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Convergence of semi-discrete approximations of Benney equations

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Abstract

In the first part of this Note we study the numerical approximation of Benney equations in the long wave-short wave resonance case. We prove the convergence of a finite-difference semi-discrete scheme in the energy space. In the second part of the Note we consider the semi-discretization of a quasilinear version of Benney equations. We prove the convergence of a finite-difference semi-discrete Lax–Friedrichs type scheme towards a weak entropy solution of the Cauchy problem. ***To cite this article: P. Amorim, M. Figueira, C. R. Acad. Sci. Paris, Ser. I 347 (2009).***

Résumé

Convergence d'une approximation semi-discrète des équations de Benney. Dans la première partie de cette Note, on étudie l'approximation numérique des équations de Benney dans le cas de résonance des ondes courtes et longues. On prouve la convergence d'un schéma aux différences finies semi-discret dans l'espace de l'énérgie. Dans la deuxième partie de cette Note, on condidère une version quasilinéaire des équations de Benney. On prouve la convergence d'un schéma du type Lax–Friedrichs semi-discret vers la solution d'entropie du problème. ***Pour citer cet article : P. Amorim, M. Figueira, C. R. Acad. Sci. Paris, Ser. I 347 (2009).***

Version française abrégée

Dans cette Note, on étudie la convergence d'approximations semi-discrètes de deux types d'équations de Benney [3] modélisant l'intéraction entre ondes courtes et ondes longues. Dans la première partie, on s'intéresse aux équations (1a), (1b), dont le problème de Cauchy a été résolu par Tsutsumi et Hatano [9] et puis par Bekiranov et al. [2] dans des espaces $H^s(\mathbb{R}) \times H^{s-1/2}(\mathbb{R})$, $s \geqslant 0$. On considère une approximation aux différences finies semi-discrètes définies par (3a)–(3c). On énonce d'abord un résultat d'existence et unicité de solution pour le problème semi-discretisé dont la preuve utilise un argument de point fixe standard.

Notre principal résultat concernant l'approximation de (1a)–(1c), le Théorème 2.2, établit la convergence de u^h, v^h vers l'unique solution forte du problème continu. Cette convergence, forte dans L^2_{loc} pour v^h et faible dans H^1 pour u^h, est une conséquence des estimations uniformes établies dans les Lemmes 2.3, 2.4 et 2.5. En particulier, on

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prouve dans le Lemme 2.4 les estimations sur les intégrales primaires associées à (3a)–(3c). Ces estimations, obtenues en utilisant les techniques du cas continu, nous permettent de montrer, par un argument du type Gronwall, l'estimation *a priori* (9). Celle-ci entraîne la compacité (faible) de la suite u^h. C'est simple alors de prouver que la limite (u, v) de (u^h, v^h) est la solution forte du problème continu.

Dans la deuxième partie de cette Note, on considère le problème (2a)–(2c), où la deuxième équation est remplacée par une loi de conservation non-linéaire couplée avec l'équation de Schrödinger par le terme de source. On utilise ici le modèle formulé par Dias et al. [4], où la loi de conservation jouit d'une sorte de principe du maximum. Ces auteurs ont par ailleurs démontré l'existence et unicité de solution d'entropie pour ce problème. On discrétise cette équation par un schéma de type Lax–Friedrichs semi-discret. Pour prouver la convergence de ce dernier vers la solution d'entropie du problème, on se place dans le cadre de la méthode de compacité par compensation, dont l'utilisation dépend de la propriété de compacité (21). La preuve de (21) repose sur les estimations *a priori* du Lemme 3.1, qui peuvent être établies à l'aide d'intégrales primaires, omis par souci de concision. Pour plus de détails, on renvoie le lecteur à [1].

