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Division of the Dickson algebra by the Steinberg unstable module ☆

Division de l'algèbre de Dickson par le module instable de Steinberg

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ABSTRACT

We compute the division of the Dickson algebra by the Steinberg unstable module in the category of unstable modules over the mod-2 Steenrod algebra.

RÉSUMÉ

On détermine la division de l'algèbre de Dickson par le module instable de Steinberg dans la catégorie des modules instables sur l'algèbre de Steenrod modulo 2.

1. Introduction

We work in the category $\mathcal{U}$ of unstable modules over the mod-2 Steenrod algebra $\mathcal{A}$ [8]. For each $H \in \mathcal{U}$ of finite type, let $(-:H)_{\mathcal{U}}$ denote the left adjoint functor of the endofunctor $- \otimes H : \mathcal{U} \to \mathcal{U}$. For V an elementary Abelian 2-group, the famous Lannes' functor T_V is the division by H^*V [5]. Here and in the sequel, H^* denotes the mod-2 singular cohomology functor. For V, W two elementary Abelian 2-groups, the purpose of this note is to determine $(D_W : L_V)_{\mathcal{U}}$ where $D_W := H^*W^{\mathrm{Aut}(W)}$ is the Dickson algebra [1] and L_V, to be defined below, is the indecomposable summand of the Steinberg summand M_V of H^*V [7]. If $\dim V = k$ then L_V is also denoted by L_k and we use the same convention for all other notations admitting an elementary Abelian 2-group as index.

Let us explain the motivation for the determination of $(D_W : L_V)_{\mathcal{U}}$. In [2] we study the cohomotopy group of a spectrum, $L'(n)$, $n \in \mathbb{N}$, whose mod-2 cohomology, L'_n, is an unstable module that has the following minimal $\mathcal{U}$-injective resolution:

$$0 \to L'_n \to L_n \to L_{n-1} \otimes J(1) \to \cdots \to L_1 \otimes J\left(2^{n-1}-1\right) \to J\left(2^n - 1\right) \to 0.$$

Here $J(k)$, $k \in \mathbb{N}$, is the Brown–Gitler module which corepresents the functor $M \mapsto \mathrm{Hom}(M^k, \mathbb{F}_2)$ [8]. We have spectral sequences computing the cohomotopy of $L'(n)$ [2]:

$$\mathrm{Ext}^r_{\mathcal{U}}\left(\mathbb{D}_s \Sigma^{-t}\mathbb{Z}/2, L'_n\right) \Longrightarrow \mathrm{Ext}^{r+s}_{\mathcal{M}}\left(\Sigma^{-t}\mathbb{Z}/2, L'_n\right) \Longrightarrow \left[L'(n), \Sigma^{r+s-t}S^0\right].$$

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Here $\mathcal{M}$ is the category of $\mathcal{A}$-modules and $\mathcal{A}$-linear maps of degree zero and $\mathbb{D}_s$ the s-th derived functor of the destabilisation functor $\mathbb{D}:\mathcal{M}\to\mathcal{U}$ [6] which is left adjoint to the inclusion $\mathcal{U}\hookrightarrow\mathcal{M}$. In order to compute $\mathrm{Ext}^*_{\mathcal{U}}(\mathbb{D}_s\Sigma^{-t}\mathbb{Z}/2, L'_n)$ using the injective resolution above, we need to know the vector space $\mathrm{Hom}_{\mathcal{U}}(\mathbb{D}_s\Sigma^{-t}\mathbb{Z}/2, L_k\otimes J(2^{n-k}-1))$. By adjunction, we need to know the division $(\mathbb{D}_s\Sigma^{-t}\mathbb{Z}/2 : L_k)_{\mathcal{U}}$. Lannes and Zarati showed in [6] that for M an unstable module, there is an isomorphism $\mathbb{D}_s\Sigma^{1-s}M\cong\Sigma R_s M$ where R_s is the Singer functor [6]. In particular, for $s-t\geqslant 1$, $\mathbb{D}_s\Sigma^{-t}\mathbb{Z}/2\cong\Sigma R_s\Sigma^{s-t-1}\mathbb{Z}/2$. The functor R_s associates to $\mathbb{Z}/2$ the Dickson algebra D_s and to an unstable module M a certain submodule of $D_s\otimes M$. One is led to the determination of $(R_s\mathbb{Z}/2 : L_k)_{\mathcal{U}}\cong(D_s : L_k)_{\mathcal{U}}$.

