Partial Differential Equations

Global solutions for the gravity water waves equation in dimension 3

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Abstract

We show existence of global solutions for the gravity water waves equation in dimension 3, in the case of small data. The proof combines energy estimates, which yield control of L^2 related norms, with dispersive estimates, which give decay in L^∞. To obtain these dispersive estimates, we use an analysis in Fourier space; the study of space and time resonances is then the crucial point. *To cite this article: P. Germain et al., C. R. Acad. Sci. Paris, Ser. I 347 (2009).*
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Résumé

Solutions globales pour les équations des ondes de surface en dimension 3. Nous montrons l'existence de solutions globales pour les équations des ondes de surface en dimension 3 avec gravité seulement, dans le cas de petites données initiales. La preuve combine des estimations d'énergie, qui donnent le contrôle de normes de type L^2, avec des estimations dispersives, qui donnent la décroissance dans L^∞. Ces estimations dispersives sont obtenues grâce à une analyse dans l'espace de Fourier, qui repose sur l'étude des résonances en temps et en espace. *Pour citer cet article : P. Germain et al., C. R. Acad. Sci. Paris, Ser. I 347 (2009).*
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Version française abrégée

Nous esquissons dans cette Note la preuve de l'existence globale pour l'équation des ondes de surface avec gravité seulement en dimension d'espace 3, et à données petites. Un fluide non visqueux, incompressible et irrotationel, occupe un domaine d'extension infinie borné par une surface libre. La surface libre, asymptotiquement plate, est paramétrisée par $S = \{(x, h(x, t)), \ x \in \mathbb{R}^2\}$. La vélocité dérive du potentiel $\tilde{\psi}$; en prenant sa trace sur la surface libre $\psi(x, t) = \tilde{\psi}(x, h(x, t), t)$ le système s'écrit

$$
\begin{cases}
\partial_t h = G(h)\psi, \\
\partial_t \psi = -h - \frac{1}{2}|\nabla\psi|^2 + \frac{1}{2(1+|\nabla h|^2)}(G(h)\psi + \nabla h \cdot \nabla\psi)^2, \\
(h, \psi)(t=0) = (h_0, \psi_0),
\end{cases}
\tag{WW}
$$

où $G(h) = \sqrt{1 + |\nabla h|^2}\,\mathcal{N}$, avec $\mathcal{N}$ l'opérateur de Dirichlet–Neumann associé au domaine borné par S.

Avant d'énoncer le théorème, il nous faut encore introduire la fonction complexe $u = h + i \Lambda^{1/2}\psi$, son profil $f = e^{it\Lambda^{1/2}}u$, et la notation $\langle a \rangle = \sqrt{1 + a^2}$.

Théorème. *Soit N un entier assez grand. Il existe $\epsilon > 0$ tel que si la donnée initial satisfait*

$$\|u_0\|_{H^{2N}} + \big\|\langle x \rangle u_0\big\|_{L^2} < \epsilon,$$

il existe une unique solution globale de (WW) *telle que*

$$\|u\|_X \overset{\text{def}}{=} \sup_t \langle t \rangle \|u\|_{W^{3,\infty}} + \langle t \rangle^{-\delta} \|u\|_{H^{2N}} + \langle t \rangle^{-\delta} \|xf\|_{L^2} \lesssim \epsilon,$$

où l'on note $W^{s,\infty}$ pour l'espace de Sobolev basé sur L^∞, H^s pour l'espace de Sobolev basésur L^2 et où $\delta > 0$ est une (petite) constante.

Esquissons la preuve. Elle se divise en deux parties.

Estimations d'énergie : contrôle de $\sup_t \langle t \rangle^{-\delta}\|u\|_{H^{2N}}$. En utilisant le formalisme intrinsèque de Shatah et Zeng [13], on montre que

$$\big\|u(t)\big\|_{H^{2N}} \leqslant \int_0^t C\big(\|u\|_X\big)\|u\|_{W^{3,\infty}}\|u\|_{H^{2N}}\,\mathrm{d}s + \big\|u(0)\big\|_{H^{2N}} \lesssim \int_0^t \frac{\|u\|_X}{\langle s \rangle}\|u\|_{H^{2N}}\,\mathrm{d}s + \big\|u(0)\big\|_{H^{2N}}$$

ce qui donne le résultat souhaité.