1. Introduction

One of the equations proposed by Benney [3] in the long wave–short wave resonance case is:

$$i\partial_t u + \partial_{xx} u = \alpha|u|^2 u + vu, \tag{1a}$$

$$\partial_t v = \partial_x\big(|u|^2\big) \tag{1b}$$

where $u = u(x,t)$ is a complex-valued function, $v = v(x,t)$ is real-valued, and $\alpha \in \mathbb{R}$. We are interested in the numerical approximation of the strong solutions of the Cauchy problem for (1a), (1b) with initial data:

$$u(x,0) = \varphi(x), \quad v(x,0) = \psi(x), \quad x \in \mathbb{R}. \tag{1c}$$

Tsutsumi and Hatano [9] proved the global well-posedness of the Cauchy problem for (1a)–(1c) in the space $H^{j+1/2}(\mathbb{R}) \times H^j(\mathbb{R})$, $j \geqslant 1$. Later on, the local well-posedness of the same Cauchy problem was strongly improved by Bekiranov et al. [2], who extended the result to the spaces $H^s(\mathbb{R}) \times H^{s-1/2}(\mathbb{R})$, $s \geqslant 0$.

Our first purpose in this Note is to prove the convergence of a numerical approximation to (1a)–(1c). We consider a semi-discrete approximation and we prove the convergence to a strong solution in the energy space $H^1(\mathbb{R}) \times L^2(\mathbb{R})$. We also refer to [6] for the numerical approximation of the nonlinear Schrödinger equation with initial data in L^2.

In a second part of this Note we consider a semi-discretization of a quasilinear version of the Benney equations:

$$i\partial_t u + \partial_{xx} u = |u|^2 u + g(v)u, \tag{2a}$$

$$\partial_t v + \partial_x f(u) = \partial_x\big(g'(v)|u|^2\big), \tag{2b}$$

$$\big(u(x,0), v(x,0)\big) = \big(\varphi(x), \psi(x)\big) \tag{2c}$$

where $f \in C^2(\mathbb{R})$, $g \in C^3(\mathbb{R})$ are real functions such that $f(0) = 0$, g' has compact support, and f, g verify a standard non-degeneracy condition (see [4]). In [4] the authors proved the global existence of a weak entropy solution for the Cauchy problem (2a)–(2c). Here we use a semi-discrete Lax–Friedrichs type scheme to approximate the quasilinear equation (2b) and we prove a convergence result towards the unique entropy solution.

Let us define the Banach spaces:

$$l_h^p(\mathbb{Z}) = \left\{(z_j):\ z_j \in \mathbb{C},\ \|z_j\|_{p,h}^p = h\sum_{j\in\mathbb{Z}} |z_j|^p < \infty\right\}, \quad h > 0.$$

For $p = 2$ we denote the usual scalar product by $(z_j, w_j)_h = h\sum_{j\in\mathbb{Z}} z_j \bar{w}_j$. We will also use the following notations for the well-known finite-difference operators: for $u = (u_j)$,

$$(D_+u)_j = (u_{j+1} - u_j)/h, \qquad (D_-u)_j = (u_j - u_{j-1})/h, \qquad (D_0u)_j = (u_{j+1} - u_{j-1})/2h,$$

$$\big(\Delta^h u\big)_j = \big(D_+(D_-u)\big)_j = \big(D_-(D_+u)\big)_j = (u_{j+1} - 2u_j + u_{j-1})/h^2.$$

2. Convergence of approximations to a short wave–long wave equation

Let us now consider the semi-discrete finite-difference approximation of (1a)–(1c):

$$i\frac{\mathrm{d}u^h}{\mathrm{d}t} + \Delta^h u^h = \alpha\left|u^h\right|^2 u^h + v^h u^h, \tag{3a}$$

$$\frac{\mathrm{d}v^h}{\mathrm{d}t} = D_0\left(\left|u^h\right|^2\right), \tag{3b}$$

$$u^h(0) = \varphi^h, \qquad v^h(0) = \psi^h \tag{3c}$$

where u^h denotes the unknown grid function $(u_j^h)_{j\in\mathbb{Z}}$, $u_j^h(t)$ being the approximation of the solution at the node $x_j = jh$. The following first result holds:

Proposition 2.1. *Let $h > 0$. Then, for each initial data $(\varphi^h, \psi^h) \in l_h^2(\mathbb{Z}) \times l_h^2(\mathbb{Z})$, there exists a unique global solution $(u^h(t), v^h(t)) \in (C(\mathbb{R}; l_h^2(\mathbb{Z})))^2$ of* (3a)–(3c).