Here is the main result of this note.

Theorem 1. *There is an isomorphism of unstable modules*: $(R_s\mathbb{Z}/2 : L_k)_{\mathcal{U}}\cong R_{s-k}(M_k)$.

Lannes and Zarati showed in [6] that there is a natural short exact sequence $0\to R_s\Sigma M\to\Sigma R_s M\to\Sigma\Phi R_{s-1}M\to 0$ for each unstable module M. By Theorem 1 and by induction on $t\in\mathbb{N}$, one gets $(R_s\Sigma^t\mathbb{Z}/2 : L_k)_{\mathcal{U}}\cong R_{s-k}(\Sigma^t M_k)$. As the functors R_s and R_{s-k} are exact and commute with colimits [6], it follows that $(R_s A : L_k)_{\mathcal{U}}\cong R_{s-k}(A\otimes M_k)$ if A is a locally finite unstable module.

Theorem 1 will be proved in Section 2, basing essentially on two technical lemmas whose proofs will be given in Section 3.

2. Proof of Theorem 1

Given an elementary Abelian 2-group V, i.e. a finite $\mathbb{F}_2$-vector space, the semi-group $\mathrm{End}(V)$ acts naturally on the left of V, and thus on the right of V^* and H^*V by *transposition*. The right action of $\mathrm{Aut}(V)$ on V^* and H^*V can be made into a left action by *contragredient duality*: $(gf)(v)=f(g^{-1}v)$, $g\in\mathrm{Aut}(V)$, $f\in V^*$, $v\in V$.

In order to calculate $(D_W : L_V)_{\mathcal{U}}$, we recall that $(H^*W : H^*V)_{\mathcal{U}}\cong\mathbb{F}_2^{V^*\otimes W}\otimes H^*W$ and this is in fact an $\mathrm{End}(V)\times\mathrm{End}(W)$-equivariant isomorphism. This can be obtained by using the commutation of Lannes' functor T_V with the universal enveloping functor $\mathbf{U}:\mathcal{U}\to\mathcal{K}$ [8]. The isomorphism is adjoint to the following composition:

$$H^*W\xrightarrow{\Delta}H^*W\otimes H^*W\xrightarrow{h\otimes Id}\left[H^*V\otimes\mathbb{F}_2^{V^*\otimes W}\right]\otimes H^*W$$

where Δ is the coproduct and h is adjoint to the natural map:

$$\mathbb{F}_2\left[\mathrm{Hom}(V,W)\right]\otimes H^*W\cong\mathrm{Hom}_{\mathcal{U}}\left(H^*W,H^*V\right)\otimes H^*W\to H^*V.$$

Now let e_λ be a primitive idempotent of $\mathbb{F}_2[\mathrm{End}(V)]$ and $L_\lambda:=(H^*V)e_\lambda$ the indecomposable direct summand of H^*V associated with e_λ. Here we use the right action of $\mathrm{End}(V)$ on H^*V. One gets then:

$$\left(H^*W : L_\lambda\right)\cong\left(e_\lambda\mathbb{F}_2^{V^*\otimes W}\right)\otimes H^*W.$$

As $(- : L_\lambda)_{\mathcal{U}}$ commutes with taking invariant (as in the case of T_V [8]), one gets:

$$(D_W : L_\lambda)_{\mathcal{U}}\cong\left[\left(e_\lambda\mathbb{F}_2^{V^*\otimes W}\right)\otimes H^*W\right]^{\mathrm{Aut}(W)}. \tag{1}$$

Here we consider the contragredient left action of $\mathrm{Aut}(W)$ on H^*W and on $\mathbb{F}_2^{V^*\otimes W}$. To rewrite the isomorphism (1) in a practical way, we use the following two simple facts.

