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Atiyah sequences, connections and characteristic forms for principal bundles over groupoids and stacks





Suites d'Ativah, connexions et formes caractéristiques pour les fibrés principaux sur les groupoïdes et les champs

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ABSTRACT

We construct connections and characteristic forms for principal bundles over groupoids and stacks in the differentiable, holomorphic and algebraic category using Atiyah exact sequences associated with transversal tangential distributions.

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RÉSUMÉ

Nous construisons les connexions et formes caractéristiques pour les fibrés principaux sur les groupoïdes et les champs dans la catégorie différentiable, holomorphe et algébrique à l'aide des suites d'Atiyah associées aux distributions transversales tangentielles.

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Version française abrégée

Dans cette note, nous commençons à développer une théorie générale des connexions et des classes caractéristiques pour les fibrés G-principaux sur les groupoïdes et les champs. Les constructions présentées ici fonctionnent aussi bien dans la catégorie différentiable, holomorphe et algébrique que pour les groupoïdes qui ne sont pas nécessairement étales. Étant donné un groupoïde $\mathbb{X} := [X_1 \Rightarrow X_0]$ tel que la source s soit une submersion, nous définissons une connexion comme une distribution particulière transversale $\mathcal{H} \subset TX_1$ dans l'espace tangent TX_1 (voir Définition 3.1).

À l'aide de présentations par des groupoïdes, il est aussi possible, de cette manière, de définir les connexions sur les champs. Pour le cas particulier des champs de Deligne-Mumford, qui sont présentés par des groupoïdes étales, les connexions existent toujours.

Étant donné un groupe de Lie G et un G-fibré principal sur un groupoïde $\mathbb{X} := [X_1 \rightrightarrows X_0]$ avec une connexion \mathcal{H} , nous contruisons une connexion induite $\hat{\mathcal{H}}$ pour le *G*-fibré principal. Un tel fibré est donné par un *G*-fibré principal $\alpha: E_G \to X_0$ sur l'espace X_0 ; nous prouvons que l'existence d'une connexion correspond à l'existence d'une section de la suite d'Atiyah associée [1] :

 $0 \rightarrow \mathrm{ad}(E_G) \rightarrow \mathrm{At}(E_G) \rightarrow TX_0 \rightarrow 0.$

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En utilisant les distributions \mathcal{H} transversales tangentielles, nous construisons aussi des formes de courbure et d'autres formes caractéristiques différentielles pour les *G*-fibrés principaux sur les groupoïdes. Dans le cas des champs de Deligne–Mumford, cette théorie correspond aux constructions de [4].

1. Introduction

In this note we outline a general theory of connections and characteristic classes for principal *G*-bundles over groupoids and stacks, which works equally well in the differentiable, holomorphic, and algebraic setting and for groupoids, which are not necessarily étale. Given a groupoid $\mathbb{X} := [X_1 \Rightarrow X_0]$ in one of these three settings with both X_0 and X_1 smooth, and the source map *s* being a submersion, we define a connection on it as a distribution $\mathcal{H} \subset TX_1$ transversal to the fibers of *s*. It is said to be flat if \mathcal{H} is integrable. Using groupoid presentations, this gives rise to connections and flat connections on stacks under certain compatibility conditions. For the particular case of Deligne–Mumford stacks presented by étale groupoids, a connection always exists, because in this case we have $\mathcal{H} = TX_1$.

Given a Lie group *G* and a principal *G*-bundle over a groupoid $\mathbb{X} := [X_1 \Rightarrow X_0]$ with connection \mathcal{H} , we can define a connection for the principal *G*-bundle. A *G*-bundle over the groupoid \mathbb{X} is basically given by a principal *G*-bundle $\alpha : E_G \rightarrow X_0$ and some extra data as explained in Section 2. A connection for the principal *G*-bundle over the groupoid \mathbb{X} corresponds to a splitting of the associated Atiyah sequence [1]:

$$0 \rightarrow \mathrm{ad}(E_G) \rightarrow \mathrm{At}(E_G) \rightarrow TX_0 \rightarrow 0$$

which is compatible with the data (see Section 4). This allows us to define curvature and characteristic differential forms. Using adequate groupoid presentations this gives a general framework for studying connections and characteristic forms for principal *G*-bundles over stacks. In the case of Deligne–Mumford stacks, this corresponds to the constructions in [4].

