\alpha, 2\rho \rangle.\end{aligned}$$

An argument similar to that of Lemma 3.2 shows that $\theta(x^{-1}) + x^{-1}\beta = \theta(x^{-1}t) = \theta(v^{-1})$, so the desired formula is established. Since $\ell(vs) > \ell(v)$, we also have $\ell(sv^{-1}) > \ell(v^{-1})$, and then Proposition 3.4 tells us that $\langle \alpha^\vee, \theta(v^{-1}) \rangle \leqslant 0$. Thus, $\langle \pi(vw), 2\rho \rangle - \langle \pi(vsw), 2\rho \rangle > 0$. □

The preceding proposition implies that for any $v, w \in W$ with $v < w$ in the Bruhat order, $\langle \theta(v), 2\rho \rangle - \langle \theta(w), 2\rho \rangle < 0$. When $v, w \in W_L$, we deduce from Theorem 2.2 the following result, and thus establish Theorem 3.1:

Proposition 3.6. *If $X_v \subset \overline{X_w}$, then* $\operatorname{scod} X_v - \operatorname{scod} X_w \geqslant 2$.

Remark 3.7. Here is a sketch of an alternate, geometric proof of this proposition, following a suggestion of the referee. It suffices to show that $v < w$ implies $\langle \theta(v), 2\rho \rangle - \langle \theta(w), 2\rho \rangle < 0$. That is equivalent to showing that the map $w \mapsto \langle w(2\rho), 2\rho \rangle$ is strictly decreasing, and then in turn to showing that the angle between the vectors $w(2\rho)$ and 2ρ strictly increases as a function of w. That can be deduced from the fact that if $v < w$ and $w = tv$ for some reflection t, then 2ρ and $v(2\rho)$ both lie on the same side of the reflecting hyperplane for t, and $w(2\rho)$ lies on the other.

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