Fact 1. Let G be a group and M,N two *left* $\mathbb{F}_2[G]$-modules with M finite dimensional. Then the linear isomorphism $M\otimes N\to\mathrm{Hom}(M^\#,N)$ given by $m\otimes n\mapsto[f\mapsto f(m)n]$, $m\in M$, $n\in N$, $f\in M^*$, is G-equivariant and induces an isomorphism $(M\otimes N)^G\cong\mathrm{Hom}_{\mathbb{F}_2[G]}(M^\#,N)$.

Here $M^\#$ denotes the contragredient dual of M which is defined to be the linear dual space M^* equipped with the *left* $\mathbb{F}_2[G]$-module structure given by $(gf)(m)=f(g^{-1}m)$, $f\in M^*$, $m\in M$.

Fact 2. Let E be a semi-group acting on the right of a finite set S. Then the composition:

$$\mathbb{F}_2[X]e\hookrightarrow\mathbb{F}_2[X]\xrightarrow{x\mapsto[f\mapsto f(x)]}\left(\mathbb{F}_2^X\right)^*\twoheadrightarrow\left(e\mathbb{F}_2^X\right)^*$$

is an isomorphism of vector spaces for each *idempotent* e in $\mathbb{F}_2[E]$.

In our case, there is an isomorphism $\mathbb{F}_2[V^*\otimes W]e_\lambda\cong(e_\lambda\mathbb{F}_2^{V^*\otimes W})^\#$, and this is actually an isomorphism of *left* $\mathbb{F}_2[\mathrm{Aut}(W)]$-modules. These above facts permit us to rewrite the isomorphism (1) as follows:

$$(D_W : L_\lambda)_{\mathcal{U}}\cong\mathrm{Hom}_{\mathbb{F}_2[\mathrm{Aut}(W)]}\left(\mathbb{F}_2\left[V^*\otimes W\right]e_\lambda, H^*W\right). \tag{2}$$

Here we consider homomorphisms between left $\mathbb{F}_2[\mathrm{Aut}(W)]$-modules.

We now specify to the division by the Steinberg summand of H^*V [7]. For this let us fix an ordered basis $(v_1, \ldots, v_k)$ of V and thus identify each endomorphism of V with its representing matrix with respect to this basis. The Steinberg idempotent [9] of $\mathbb{F}_2[\mathrm{Aut}(V)]$ is given by:

$$\mathbf{e}_V := \sum_{S \in \Sigma_V, B \in \mathrm{B}_V} SB,$$

where B_V denotes the Borel subgroup of *lower* triangular matrices in $\mathrm{Aut}(V)$ and Σ_V the symmetric group on k letters considered as the subgroup of monomial matrices in $\mathrm{Aut}(V)$.

Let M_V be the direct summand of H^*V associated with $\mathbf{e}_V$. This unstable module can be further decomposed by decomposing the Steinberg idempotent $\mathbf{e}_V$ in $\mathbb{F}_2[\mathrm{End}(V)]$. Set $\tilde{\mathbf{e}}_V := \mathbf{e}_V - \mathbf{e}_V \tilde{I}_V \mathbf{e}_V$ where $\tilde{I}_V$ denotes the diagonal matrix $\mathrm{diag}(1, \ldots, 1, 0) \in \mathrm{End}(V)$. Then according to [4, Remark 2.5], $\mathbf{e}_V = \tilde{\mathbf{e}}_V + \mathbf{e}_V \tilde{I}_V \mathbf{e}_V$ is a decomposition of $\mathbf{e}_V$ into a sum of primitive idempotents in $\mathbb{F}_2[\mathrm{End}(V)]$.

Let L_V denote the indecomposable direct summand of H^*V associated with $\tilde{\mathbf{e}}_V$. It follows from the isomorphism (2) that:

$$(D_W : L_V)_{\mathcal{U}} \cong \mathrm{Hom}_{\mathbb{F}_2[\mathrm{Aut}(W)]}\big(\mathbb{F}_2\big[\mathrm{Hom}(V, W)\big]\tilde{\mathbf{e}}_V, H^*W\big). \tag{3}$$

The following technical lemma, which is crucial for the proof of Theorem 1, implies in particular that the division $(D_W : L_V)_{\mathcal{U}}$ is trivial if $\dim V > \dim W$.

