



Analytic Geometry

On the Ohsawa–Takegoshi L^2 extension theorem and the twisted Bochner–Kodaira identity

Sur le théorème d'extension L^2 de Ohsawa–Takegoshi et l'identité tordue de Bochner–Kodaira

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ABSTRACT

In this Note, we improve the estimate in Ohsawa's generalization of the Ohsawa–Takegoshi L^2 extension theorem for holomorphic functions by finding a smaller constant, and apply the result to the Suita conjecture. We also present a remark allowing to generalize the Ohsawa–Takegoshi extension theorem to the case of $\bar{\partial}$ -closed smooth $(n-1, q)$ -forms. Finally, we prove that the twist factor in the twisted Bochner–Kodaira identity can be a non-smooth plurisuperharmonic function.

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RÉSUMÉ

Dans cette Note, nous améliorons l'estimation des constantes dans la généralisation par Ohsawa du théorème d'extension L^2 de Ohsawa–Takegoshi concernant les fonctions holomorphes, et nous appliquons ce résultat à l'étude de la conjecture de Suita. Nous présentons également une remarque permettant de généraliser le théorème d'extension de Ohsawa–Takegoshi au cas des $(n-1, q)$ -formes lisses $\bar{\partial}$ -fermées. Enfin, nous montrons que le facteur tordu dans l'identité tordue de Bochner–Kodaira peut être une fonction plurisuperharmonique non lisse.

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1. Introduction and main results

In [16], Ohsawa and Takegoshi proved the famous Ohsawa–Takegoshi L^2 extension theorem, which has become an important tool in several complex variables and complex geometry. Several years later, Ohsawa [15] made the following generalization, which can imply the original Ohsawa–Takegoshi L^2 extension theorem in [16] by taking $\psi = 0$.

Theorem 1.1. *Let D be a pseudoconvex domain in \mathbb{C}^n , φ be a plurisubharmonic function on D and $H = \{z \in \mathbb{C}^n : z^n = 0\}$. Then, for any plurisubharmonic function ψ on D such that $\sup_{z \in D} (\psi(z) + 2 \log |z^n|) \leq 0$, there exists a uniform constant \mathbf{C} independent of D , φ and ψ such that, for any holomorphic function f on $D \cap H$ satisfying*

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$$\int_{D \cap H} |f|^2 e^{-\varphi - \psi} (\sqrt{-1})^{n-1} dz^1 \wedge d\bar{z}^1 \wedge \cdots \wedge dz^{n-1} \wedge d\bar{z}^{n-1} < \infty,$$

there exists a holomorphic function F on D satisfying $F = f$ on $D \cap H$ and

$$\begin{aligned} & \int_D |F|^2 e^{-\varphi} (\sqrt{-1})^n dz^1 \wedge d\bar{z}^1 \wedge \cdots \wedge dz^n \wedge d\bar{z}^n \\ & \leq 2C\pi \int_{D \cap H} |f|^2 e^{-\varphi - \psi} (\sqrt{-1})^{n-1} dz^1 \wedge d\bar{z}^1 \wedge \cdots \wedge dz^{n-1} \wedge d\bar{z}^{n-1}. \end{aligned}$$

Since the canonical line bundle on \mathbb{C}^n is trivial, Theorem 1.1 gives the extension of L^2 $\bar{\partial}$ -closed smooth $(n-1, q)$ -forms with an estimate for $q=0$. Several years later, the extension with an estimate for the case $q \geq 1$ was presented in [13,3,5,7]. In this case, one has a regularity trouble. Here, we present the following simpler remark concerning the case $q \geq 1$.

Theorem 1.2. Let (X, g) be a Stein manifold of dimension n with a Kähler metric g and $D \Subset X$ be a domain in X . Assume D is weakly pseudoconvex. Let w be a holomorphic function on X such that dw is never zero on the set $H = \{w=0\}$. Then there exists a uniform constant E independent of X, D and w such that, for any $\bar{\partial}$ -closed form $f \in C^\infty(H, \bigwedge^{n-1, q} T_H^*)$, where $0 \leq q \leq n-1$, there exists a $\bar{\partial}$ -closed form $F \in C^\infty(D, \bigwedge^{n, q} T_X^*)$ satisfying $F = dw \wedge \tilde{f}$ on $D \cap H$ with $\theta^* \tilde{f} = f$ and $\int_D |F|^2 dV_X \leq E \sup_D |w|^2 \int_{D \cap H} |f|^2 dV_H$, where $\theta: H \rightarrow X$ is the inclusion map and H is endowed with the Kähler metric h induced from g by θ .