Proof. The classical fixed-point theorem leads to the local existence of solution to (3a)–(3c). The global existence is now an immediate consequence of the conservation of the $l_h^2(\mathbb{Z})$ norm of u^h established below in Lemma 2.4. □

Now let $\mathbf{P}_1^h$ denote the piecewise linear and continuous interpolator and let $(\varphi, \psi) \in H^1(\mathbb{R}) \times L^2(\mathbb{R})$ be the initial data for the problem (1a)–(1c). Our main result in this section is the following:

Theorem 2.2. *Let (φ^h, ψ^h) be the initial data for the discretized problem* (3a)–(3c) *such that $\mathbf{P}_1^h\varphi^h \rightharpoonup \varphi$ weakly in $H^1(\mathbb{R})$ and $\mathbf{P}_1^h\psi^h \rightharpoonup \psi$ weakly in $L^2(\mathbb{R})$ when $h \to 0$. Then, for each $T > 0$, the sequence $\mathbf{P}_1^h u^h$ satisfies*:

$$\mathbf{P}_1^h u^h \rightharpoonup u \quad \text{in } L^\infty\left([-T,T]; H^1(\mathbb{R})\right), \qquad \mathbf{P}_1^h u^h \to u \quad \text{in } L^\infty\left([-T,T]; L^2_{\mathrm{loc}}(\mathbb{R})\right) \quad \text{weak } *,$$
$$\mathbf{P}_1^h v^h \rightharpoonup v \quad \text{in } L^\infty\left([-T,T]; L^2(\mathbb{R})\right) \quad \text{weak } *$$

with (u, v) the unique strong solution of (1a)–(1c),

$$(u,v) \in \left(C\left([-T,T]; L^2\right) \cap L^\infty\left([-T,T]; H^1\right) \cap C\left([-T,T]; H^1_{\mathrm{weak}}\right)\right) \times C\left([-T,T]; L^2\right).$$

Moreover, for the more regular data $(\varphi, \psi) \in H^2(\mathbb{R}) \times H^1(\mathbb{R})$ we obtain the locally strong convergence

$$\mathbf{P}_1^h u^h \to u \quad \text{in } L^\infty\left([-T,T]; H^1_{\mathrm{loc}}(\mathbb{R})\right), \qquad \mathbf{P}_1^h v^h \to v \quad \text{in } L^\infty\left([-T,T]; L^2_{\mathrm{loc}}(\mathbb{R})\right).$$

2.1. Main estimates

The following inequalities hold:

Lemma 2.3. *Let $\phi = (\phi_j) \in l_h^2(\mathbb{Z})$. Then*

$$\|\phi\|_\infty \leqslant C\|\phi\|_{2,h}^{1/2}\|D_+\phi\|_{2,h}^{1/2}, \tag{4}$$

$$\|\phi\|_{4,h} \leqslant C\|\phi\|_{2,h}^{3/4}\|D_+\phi\|_{2,h}^{1/4}. \tag{5}$$

Proof. The inequality (4) is a consequence of the Gagliardo–Niremberg inequality:

$$\|\phi\|_\infty = \left\|\mathbf{P}_1^h\phi\right\|_\infty \leqslant C\left\|\mathbf{P}_1^h\phi\right\|_2^{1/2}\left\|\nabla\mathbf{P}_1^h\phi\right\|_2^{1/2} \leqslant C\|\phi\|_{2,h}^{1/2}\|D_+\phi\|_{2,h}^{1/2}.$$

Using (4) we easily obtain the inequality (5). □

Lemma 2.4. *Let u^h be the global solution of the problem* (3a)–(3c). *Then it holds*:

$$\frac{\mathrm{d}}{\mathrm{d}t}\|u^h\|_{2,h}^2 = 0, \tag{6}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left\{\frac{1}{2}\|D_+u^h\|_{2,h}^2 + \frac{\alpha}{4}\|u^h\|_{4,h}^4 + \frac{1}{2}\big(v, |u^h|^2\big)_h\right\} = 0, \tag{7}$$

$$\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\|v^h\|_{2,h}^2 = \big(D_0|u^h|^2, v^h\big)_h. \tag{8}$$