Lemma 2. *Let $M \in \mathrm{Hom}(V, W)$ with $\mathrm{rank}(M) < \dim V$. Then $M\tilde{\mathbf{e}}_V = 0$.*

We consider now the case where $\dim V \leqslant \dim W$. By Lemma 2, we have:

$$\mathbb{F}_2\big[\mathrm{Hom}(V, W)\big]\tilde{\mathbf{e}}_V = \mathbb{F}_2\big[\mathrm{Inj}(V, W)\big]\tilde{\mathbf{e}}_V,$$

where $\mathrm{Inj}(V, W) \subset \mathrm{Hom}(V, W)$ is the subset of monomorphisms $V \hookrightarrow W$. Now it is clear that the left $\mathrm{Aut}(W)$-set $\mathrm{Inj}(V, W)$ is transitive. By fixing a monomorphism $\alpha : V \hookrightarrow W$, one has $\mathrm{Inj}(V, W) = \mathrm{Aut}(W)\alpha$. By Lemma 2 and by transitivity of $\mathrm{Inj}(V, W)$, one gets:

$$\mathbb{F}_2\big[\mathrm{Hom}(V, W)\big]\tilde{\mathbf{e}}_V = \mathbb{F}_2\big[\mathrm{Inj}(V, W)\big]\tilde{\mathbf{e}}_V = \mathbb{F}_2\big[\mathrm{Aut}(W)\big]\alpha\tilde{\mathbf{e}}_V,$$

that is, $\mathbb{F}_2[\mathrm{Hom}(V, W)]\tilde{\mathbf{e}}_V$ is generated by $\alpha\tilde{\mathbf{e}}_V$ as a left $\mathbb{F}_2[\mathrm{Aut}(W)]$-submodule of $\mathbb{F}_2[\mathrm{Hom}(V, W)]$. The isomorphism (3) is then rewritten as follows:

$$(D_W : L_V)_{\mathcal{U}} \cong \mathrm{Hom}_{\mathbb{F}_2[\mathrm{Aut}(W)]}\big(\mathbb{F}_2\big[\mathrm{Aut}(W)\big]\alpha\tilde{\mathbf{e}}_V, H^*W\big). \tag{4}$$

Let $\mathrm{Ann}(\alpha\tilde{\mathbf{e}}_V) := \{f \in \mathbb{F}_2[\mathrm{Aut}(W)] \mid f\alpha\tilde{\mathbf{e}}_V = 0\}$ denote the annihilator ideal of $\alpha\tilde{\mathbf{e}}_V$. In order to describe this ideal, let $G_\alpha = \{g \in \mathrm{Aut}(W) \mid g\alpha = \alpha\}$ be the stabiliser subgroup of α and let $\mathbf{e}_\alpha \in \mathbb{F}_2[\mathrm{Aut}(W)]$ be an idempotent which lifts $\mathbf{e}_V \in \mathbb{F}_2[\mathrm{Aut}(V)]$ through α,

$$\begin{array}{ccc} V & \xrightarrow{\mathbf{e}_V} & V \\ \big\downarrow{\scriptstyle\alpha} & & \big\downarrow{\scriptstyle\alpha} \\ W & \xrightarrow{\mathbf{e}_\alpha} & W, \end{array}$$

that is $\alpha\mathbf{e}_V = \mathbf{e}_\alpha\alpha$.