Full details of the results presented here will appear in [5]. There we also discuss relations of our approach with Behrend's theory of cofoliations on stacks [2], the various approaches to Chern–Weil theory for étale groupoids via simplicial manifolds [6,9,14] and the infinitesimal theory via Lie algebroids [8,16]. Another aspect of [5] is the systematic construction of secondary characteristic classes and Deligne cohomology for principal *G*-bundles with connections over general groupoids and stacks. This extends the theory of secondary characteristic classes of Cheeger–Simons differential characters to stacks and general groupoids, which for differentiable manifolds were originally constructed in [7], for algebraic varieties in [10] and recently in the case of étale groupoids in [11].

2. Groupoids, stacks and principal bundles

We will consider groupoids and stacks over the following three categories: the differentiable category of C^{∞} -manifolds, the holomorphic category of complex analytic manifolds and the algebraic category of smooth schemes of finite type over a field *k* of characteristic zero. We will refer to any of these categories as the *category* \mathfrak{S} of smooth spaces. Any smooth space *X* has a structure sheaf \mathcal{O}_X . Vector bundles are identified with their locally free sheaves of \mathcal{O}_X -modules. The tangent bundle of *X* will be denoted by *TX*.

A groupoid $\mathbb{X} := [X_1 \rightrightarrows X_0]$ will mean a groupoid internal to the category \mathfrak{S} of smooth spaces, i.e., the space X_1 of arrows and the space X_0 of objects are objects of \mathfrak{S} and all structure morphisms:

$$m: X_1 \times_{X_0} X_1 \to X_1, \quad s, t: X_1 \to X_0, \quad i: X_1 \to X_1, \quad e: X_0 \to X_1$$

are morphisms in \mathfrak{S} and the source map *s* is a submersion. Here *t* is the target map, *m* denotes the multiplication map, *e* is the identity section and *i* is the inversion map of the groupoid. If in addition *s* and *t* are étale, the groupoid $\mathbb{X} = [X_1 \rightrightarrows X_0]$ is called *étale*. If in addition, the *anchor map*:

$$(s,t): X_1 \rightarrow X_0 \times X_0$$

is proper, the groupoid is called a proper groupoid.

Every groupoid $\mathbb{X} = [X_1 \rightrightarrows X_0]$ has an induced tangent groupoid $T\mathbb{X} := [TX_1 \rightrightarrows TX_0]$. A *Lie group G* is a groupoid $[G \rightrightarrows *]$ with one object, i.e., the space X_0 is just a point.

A *stack* \mathfrak{X} will mean a stack over the category \mathfrak{S} (endowed with the "submersion" Grothendieck topology) associated with a groupoid $[X_1 \rightrightarrows X_0]$, such that \mathfrak{X} has a *presentation* or *atlas* $X_1 \rightrightarrows X_0 \rightarrow \mathfrak{X}$, which is a representable surjective submersion. A stack \mathfrak{X} is called a *Deligne–Mumford stack* if there is a presentation $X_1 \rightrightarrows X_0 \rightarrow \mathfrak{X}$ such that the groupoid $[X_1 \rightrightarrows X_0]$ is étale. Orbifolds correspond to proper Deligne–Mumford stacks (see [2,15]). Different presentations of the same stack are given by *Morita equivalent* groupoids and stacks can be seen as Morita equivalence classes of groupoids [3]. Given a groupoid \mathbb{X} , the associated category $\mathfrak{B}\mathbb{X}$ of \mathbb{X} -torsors is a stack [3,5].