Proof. Taking the scalar product of (3a) by $\bar{u}^h$ and taking the imaginary part we obtain (6). To obtain (7), take the scalar product of (3a) and $\mathrm{d}\bar{u}^h/\mathrm{d}t$, take the real part and use Eq. (3b). Similarly we deduce (8). □

Lemma 2.5. *For all $h > 0$ and $t \in \mathbb{R}$ there exist positive constants $C_1 = C_1(\varphi^h, \psi^h)$, $C_2 = C_2(\varphi^h, \psi^h)$, $\tilde{C}_1 = \tilde{C}_1(\varphi^h, \psi^h)$, $\tilde{C}_2 = \tilde{C}_2(\varphi^h, \psi^h)$, such that*

$$\|D_+u^h(t)\|_{2,h} \leqslant C_1 e^{C_2 t}, \tag{9}$$

$$\|\Delta^h u^h(t)\|_{2,h} \leqslant \tilde{C}_1 e^{\tilde{C}_2 t}. \tag{10}$$

Proof. From the conservation of the discrete energy (7) it follows that

$$\|D_+u^h\|_{2,h}^2 + \frac{\alpha}{4}\|u^h\|_{4,h}^4 \leqslant C_1 + C_2\|v^h\|_{2,h}\|u^h\|_{4,h}^2.$$

Hence, by (5) and (6) we obtain

$$\|D_+u^h\|_{2,h}^2 \leqslant C_1 + C_2\|v^h\|_{2,h}\|D_+u^h\|_{2,h}^{1/2}. \tag{11}$$

On the other hand, using (4), (6) and (8) we derive

$$\frac{\mathrm{d}}{\mathrm{d}t}\|v^h\|_{2,h} \leqslant C\|D_0u^h\|_{2,h}\|u^h\|_\infty \leqslant C\|D_+u^h\|_{2,h}^{3/2}$$

and so

$$\|v^h\|_{2,h} \leqslant C + C\int_0^t \|D_+u^h(s)\|_{2,h}^{3/2}\,\mathrm{d}s.$$

Hence, by (11)

$$\|D_+u^h\|_{2,h}^2 \leqslant C_1 + C_2\|D_+u^h\|_{2,h}^{1/2} + C_2\left(\int_0^t \|D_+u^h(s)\|_{2,h}^{3/2}\,\mathrm{d}s\right)\|D_+u^h\|_{2,h}^{1/2}.$$

From this it follows easily that

$$\big(1 + \|D_+u^h\|_{2,h}\big)^{3/2} \leqslant C_1 + C_2\int_0^t \big(1 + \|D_+u^h(s)\|_{2,h}\big)^{3/2}\,\mathrm{d}s$$

and (9) follows by Gronwall's inequality. To prove (10) we differentiate in t Eq. (3a), and take the inner product by $\bar{u}_t^h$. Taking the imaginary part, we obtain the suitable estimation for $\|u_t^h\|_{2,h}$ and the conclusion follows. □

Sketch of the proof of Theorem 2.2. We apply the piecewise linear interpolator $\mathbf{P}_1^h$ to Eqs. (3a), (3b):

$$i\frac{\mathrm{d}\mathbf{P}_1^h u^h}{\mathrm{d}t} + \Delta^h\mathbf{P}_1^h u^h = \alpha\mathbf{P}_1^h\big(|u^h|^2u^h\big) + \mathbf{P}_1^h\big(v^hu^h\big), \tag{12}$$

$$\frac{\mathrm{d}\mathbf{P}_1^h v^h}{\mathrm{d}t} = D_0\big(\mathbf{P}_1^h|u^h|^2\big). \tag{13}$$