Remark 1.3. In the above theorem, D is called weakly pseudoconvex if there is a smooth plurisubharmonic exhaustion function on D . T_X and T_H are the holomorphic tangent bundles of X and H respectively. T_X^* and T_H^* are the dual bundles of them. dV_X and dV_H are the volume elements of X and H respectively.

In [18], Siu got an explicit constant \mathbf{C} in Theorem 1.1. Berndtsson in [1] reduced the constant \mathbf{C} in Theorem 1.1 to 4. Later, Chen (see [4]) reduced the constant \mathbf{C} in Theorem 1.1 to 3.3155. In the following theorem, we generalize Theorem 1.1 to Stein manifolds and get the constant \mathbf{C} given by $\inf_{\chi \in \mathcal{M}} \sup_{s \leq 0} \{(-\chi(s) + \frac{\chi'(s)^2}{\chi''(s)})e^s\}$, where \mathcal{M} is the class of functions defined by $\{\chi \in C^\infty(-\infty, 0]: \chi(s) \leq 0, \chi'(s) \geq 1 \text{ and } \chi''(s) > 0 \text{ for all } s \in (-\infty, 0]\}$, and make the constant \mathbf{C} smaller than 2.14 by choosing an explicit function $\chi(s)$.

Remark 1.4. According to a suggestion of Prof. Demailly, the constant \mathbf{C} can be estimated by solving the ordinary differential equation $(-\chi(s) + \chi'(s)^2/\chi''(s))e^s = \mathbf{C}$ with $\chi'(0)$ and \mathbf{C} adjusted so that $\lim_{s \rightarrow -\infty} \chi'(s) = 1$. A standard Runge–Kutta integration with a computer then shows that one can obtain $\mathbf{C} < 1.954$ when $\chi(0) = 0$ and $\chi'(0) = 4.331$. For the details, we refer to [11].

Theorem 1.5. Let X be a Stein manifold of dimension n . Let φ and ψ be two plurisubharmonic functions on X . Assume that w is a holomorphic function on X such that $\sup_X (\psi + 2 \log |w|) \leq 0$ and dw does not vanish identically on any branch of $w^{-1}(0)$. Put $H = w^{-1}(0)$ and $H_0 = \{x \in H: dw(x) \neq 0\}$. Then there exists a uniform constant $\mathbf{C} < 1.954$ independent of X, φ, ψ and w such that, for any holomorphic $(n-1)$ -form f on H_0 satisfying $c_{n-1} \int_{H_0} e^{-\varphi - \psi} f \wedge \bar{f} < \infty$, where $c_k = (-1)^{\frac{k(k-1)}{2}} i^k$ for $k \in \mathbb{Z}$, there exists a holomorphic n -form F on X satisfying $F = dw \wedge \tilde{f}$ on H_0 with $\theta^* \tilde{f} = f$ and $c_n \int_X e^{-\varphi} F \wedge \bar{F} \leq 2C\pi c_{n-1} \int_{H_0} e^{-\varphi - \psi} f \wedge \bar{f}$, where $\theta: H_0 \rightarrow X$ is the inclusion map.

One motivation to estimate the constant \mathbf{C} in Theorem 1.1 comes from the Suita conjecture [19], which is stated below.

Let Ω be a bounded domain in \mathbb{C} , K_Ω be the Bergman kernel function on Ω and $c_\Omega(z)$ be the logarithmic capacity of the complement $\mathbb{C} \setminus \Omega$ with respect to z defined by

$$c_\Omega(z) = \exp \lim_{\xi \rightarrow z} (G_\Omega(\xi, z) - \log |\xi - z|),$$

where G_Ω is the negative Green function on Ω . Then $(c_\Omega(z))^2 \leq \pi K_\Omega(z)$ for any $z \in \Omega$.

Using Theorem 1.5 and Remark 1.4, we get

Corollary 1.6. $(c_\Omega(z))^2 \leq C\pi K_\Omega(z)$, where the constant satisfies $\mathbf{C} < 1.954$.

It should be noted that Blocki in [2] proved that $(c_\Omega(z))^2 \leq C\pi K_\Omega(z)$ for $\mathbf{C} = 2$ using a different method.