Décroissance et estimation à poids : contrôle de $\sup_t \langle t \rangle\|u\|_{W^{3,\infty}} + \langle t \rangle^{-\delta}\|xf\|_{H^N}$. Ces estimations sont obtenues en utilisant la notion de résonance en temps et en espace, introduite dans le cadre de l'équation de Schrödinger par Germain, Masmoudi et Shatah [8]. L'idée consiste à écrire la formule de Duhamel pour le profil f dans l'espace de Fourier. On voit alors apparaître un problème de phase stationnaire, dont l'analyse est la clé de la décroissance dans L^∞ de u, ainsi que du contrôle de f dans l'espace à poids.

1. Introduction

We consider the three-dimensional irrotational water wave problem in the presence of gravity. We assume that the domain of the fluid is given by $\Omega = \{(x, y) = (x_1, x_2, y) \in \mathbb{R}^3,\ y \leqslant h(x, t)\}$, where the graph of the function h, i.e., $S = \{(x, h(x, t)),\ x \in \mathbb{R}^2\}$ represents the free boundary of the fluid which moves by the normal velocity of the fluid. The function h is assumed to be asymptotically flat. In this setting the Euler equation and the boundary conditions are given by

$$\begin{cases} \boldsymbol{D}_t v \overset{\text{def}}{=} \partial_t v + \nabla_v v = -\nabla p - ge_3, & (x, y) \in \Omega, \\ \nabla \cdot v = 0, & \Omega, \end{cases} \tag{E}$$

$$\begin{cases} \partial_t h + v \cdot \nabla_{(x,y)}(h - y) = 0, & x \in \mathbb{R}^2, \\ p|_S = 0, \end{cases} \tag{BC}$$

where g is the gravity constant. Since the flow is assumed to be irrotational, the Euler equation can be reduced to an equation on the boundary and thus with the boundary condition we end up with a system defined on S. This is achieved by introducing the potential $\tilde{\psi}$ where $v = \nabla\tilde{\psi}$. Denoting the trace of the potential on the free boundary by $\psi(x, t) = \tilde{\psi}(x, h(x, t), t)$, the system of equations for ψ and h are [14]

$$\begin{cases} \partial_t h = G(h)\psi, \\ \partial_t \psi = -h - \frac{1}{2}|\nabla\psi|^2 + \frac{1}{2(1+|\nabla h|^2)}(G(h)\psi + \nabla h \cdot \nabla\psi)^2, \\ (h, \psi)(t = 0) = (h_0, \psi_0), \end{cases} \tag{WW}$$

where $G(h) = \sqrt{1 + |\nabla h|^2}\,\mathcal{N}$ and $\mathcal{N}$ is the Dirichlet–Neumann operator associated with Ω.

There is an extensive body of literature on local well posedness and energy estimates for (E) in Sobolev spaces starting with the work of Nalimov [12] for small data and the work of S.J. Wu [15,16] for arbitrary data; we also mention [1–7,9,10,13,18,19]. Recently S.J. Wu proved almost global existence for small data for (WW) in two space dimensions (one-dimensional surface) [17]. Here we will show that in three dimensions we have global solutions. Our proof is based on Fourier space analysis of the Space Time resonant set introduced in [8]. To state our result we introduce the Calderon operator $\Lambda = |D|$, the complex function $u = h + i\Lambda^{1/2}\psi$, its profile $f = e^{it\Lambda^{1/2}}u$, and the notation $\langle a \rangle = \sqrt{1 + a^2}$, then

Theorem. *Let N be a large integer. There exists an $\epsilon > 0$ such that if the data satisfies*

$$\|u_0\|_{H^{2N}} + \|\langle x \rangle^{3/2} u_0\|_{H^N} < \epsilon,$$

then there exists a unique global solution to (WW) *such that*

$$\|u\|_X \stackrel{\text{def}}{=} \sup_t \langle t \rangle \|u\|_{W^{3,\infty}} + \langle t \rangle^{-\delta} \|u\|_{H^{2N}} + \langle t \rangle^{-\delta} \|xf\|_{H^N} \lesssim \epsilon,$$

where H^s and $W^{s,\infty}$ stand respectively for the L^2 and L^∞ based Sobolev spaces, and where $\delta > 0$ is a small constant.