Lemma 3. *The left ideal $\mathrm{Ann}(\tilde{\mathbf{e}}_V\alpha)$ of $\mathbb{F}_2[\mathrm{Aut}(W)]$ is generated by $(1 - \mathbf{e}_\alpha)$ and $\{1 - g \mid g \in G_\alpha\}$.*

Combining the isomorphism (4) with this lemma gives $(D_W : L_V)_{\mathcal{U}} \cong [\mathbf{e}_\alpha H^*W] \cap [H^*W^{G_\alpha}]$. But it is shown in [6] that $R_U(H^*V) \cong H^*W^{G_\alpha}$ and $R_U(M) \cong [H^*U \otimes M] \cap R_U(N)$ if N is an unstable module and M is a submodule of N. It follows that:

$$(D_W : L_V)_{\mathcal{U}} \cong \big[H^*U \otimes \mathbf{e}_V H^*V\big] \cap \big[R_U\big(H^*V\big)\big] \cong R_U\big(\mathbf{e}_V H^*V\big) \cong R_U(M_V).$$

Theorem 1 is proved.

3. Proof of Lemmas 2 and 3

Using the ordered basis $(v_1,\dots,v_k)$ of V, we identify the group $\mathrm{Aut}(V)$ with the general linear group $\mathrm{GL}_k := \mathrm{GL}_k(\mathbb{F}_2)$. Recall that $\tilde{\mathbf{e}}_k = \mathbf{e}_k - \mathbf{e}_k\tilde{I}_k\mathbf{e}_k$ where $\tilde{I}_k$ is the diagonal $k\times k$-matrix $\mathrm{diag}(1,\dots,1,0)$ and $\mathbf{e}_k$ is the Steinberg idempotent of $\mathbb{F}_2[\mathrm{GL}_k]$ defined by $\mathbf{e}_k = \sum_{S\in\Sigma_k, B\in\mathrm{B}_k} SB$, B_k denoting the subgroup of *lower* triangular matrices in GL_k and Σ_k the symmetric group on k letters. We consider the Steinberg idempotent $\mathbf{e}_{k-1}$ of $\mathbb{F}_2[\mathrm{GL}_{k-1}]$ as an element of $\mathbb{F}_2[\mathrm{GL}_k]$ by considering GL_{k-1} as the subgroup of automorphisms of V preserving v_k. It was proved in [3] that $\tilde{I}_k\mathbf{e}_k\tilde{I}_k = \mathbf{e}_{k-1}\tilde{I}_k\mathbf{e}_{k-1}$ and $\mathbf{e}_{k-1}\mathbf{e}_k = \mathbf{e}_k$.

Proof of Lemma 2. We need to prove that if M is an $m\times k$-matrix of rank less than k, then $M\tilde{\mathbf{e}}_k = 0$. Suppose first that the last column of M is zero. Then $M\mathbf{e}_{k-1}$ is a sum of matrices with trivial last column. So $M\tilde{I}_k = M$ and $(M\mathbf{e}_{k-1})\tilde{I}_k = M\mathbf{e}_{k-1}$. We have then:

$$\begin{aligned} M\mathbf{e}_k\tilde{I}_k\mathbf{e}_k &= M\tilde{I}_k\mathbf{e}_k\tilde{I}_k\mathbf{e}_k \quad (\text{as } M\tilde{I}_k = M)\\ &= M\mathbf{e}_{k-1}\tilde{I}_k\mathbf{e}_{k-1}\mathbf{e}_k \quad (\text{as } \tilde{I}_k\mathbf{e}_k\tilde{I}_k = \mathbf{e}_{k-1}\tilde{I}_k\mathbf{e}_{k-1})\\ &= M\mathbf{e}_{k-1}\mathbf{e}_{k-1}\mathbf{e}_k \quad (\text{as } M\mathbf{e}_{k-1}\tilde{I}_k = M\mathbf{e}_{k-1})\\ &= M\mathbf{e}_k \quad \big(\text{as } \mathbf{e}_{k-1}^2 = \mathbf{e}_{k-1} \text{ and } \mathbf{e}_{k-1}\mathbf{e}_k = \mathbf{e}_k\big). \end{aligned}$$

Hence $M\tilde{\mathbf{e}}_k = M\mathbf{e}_k - M\mathbf{e}_k\tilde{I}_k\mathbf{e}_k = 0$.