The twisted Bochner–Kodaira identity with a smooth twist factor was widely discussed (see [8,9,17,14] and references therein). In [16], it is the key tool to prove the Ohsawa–Takegoshi L^2 extension theorem. Berndtsson [1] gave a proof of the Ohsawa–Takegoshi L^2 extension theorem by using the twisted Bochner–Kodaira identity with special non-smooth

plurisuperharmonic twist factors $-\log|h|$ and $1 - |h|^{2\delta}$ ($\delta < 1$), where h is a holomorphic function having zeros. His proof involves in solving $\bar{\partial}$ -equations for $\bar{\partial}$ -closed currents. Motivated by his use of the special non-smooth plurisuperharmonic functions, we shall prove that the twist factor can be chosen as any non-smooth plurisuperharmonic function as follows.

Theorem 1.7. *Let Ω be a domain in \mathbb{C}^n , φ be a C^2 function on Ω and $D \Subset \Omega$ be a domain with C^2 boundary bD . Assume that η is a plurisuperharmonic function on Ω (i.e., $-\eta$ is a plurisubharmonic function) and ρ is a C^2 defining function for D such that $|\mathrm{d}\rho| = 1$ on bD . Then for any $\beta = \sum_{i=1}^n \beta_i \mathrm{d}\bar{z}^i \in C_{(0,1)}^2(\bar{D}) \cap \mathrm{Dom}_D(\bar{\partial}_\varphi^*)$,*

$$\begin{aligned} & \int_D \eta |\bar{\partial}\beta|^2 e^{-\varphi} \mathrm{d}V + \int_D \eta |\bar{\partial}_\varphi^* \beta|^2 e^{-\varphi} \mathrm{d}V \\ &= \int_D \sum_{j,k=1}^n (-\partial_j \bar{\partial}_k \eta) \beta_j \bar{\beta}_k e^{-\varphi} \mathrm{d}V + \int_D \sum_{j,k=1}^n \eta (\partial_j \bar{\partial}_k \varphi) \beta_j \bar{\beta}_k e^{-\varphi} \mathrm{d}V + \int_D \sum_{j,k=1}^n \eta |\bar{\partial}_j \beta_k|^2 e^{-\varphi} \mathrm{d}V \\ &+ \int_{bD} \sum_{j,k=1}^n \eta (\partial_j \bar{\partial}_k \rho) \beta_j \bar{\beta}_k e^{-\varphi} \mathrm{d}S + 2\mathrm{Re} \int_D \sum_{j=1}^n (\partial_j \eta) \beta_j \bar{\partial}_\varphi^* \beta e^{-\varphi} \mathrm{d}V. \end{aligned}$$

Theorem 1.7 seems not to be obvious. Its proof actually involves some results on normal currents. By modifying the proof of Theorem 1.7, we can get the following theorem with non-smooth twist factors, which also includes the twist factors used in [1].

Theorem 1.8. *Let Ω be a domain in \mathbb{C}^n , φ be a C^2 function on Ω and $D \Subset \Omega$ be a domain with C^2 boundary bD . Assume that H is a closed set in Ω such that the $(2n - 1)$ -dimensional Hausdorff measure of $H \cap bD$ is zero. Let η be a real function in $L^1_{\mathrm{loc}}(\Omega) \cap C^0(\Omega \setminus H)$ such that $\partial\bar{\partial}\eta$ is a current of order zero and ρ be a C^2 defining function for D such that $|\mathrm{d}\rho| = 1$ on bD . Then the conclusion of Theorem 1.7 holds.*

2. Ideas of the proofs

In this section, we shall give the main ideas used in the proofs of the theorems. For complete proofs of the theorems, we refer to [11].