2. Local existence and energy estimates

The conserved energy for (WW) is given by

$$\int_S \frac{1}{2}(\psi \mathcal{N} \psi + h^2) \, dS = constant,$$

which is equivalent to $\int |u|^2 \, dx$ being bounded. To obtain higher order energy estimates we follow the method developed in [13]. The idea is the following: since the system is reduced to the boundary we apply the surface Laplacian Δ_S to (BC) instead of partial derivatives. The mean curvature κ is given by $\kappa = (1 + |\nabla h|^2)^{1/2} \Delta_S h$, thus this leads to an equation for $D_t \kappa$ in terms of the velocity v (Eq. (3.7) in [13]). Applying D_t again we obtain (Eq. (3.17) in [13])

$$D_t^2 \kappa = -N \cdot \Delta_S D_t v + \tilde{R},$$

where N is the unit normal to the surface S and $\tilde{R} = O(\sum_0^3 |D^j \psi|^2 + \kappa^2)$ in norm. Using the Euler equation to substitute for $D_t v$ we obtain

$$D_t^2 \kappa + \mathcal{N}^{1/2} A \mathcal{N}^{1/2} \kappa + g\mathcal{N}\kappa = R,$$

where A is a zeroth order operator, $A = O(\sum_0^3 |D^j \psi| + |h| + |Dh|)$ in operator norm, and $R = O(\sum_0^3 |D^j \psi|^2 + |\kappa|^2 + |D^{1/2}\kappa|^2)$ in norm. Note that in terms of regularity $\kappa \sim \Delta_S h$, $D_t \kappa \sim D^3 \psi$, and $\|u\|_{H^s} \sim \|D_t \kappa\|_{H^{s-5/2}} + \|\kappa\|_{H^{s-2}} + \|h\|_{L^2}$. Thus for sufficiently large N standard energy methods imply

$$\|u(t)\|_{H^{2N}} \leqslant \int_0^t C(\|u\|_X) \|u\|_{W^{3,\infty}} \|u\|_{H^{2N}} \, ds + \|u(0)\|_{H^{2N}}$$

$$\lesssim C(\|u\|_X) \|u\|_X \int_0^t \frac{1}{\langle s \rangle} \|u\|_{H^{2N}} \, ds + \|u(0)\|_{H^{2N}},$$

which implies the control of the second term in the definition of $\|u\|_X$.

3. The decay and weighted estimates

Expanding (WW) in powers of h and ψ, setting $g = 1$, and keeping track of quadratic and cubic terms we obtain

$$\begin{cases} \partial_t h = \Lambda \psi - \nabla \cdot (h \nabla \psi) - \Lambda(h \Lambda \psi) - \frac{1}{2}(\Lambda(h^2 \Lambda^2 \psi) + \Lambda^2(h^2 \Lambda \psi) - 2\Lambda(h\Lambda(h\Lambda\psi))) + R_1, \\ \partial_t \psi = -h - \frac{1}{2}|\nabla \psi|^2 + \frac{1}{2}|\Lambda \psi|^2 + \Lambda \psi(h\Lambda^2\psi - \Lambda(h\Lambda\psi)) + R_2, \end{cases} \tag{1}$$

where R_1 and R_2 are of order 4. We refer to the book of Sulem and Sulem [14] for the above expansion. In what follows we will ignore derivative loss in the decay estimate since any such loss can be controlled by energy. Moreover we will drop all quartic and higher terms since these terms have strong time decay which make all the estimates immediate.

3.1. Writing the equation in Fourier space

Writing Duhamel formula for $f = e^{it\Lambda^{1/2}}(h + i\Lambda^{1/2}\psi)$ in Fourier space yields

$$\hat{f}(t, \xi) = \hat{f}_0 + \sum_{\pm, \pm} \sum_{i=1,2} c_{i,\pm,\pm} \iint_0^t e^{is\phi_{\pm,\pm}} m_i(\xi, \eta) \widehat{f_{\mp}}(s, \eta) \widehat{f_{\mp}}(s, \xi - \eta) \, d\eta \, ds$$

$$+ \sum_{\pm, \pm, \pm} \sum_{i=3,4} c_{i,\pm,\pm,\pm} \iiint_0^t e^{is\phi_{\pm,\pm,\pm}} m_i(\xi, \eta, \sigma) \widehat{f_{\mp}}(s, \eta) \widehat{f_{\mp}}(s, \sigma) \widehat{f_{\mp}}(s, \xi - \eta - \sigma) \, d\eta \, d\sigma \, ds, \qquad (2)$$

where $c_{i,\pm,\pm}$ and $c_{i,\pm,\pm,\pm}$ are complex coefficients, $f_+ \stackrel{\text{def}}{=} f$ and $f_- \stackrel{\text{def}}{=} \bar{f}$. The phases are given by

$$\phi_{\pm,\pm}(\xi, \eta) = |\xi|^{1/2} \pm |\eta|^{1/2} \pm |\xi - \eta|^{1/2},$$
$$\phi_{\pm,\pm,\pm}(\xi, \eta, \sigma) = |\xi|^{1/2} \pm |\eta|^{1/2} \pm |\sigma|^{1/2} \pm |\xi - \eta - \sigma|^{1/2},$$