Now let M be an arbitrary $m\times k$-matrix of rank less than k. One chooses $g\in\mathrm{GL}_k$ such that the last column of $N := Mg$ is trivial. So $M\mathbf{e}_k \in N\mathbb{F}_2[\mathrm{GL}_k]\mathbf{e}_k$. But it is well known from the work of Steinberg [9] that $\mathbb{F}_2[\mathrm{GL}_k]\mathbf{e}_k = \mathbb{F}_2[\mathrm{B}_k]\mathbf{e}_k$. Hence $M\mathbf{e}_k \in N\mathbb{F}_2[\mathrm{B}_k]\mathbf{e}_k$. Since $\mathbf{e}_k\tilde{\mathbf{e}}_k = \tilde{\mathbf{e}}_k$, it follows that $M\tilde{\mathbf{e}}_k \in N\mathbb{F}_2[\mathrm{B}_k]\tilde{\mathbf{e}}_k$. The space $N\mathbb{F}_2[\mathrm{B}_k]\tilde{\mathbf{e}}_k$ is trivial because, for each $B\in\mathrm{B}_k$, the last column of NB is zero, which implies $NB\tilde{\mathbf{e}}_k = 0$ as verified above. The lemma is proved. □

We prove now Lemma 3. For this we need the following elementary fact.

Fact 3. Let G be a finite group acting on the left of a finite set S. For $s\in S$, let $\mathrm{Ann}(s) := \{f\in\mathbb{F}_2[G] \mid fs = 0\}$ denote the annihilator ideal of s and $G_s := \{g\in G \mid gs = s\}$ the stabiliser subgroup of s. Then $\mathrm{Ann}(s)$ is the left ideal generated by $\{1-g \mid g\in G_s\}$.

Proof of Lemma 3. Let $f\in\mathbb{F}_2[\mathrm{Aut}(W)]$ be an element of $\mathrm{Ann}(\alpha\tilde{\mathbf{e}}_V)$, that is $f\alpha\tilde{\mathbf{e}}_V = 0$ in $\mathbb{F}_2[\mathrm{End}(V,W)]$. So $f\alpha\mathbf{e}_V - f\alpha\mathbf{e}_V\tilde{I}_V\mathbf{e}_V = 0$. The first term of the left-hand side is a linear combination of monomorphisms in $\mathrm{Hom}(V,W)$, while the second is a combination of homomorphisms of rank $\dim V - 1$; so each term vanishes, thus $f\alpha\mathbf{e}_V = 0$. But $\alpha\mathbf{e}_V = \mathbf{e}_\alpha\alpha$, so $f\mathbf{e}_\alpha\alpha = 0$. This means that $f\mathbf{e}_\alpha$ belongs to the annihilator ideal $\mathrm{Ann}(\alpha)\subset\mathbb{F}_2[\mathrm{Aut}(W)]$ of α. Hence $f\equiv f(1-\mathbf{e}_\alpha) \bmod \mathrm{Ann}(\alpha)$. By the above fact, $\mathrm{Ann}(\alpha)$ is the left ideal of $\mathbb{F}_2[\mathrm{Aut}(W)]$ generated by $\{1-g \mid g\in G_\alpha\}$, so f belongs to the left ideal of $\mathbb{F}_2[\mathrm{Aut}(W)]$ generated by $(1-\mathbf{e}_\alpha)$ and $\{1-g \mid g\in G_\alpha\}$.

The reverse inclusion is verified easily: that $1-\mathbf{e}_\alpha$ belongs to $\mathrm{Ann}(\alpha\tilde{\mathbf{e}}_V)$ is because $(1-\mathbf{e}_\alpha)\alpha\tilde{\mathbf{e}}_V = \alpha\tilde{\mathbf{e}}_V - \alpha\mathbf{e}_V\tilde{\mathbf{e}}_V = \alpha\tilde{\mathbf{e}}_V - \alpha\tilde{\mathbf{e}}_V = 0$ and that $1-g$, $g\in G_\alpha$, belongs to $\mathrm{Ann}(\alpha\tilde{\mathbf{e}}_V)$ is because $(1-g)\alpha\tilde{\mathbf{e}}_V = (\alpha - g\alpha)\tilde{\mathbf{e}}_V = (\alpha-\alpha)\tilde{\mathbf{e}}_V = 0$. The lemma is proved. □

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