In the proof of Theorem 1.2, we first use a special partition of unity to obtain a local extension \tilde{f} of f in a neighborhood of H in X . Choose a C^∞ function $\chi : \mathbb{R} \rightarrow \mathbb{R}$ satisfying $\mathrm{supp} \chi \subset (-1, 1)$, $0 \leq \chi \leq 1$, $\chi|_{(-\frac{1}{4}, \frac{1}{4})} \equiv 1$ and $|\chi'| < 2$. Define $\chi_\varepsilon(w) = \chi(\frac{|w|^2}{\varepsilon^2})$ ($\varepsilon > 0$) and define $\lambda_\varepsilon = \mathrm{d}w \wedge \frac{\bar{\partial}(\chi_\varepsilon \tilde{f})}{w}$. Then we approximate D by a sequence of subdomains $\{D_v\}_{v=1}^\infty$ and on each D_v we solve twisted $\bar{\partial}$ -equations $\bar{\partial}(\sqrt{\tau_\varepsilon + \bar{A}_\varepsilon} u_{v,\varepsilon}) = \lambda_\varepsilon$ with $u_{v,\varepsilon} \in \overline{\mathrm{Ran}(\sqrt{\tau_\varepsilon + \bar{A}_\varepsilon} \bar{\partial}^*)}$ and $\int_{D_v} |u_{v,\varepsilon}|^2 \mathrm{d}V_X$ uniformly bounded. Let u_ε be the weak limit of some weakly convergent subsequence of $\{u_{v,\varepsilon}\}_{v=1}^\infty$. Then $\bar{\partial}(\sqrt{\tau_\varepsilon + \bar{A}_\varepsilon} u_\varepsilon) = \lambda_\varepsilon$ and $\vartheta(\frac{u_\varepsilon}{\sqrt{\tau_\varepsilon + \bar{A}_\varepsilon}}) = 0$. Hence u_ε is a smooth form. Define $F_\varepsilon = \mathrm{d}w \wedge (\chi_\varepsilon \tilde{f}) + w \sqrt{\tau_\varepsilon + \bar{A}_\varepsilon} u_\varepsilon$. Then it is easy to check that F_ε is the desired extension in Theorem 1.2.

Unlike holomorphic n -forms, the regularity of $\bar{\partial}$ -closed (n, q) -forms ($q \geq 1$) can't be obtained just from the equation $\bar{\partial}F = 0$. This is also the main difficulty for us to get a $\bar{\partial}$ -closed smooth extension F on the whole manifold X with an estimate since we can't check the regularity after taking a weak limit as ε approaches 0.

The main goal in the proof of Theorem 1.5 is to reduce the constant **C**. The proof of the theorem is a combination of the methods used in [1,4,5,14,18]. There are lots of papers which have given proofs of the Ohsawa–Takegoshi L^2 extension theorem. In all of them, two tools are essentially used. One is an inequality deduced from the twisted Bochner–Kodaira identity with smooth or plurisubharmonic twisted factor, and the other is a proposition to solve $\bar{\partial}$ -equations with universal constant estimates. We find that the following versions of the two tools are suitable for us to reduce the constant **C**.

Lemma 2.1. *Let (X, g) be a Kähler manifold of dimension n with a Kähler metric g , $\Omega \Subset X$ be a pseudoconvex domain with C^∞ boundary $b\Omega$ and $\phi \in C^\infty(\bar{\Omega})$ be plurisubharmonic in Ω . Assume that $\tau, A \in C^\infty(\bar{\Omega})$ are positive functions. Then for any $(n, q + 1)$ -form $\alpha = \sum_{|I|=n, |J|=q+1} \alpha_{I\bar{J}} \mathrm{d}z^I \wedge \mathrm{d}\bar{z}^{\bar{J}} \in \mathrm{Dom}_\Omega(\bar{\partial}^*) \cap C_{(n,q+1)}^\infty(\bar{\Omega})$,*

$$\int_\Omega (\tau + A) |\bar{\partial}_\phi^* \alpha|^2 e^{-\phi} \mathrm{d}V_X + \int_\Omega \tau |\bar{\partial}\alpha|^2 e^{-\phi} \mathrm{d}V_X \geq \sum'_{|I|=n, |K|=q} \sum_{i,j=1}^n \int_\Omega \left(-\partial_i \bar{\partial}_j \tau - \frac{\partial_i \tau \bar{\partial}_j \tau}{A} \right) \alpha_{I\bar{K}}^i \bar{\alpha}^{\bar{I}j\bar{K}} e^{-\phi} \mathrm{d}V_X.$$

Lemma 2.2. *Let (X, g) be a Kähler manifold of dimension n with a Kähler metric g , $\Omega \Subset X$ be a strictly pseudoconvex domain in X with C^∞ boundary $b\Omega$ and $\phi \in C^\infty(\bar{\Omega})$. Let λ be the current $\bar{\partial}\frac{1}{w} \wedge \bar{F}$, where \bar{F} is a holomorphic n -form on X . Assume the inequality*

$$|(\lambda, \alpha)_{\Omega, \phi}|^2 \leq C \int_{\Omega} |\bar{\partial}_{\phi}^* \alpha|^2 \frac{e^{-\phi}}{\mu} dV_X,$$

where $\frac{1}{\mu}$ is an integrable positive function on Ω and C is a constant, holds for all $(n, 1)$ -form $\alpha \in \text{Dom}_{\Omega}(\bar{\partial}^*) \cap \text{Ker}(\bar{\partial}) \cap C_{(n,1)}^{\infty}(\bar{\Omega})$. Then there is a solution u to the equation $\bar{\partial}u = \lambda$ such that $\int_{\Omega} |u|^2 \mu e^{-\phi} dV_X \leq C$.