and the multilinear symbols are defined by

$$m_1(\xi, \eta) = \frac{1}{|\eta|^{1/2}}\big(\xi \cdot \eta - |\xi||\eta|\big),$$

$$m_2(\xi, \eta) = \frac{1}{2} \frac{|\xi|^{1/2}}{|\eta|^{1/2}|\xi - \eta|^{1/2}}\big(\eta \cdot (\xi - \eta) + |\eta||\xi - \eta|\big),$$

$$m_3(\xi, \eta, \sigma) = -\frac{1}{2}|\xi|\big(|\xi - \eta - \sigma|^{3/2} + |\xi||\xi - \eta - \sigma|^{1/2} - 2|\xi - \eta||\xi - \eta - \sigma|^{1/2}\big),$$

$$m_4(\xi, \eta, \sigma) = -|\xi|^{1/2}\big(-|\eta|^{1/2}|\xi - \eta - \sigma|^{3/2} + |\eta|^{1/2}|\xi - \eta||\xi - \eta - \sigma|^{1/2}\big).$$

3.2. Space and time resonances

The key notion to understand the long time behavior of (2) is that of space–time resonance which we now proceed to explain (see [8] for an application of this idea in the context of nonlinear Schrödinger equations).

• *Time resonances* correspond to the classical notion of resonances, which is well known from ODE theory. Time resonances between different frequencies correspond to the vanishing of the phase, $\phi_{\pm,\pm}$ or $\phi_{\pm,\pm,\pm}$; in other words it corresponds to stationary phase in s.
• *Space resonances* only occur in a dispersive PDE setting. The physical phenomenon underlying this notion is the following: wave packets corresponding to different frequencies may have the same group velocity; if they do, these wave packets are called space resonant and they might interact since they are located in the same area in space. If they are not space resonant then their space–time supports are (asymptotically) disjoint, and they cannot interact. Space resonances correspond to the vanishing of the gradient of the phase, $\nabla_\eta \phi_{\pm,\pm}$ or $\nabla_{\eta,\sigma} \phi_{\pm,\pm,\pm}$; in other words it corresponds to stationary phase in η, σ.

3.3. Treating the quadratic terms

The quadratic terms can be dealt with using the following observation: there are relatively few quadratic time resonances, namely

$$|\phi_{\pm,\pm}| \geqslant \inf\big(|\xi|^{1/2}, |\eta|^{1/2}, |\xi - \eta|^{1/2}\big);$$

moreover they are somehow canceled by the structure of the nonlinearity in the sense that for $i = 1, 2$,

$$\left| \frac{m_i}{\phi_{\pm,\pm}} \right|$$

is bounded on a sphere and homogeneous of degree $3/2$ in (ξ, η), and sufficiently smooth to allow for estimates (null condition). Integrating by parts in time (which amounts to a normal form transform), the quadratic term becomes

$$\int_0^t \iint e^{is\phi_{\pm,\pm}} m_i(\xi, \eta) \widehat{f_{\mp}}(s, \eta) \widehat{f_{\mp}}(s, \xi - \eta) \, d\eta \, ds \tag{3a}$$

$$= \int e^{it\phi_{\pm,\pm}} \frac{m_i(\xi, \eta)}{i\phi_{\pm,\pm}} \widehat{f_{\mp}}(s, \eta) \widehat{f_{\mp}}(s, \xi - \eta) \, d\eta \Big|_0^t \tag{3b}$$

$$- \int_0^t \iint e^{is\phi_{\pm,\pm}} m_i(\xi, \eta) \partial_s \big(\widehat{f_{\mp}}(s, \eta) \widehat{f_{\mp}}(s, \xi - \eta) \big) \, d\eta \, ds. \tag{3c}$$

The term (3b) corresponds essentially to a pseudo-product with non-smooth symbol, as described in Section 3.5, between $f_{\mp}$ and $f_{\mp}$. Thus one can prove directly the L^∞ and weighted L^2 estimates on this term.

As for (3c), a small computation shows that it can be written as a sum of terms of the type

$$\int_0^t \iiint e^{is\phi_{\pm,\pm,\pm}} \frac{m_i(\xi, \eta) m_j(\eta, \sigma)}{\phi_{\pm,\pm}} \widehat{f_{\mp}}(s, \eta) \widehat{f_{\mp}}(s, \sigma) \widehat{f_{\mp}}(s, \xi - \eta - \sigma) \, d\eta \, d\sigma \, ds, \tag{4}$$

and it will thus be analyzed with the cubic terms.