From the estimate (9) we deduce that there exist (u, v) such that, up to a subsequence,

$$\mathbf{P}_1^h u^h \overset{*}{\rightharpoonup} u \quad \text{in } L^\infty\big([-T,T]; H^1(\mathbb{R})\big), \qquad \mathbf{P}_1^h u^h \to u \quad \text{in } L^\infty\big([-T,T]; L^2_{\mathrm{loc}}(\mathbb{R})\big),$$
$$\mathbf{P}_1^h v^h \overset{*}{\rightharpoonup} v \quad \text{in } L^\infty\big([-T,T]; L^2(\mathbb{R})\big).$$

Now, we consider the piecewise constant interpolator, $\mathbf{P}_0^h$, which commutes with the nonlinearity. Since

$$\mathbf{P}_1^h\big(\big|u^h\big|^2 u^h\big) - \mathbf{P}_0^h\big(\big|u^h\big|^2 u^h\big) \to 0, \quad \mathbf{P}_1^h\big(v^h u^h\big) - \mathbf{P}_0^h\big(v^h u^h\big) \overset{*}{\rightharpoonup} 0 \quad \text{in } L^\infty\big([-T,T]; L^2(\mathbb{R})\big),$$

and

$$\mathbf{P}_0^h\big(\big|u^h\big|^2 u^h\big) = \big|\mathbf{P}_0^h u^h\big|^2 \mathbf{P}_0^h u^h \to |u|^2 u \quad \text{in } L^\infty\big([-T,T]; L^2_{\mathrm{loc}}(\mathbb{R})\big)$$

we deduce that

$$\mathbf{P}_1^h\big(\big|u^h\big|^2 u^h\big) \overset{*}{\rightharpoonup} |u|^2 u \quad \text{in } L^\infty\big([-T,T]; H^1(\mathbb{R})\big), \qquad \mathbf{P}_1^h\big(v^h u^h\big) \overset{*}{\rightharpoonup} vu \quad \text{in } L^\infty\big([-T,T]; L^2(\mathbb{R})\big).$$

We now obtain the conclusion by taking the limit $h \to 0$ in Eqs. (12), (13).

3. Convergence of approximations of a quasilinear version of Benney equations

We propose here the following semi-discrete approximation to Eqs. (2a), (2b). The first equation is approximated by a standard finite-difference scheme and the second equation by a Lax–Friedrichs type scheme:

$$i\partial_t u^h + \Delta^h u^h = \big|u^h\big|^2 u^h + g\big(v^h\big)u^h, \tag{14}$$

$$\partial_t v^h + D_0 f\big(v^h\big) = D_0\big(g'\big(v^h\big)\big|u^h\big|^2\big) + \frac{h}{2\lambda}\Delta^h v^h + \frac{1}{2\gamma}\big(\big|u^h\big|_+^2 D_+ v^h - \big|u^h\big|_-^2 D_- v^h\big), \tag{15}$$

$$u^h(0) = \varphi^h(0), \qquad v^h(0) = \psi^h(0). \tag{16}$$

Here, λ, γ are some constants ensuring the stability of the scheme via a CFL condition, and we have set $(|u^h|_\pm^2)_j = |u|^2_{j+1/2} = (|u_{j\pm1}|^2 + |u_j|^2)/2$. In what follows, we use the notation u^h, v^h to denote the piecewise constant interpolator of the grid functions u^h, v^h. Note that the existence (for each h) of a solution to Eqs. (14)–(16) with initial data in l_h^2 is guaranteed by a simple fixed-point argument.

The following estimates are crucial to the proof of convergence, and follow from the approximate first integrals associated with Eqs. (14), (15). For the proof, see [1].

Lemma 3.1. *Let* (u^h, v^h) *be defined by* (14)–(16)*. Then, under an appropriate CFL-like condition involving* λ*,* γ *(see* [1]*), there exist constants* $k, C_{1,2} > 0$ *depending only on the initial data, and non-negative functions* $a(t)$*,* $b(t)$ *continuous on* $[0,\infty)$ *such that for every* $t > 0$ *we have, uniformly in h,*

$$\big\|u^h(t)\big\|_2 \leqslant C_1, \tag{17}$$

$$\big\|v^h(t)\big\|_\infty \leqslant C_2, \tag{18}$$

$$\big\|v^h(t)\big\|_2^2 + k\int_0^t \sum_{j\in\mathbb{Z}}\big(1+|u_j|^2\big)(v_{j+1}-v_j)^2\,\mathrm{d}s \leqslant a(t), \tag{19}$$

$$\big\|D_+ u^h\big\|_2 \leqslant b(t). \tag{20}$$

Our main result in this section is the following.