Lemma 2.1 is a variation of a lemma in [14]. Lemma 2.2 is a variation of a lemma in [1] and it is in fact a tool to solve $\bar{\partial}$ -equations for $\bar{\partial}$ -closed currents.

Since X is Stein, there is a holomorphic n -form \tilde{F} on X such that $\tilde{F} = dw \wedge \tilde{f}$ on H with $\theta^* \tilde{f} = f$. Define $\lambda = \bar{\partial} \frac{1}{w} \wedge \tilde{F}$. Then by the above lemmas and a careful calculation, we can solve equations of the form $\bar{\partial}u_{\gamma, v} = \lambda$ with uniform constant estimates on suitable subdomains $\Omega_v \Subset \Omega$, where $\gamma \in (0, 1)$ is a technical parameter. Define $F_{\gamma, v} = wu_{\gamma, v}$. The weak limit of some weakly convergent subsequence of $\{F_{\gamma, v}\}_{\gamma, v}$ gives us the desired extension F with the constant C by the formula stated before Remark 1.4. Then by means of a careful selection of the function χ and some elementary estimates (we first choose a family of the functions $\chi_{\delta}(s) = s - \log(1 + \delta - \delta e^s)$ depending on a positive parameter δ and then choose a suitable δ), one can show that $C < 2.14$. According to a suggestion of Prof. Demailly, one can get even smaller constant $C < 1.954$ with the help of a computer.

To prove Theorem 1.7, we first use convolutions to construct smooth functions $\{\eta_{\varepsilon}\}_{\varepsilon > 0}$ such that $\lim_{\varepsilon \rightarrow 0} \eta_{\varepsilon} = \eta$. Then we use the well-known twisted Bochner-Kodaira identity with the smooth twist factor η_{ε} . The main difficulty is to prove

$$\lim_{\varepsilon \rightarrow 0} \int_D \sum_{j,k=1}^n (-\partial_j \bar{\partial}_k \eta_{\varepsilon}) \beta_j \bar{\beta}_k e^{-\varphi} dV = \int_D \sum_{j,k=1}^n (-\partial_j \bar{\partial}_k \eta) \beta_j \bar{\beta}_k e^{-\varphi} dV$$

since β may not vanish on bD . Let λ be a smooth cut-off function such that $\lambda = 1$ on an open neighborhood of \bar{D} and $\text{supp } \lambda \Subset \Omega$. Then $\lambda \partial \bar{\partial} \eta$ is a normal current on Ω . We need to use some results on normal currents (see [10] and [12]) to obtain the following lemma. Then we can overcome the difficulty and get Theorem 1.7. For the theory of currents and normal currents, the reader is referred to [6].

Lemma 2.3. *Let Ω be a domain in \mathbb{C}^n and $D \Subset \Omega$ be a domain with C^2 boundary bD . Let $\xi = \sum_{i=1}^n \xi_i d\bar{z}^i$ and $\zeta = \sum_{i=1}^n \zeta_i d\bar{z}^i$ be two $(0, 1)$ -forms in $C_{(0,1)}^2(\bar{D}) \cap \text{Dom}_D(\bar{\partial}_{\phi}^*)$. Assume that $T = \sum_{i,j=1}^n T_{ij} dz^i \wedge d\bar{z}^j$ is a normal current on Ω . Then the measure $\sum_{i,j=1}^n \mathbb{1}_{bD} \xi_i \bar{\zeta}_j T_{ij} = 0$ on Ω , where $\mathbb{1}_{bD}$ is the characteristic function of bD .*

In order to prove Theorem 1.8, we need to use some arguments in the proof of Theorem 1.7. The main difficulty is to prove $\lim_{\varepsilon \rightarrow 0} \int_{bD} \sum_{j,k=1}^n \eta_{\varepsilon} (\partial_j \bar{\partial}_k \rho) \beta_j \bar{\beta}_k e^{-\varphi} dS = \int_{bD} \sum_{j,k=1}^n \eta (\partial_j \bar{\partial}_k \rho) \beta_j \bar{\beta}_k e^{-\varphi} dS$ since the integrable property of η on bD is not obvious. This can be done by proving that η is the difference of two subharmonic functions. For the details, we refer to [11].

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