3.4. Treating the cubic terms

Cubic terms are either of the form appearing in (2), or of the form (4). In either case, they can be written as

$$\int_0^t \iiint e^{is\phi_{\pm,\pm,\pm}} \mu(\xi, \eta, \sigma) \widehat{f_{\mp}}(s, \eta) \widehat{f_{\mp}}(s, \sigma) \widehat{f_{\mp}}(s, \xi - \eta - \sigma) \, d\eta \, d\sigma \, ds. \tag{5}$$

Once again, the analysis of this term will depend on the resonant structure given by $\phi_{\pm,\pm,\pm}$.

● *The case of* $\phi_{+,+,+}$, $\phi_{-,+,+}$, $\phi_{+,-,+}$, $\phi_{+,+,-}$, $\phi_{-,-,-}$. For these phases, time resonances represent a small set in (ξ, η, σ), one has namely

$$|\phi_{\pm,\pm,\pm}| \geqslant \inf\big(|\xi|^{1/2}, |\eta|^{1/2}, |\sigma|^{1/2}, |\xi - \eta - \sigma|^{1/2} \big).$$

Thus, as in Section 3.3 an integration by parts in time gives the desired estimates; there is a difference though: this time, this manipulation produces quartic terms, which can be directly estimated, without any further study of resonances.

● *The case of* $\phi_{-,-,+}$, $\phi_{-,+,-}$, $\phi_{+,-,-}$. For these phases, the phase and its derivatives vanish together on a large set, so the situation seems hopeless. However there is a redeeming feature which is the following. When trying to establish the weighted L^2 estimate, one differentiates the expression (5) in ξ, which corresponds to adding an x weight in physical space. The worst term arises when the ξ derivative hits the phase which introduces a factor of s

$$\int_0^t \iiint e^{is\phi_{\pm,\pm,\pm}} s \nabla_\xi \phi_{\pm,\pm,\pm} \mu(\xi, \eta, \sigma) \widehat{f_{\mp}}(s, \eta) \widehat{f_{\mp}}(s, \sigma) \widehat{f_{\mp}}(s, \xi - \eta) \, d\eta \, d\sigma \, ds.$$

The key observation is that close to the set where $\nabla_{\sigma,\eta} \phi_{\pm,\pm,\pm} = 0$, one has

$$2\nabla_\xi \phi_{\pm,\pm,\pm} \sim \nabla_\sigma \phi_{\pm,\pm,\pm} + \nabla_\eta \phi_{\pm,\pm,\pm}.$$

Thus one can essentially replace $\nabla_\xi \phi_{\pm,\pm,\pm}$ by $\frac{1}{2}[\nabla_\sigma \phi_{\pm,\pm,\pm} + \nabla_\eta \phi_{\pm,\pm,\pm}]$. This allows one to perform an integration by parts in η or σ which eliminates the s factor and allows us to close the estimates.

The manipulation which has just been checked gives the control of the weighted L^2 norm; working a little more gives control of $|x| f$ in $L^{2-\gamma}$, where γ a small constant. The decay of u in L^∞ follows.

3.5. Pseudo-product operators with non-smooth symbols

We would like to say a word here about the bi- and tri-linear operators which appear in (2). These operators belong to the class of pseudo-product operators, first introduced by Coifman and Meyer [5]. In the case of bilinear pseudo-product operators (to which we restrict the discussion for the sake of simplicity), they are given in Fourier coordinates by

$$\widehat{B(f,g)}(\xi) = \int m(\xi,\eta) \hat{f}(\eta) \hat{f}(\xi-\eta) \, d\eta.$$

The classical theorem of Coifman and Meyer states that if the symbol satisfies $\partial_\xi^\alpha \partial_\eta^\beta m(\xi,\eta) \lesssim (|\xi| + |\eta|)^{-|\alpha|-|\beta|}$ for sufficiently many multi-indices, then the operator B enjoys the same bounds as the ones given by Hölder's inequalities for the standard point wise product.

A quick examination of the m_i that we have to deal with reveals that they do not satisfy such a strong assumption; and the manipulations described in the previous sections lead to even more singular symbols. In particular, flag singularities [11] appear: roughly speaking, this corresponds to symbols which are products of bilinear and trilinear symbols and, as such, do not satisfy the Coifman–Meyer condition. But one notices that the m_i are only singular when one of the Fourier coordinates (i.e.: $|\eta|$, $|\xi|$, $|\xi - \eta|$, $|\sigma|$, $|\xi - \eta - \sigma|$) is zero. By developing the symbol in series around this zero coordinate, one can prove bounds almost as good as Hölder's inequalities for the associated pseudo-product operators, which can then be handled.

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