Theorem 3.2. *Let* (u^h, v^h) *be defined by the semi-discrete approximation* (14)–(16)*. Then there exist functions* $u \in C([0,\infty); H^1(\mathbb{R}))$*,* $v \in L^\infty(\mathbb{R}\times[0,\infty))$*, solutions to the Cauchy problem* (2a)–(2c) *in the sense of* [4] *such that, up to a subsequence,* (u^h, v^h) *converge to* (u, v) *in* $L^1_{\mathrm{loc}}(\mathbb{R}\times[0,\infty))$*.*

We make some observations regarding the proof of Theorem 3.2. First, the compactness of u^h follows from the estimates in Lemma 3.1 and arguments similar to the ones in [4]. The details can be found in [1].

To prove the convergence of v^h, we rely on the compensated compactness method [7,8], according to which the strong compactness of a sequence of approximate solutions (v^h) is a consequence of the following property (see [4]):

$$\partial_t \eta\big(v^h\big) + \partial_x\big(q_1\big(v^h\big) - \big|u^h\big|^2 q_2\big(v^h\big)\big) \in \big\{\text{compact of } W^{-1,2}_{\mathrm{loc}}\big\}, \tag{21}$$

where $\eta(v)$ is a convex function (the *entropy*), and the *entropy fluxes* $q_{1,2}$ verify $q_1'(v) = \eta'(v) f'(v)$ and $q_2'(v) = \eta'(v) g''(v)$, see [4]. In practice, one may use the following result to establish (21): If $1 < q < 2 < r \leqslant \infty$, then

$$\big\{\text{compact of } W^{-1,q}_{\mathrm{loc}}\big\} \cap \big\{\text{bounded in } W^{-1,r}_{\mathrm{loc}}\big\} \subset \big\{\text{compact of } W^{-1,2}_{\mathrm{loc}}\big\}. \tag{22}$$

Lemma 3.3. *Let* (u^h, v^h) *be defined by the semi-discrete approximation* (14), (15). *Then, the compactness property in* (21) *is valid.*

Sketch of the proof. Let ϕ be α-Hölder continuous (for some $\alpha \in (1/2, 1)$) and compactly supported on $\mathbb{R} \times [0, \infty)$. Then, one considers the following functional:

$$E^h(\phi) = \iint\limits_{\mathbb{R}\times[0,\infty)} \phi(x,t)\partial_t \eta\big(v^h\big) + \phi(x,t)\partial_x\big(q_1\big(v^h\big) - q_2\big(v^h\big)\big|u^h\big|^2\big)\,\mathrm{d}x\,\mathrm{d}t. \tag{23}$$

We then multiply Eq. (15) by $\eta'(v^h)\phi^h$ (where ϕ^h is a piecewise constant approximation of ϕ), sum in $j \in \mathbb{Z}$ and integrate in time to deduce an expression for $E^h(\phi)$. Although this expression is complicated (and so we omit it for the sake of brevity – see [1] for the details), it is then straightforward to use the bounds in Lemma 3.1 to deduce as in [5] that there is a constant independent of h such that $E^h(\phi) \leqslant C(\|\phi\|_{C^{0,\alpha}} + \|\phi\|_\infty)$. The compactness property in (22) is then a consequence of the compact embedding of $W^{1,q'}$ into $C^{0,\alpha}$ for some suitable q', and the compact embedding of $\mathcal{M}$ (the space of bounded Radon measures on the support of ϕ) into $W^{-1,q}$ for a suitable q. The boundedness property in (21) is established similarly. It is then standard to deduce using the bounds in Lemma 3.1 that the functions (u, v) thus obtained are the unique entropy solutions of (2a)–(2c). □

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