Definition 2.1. Let $\mathbb{X} = [X_1 \Rightarrow X_0]$ be a groupoid. An \mathbb{X} -space is given by an object P of \mathfrak{S} together with a smooth map $\pi : P \to X_0$ and a map $\sigma : Q \to P$, $\sigma(\gamma, x) := \gamma \cdot x$, where $Q := X_1 \times_{s, X_0, \pi} P$ is the fiber product such that:

(i) for all $x \in P$, we have: $e(\pi(x)) \cdot x = x$,

(ii) for all $x \in P$ and all $\gamma, \delta \in X_1$ such that $\pi(x) = s(\gamma)$ and $t(\gamma) = s(\delta)$ we have: $(\delta \cdot \gamma) \cdot x = \delta \cdot (\gamma \cdot x)$.

Let *G* be a Lie group and $\mathbb{X} = [X_1 \rightrightarrows X_0]$ a groupoid. A *principal G-bundle* over \mathbb{X} , denoted by $[s^*E_G \rightrightarrows E_G]$ is a principal right *G*-bundle $\pi : E_G \rightarrow X_0$, which is also an \mathbb{X} -space such that for all $x \in E_G$ and all $\gamma \in X_1$ with $s(\gamma) = \pi(x)$, we have $(\gamma \cdot x) \cdot g = \gamma \cdot (x \cdot g)$ for all $g \in G$.

It can be shown that if $\mathbb{X} = [X_1 \rightrightarrows X_0]$ and $\mathbb{Y} = [Y_1 \rightrightarrows Y_0]$ are Morita equivalent groupoids, then there is an equivalence between their categories of principal *G*-bundles (see [3,14]).

Principal *G*-bundles over a stack \mathfrak{X} can be defined directly using a presentation. A principal *G*-bundle \mathfrak{E}_G over a stack \mathfrak{X} is then given by a principal *G*-bundle $E_G \to X_0$ for an atlas $X_0 \to \mathfrak{X}$ together with an isomorphism of the pullbacks $p_1^*E_G \xrightarrow{\sim} p_2^*E_G$ on the fiber product $X_0 \times_{\mathfrak{X}} X_0$ satisfying the cocycle condition on $X_0 \times_{\mathfrak{X}} X_0 \times_{\mathfrak{X}} X_0$. It turns out that for any submersion $f: U \to \mathfrak{X}$, this datum defines a principal *G*-bundle $E_G \to U$ over *U*, because $X_0 \times_{\mathfrak{X}} U \to U$ has local sections. Therefore, we get a stack \mathfrak{E}_G and the *G*-multiplication map glues and there is a natural morphism $G \times \mathfrak{E}_G \to \mathfrak{E}_G$ (see [4,12]). Similarly, we can define vector bundles over a groupoid \mathbb{X} and vector bundles over a stack \mathfrak{X} . They can be identified with respective principal GL_n -bundles.

Let *G* be a Lie group, \mathfrak{X} a stack and $\mathbb{X} = [X_1 \Rightarrow X_0]$ a groupoid presenting \mathfrak{X} . Then there is a canonical equivalence between the category of principal *G*-bundles over \mathfrak{X} and the category of principal *G*-bundles over \mathbb{X} (see [3,12]).

3. Connections on groupoids and stacks

Let $(X = [X_1 \Rightarrow X_0], s, t, m, e, i)$ be a groupoid. The source map $s : X_1 \rightarrow X_0$ is a submersion and therefore the differential $ds : TX_1 \rightarrow s^*TX_0$ of s is surjective. Let:

$$\mathscr{K} := \operatorname{kernel}(\operatorname{ds}) \subset TX_1$$

be the vertical tangent bundle for *s*. Fix a distribution given by a subbundle $\mathscr{H} \subset TX_1$ such that the natural map $\mathscr{H} \oplus \mathscr{H} \to TX_1$ is an isomorphism. Define:

$$d_{\mathscr{H}}s := (ds)|_{\mathscr{H}} : \mathscr{H} \to s^*TX_0$$

to be the restriction of ds to \mathscr{H} . Note that $d_{\mathscr{H}}s$ is an isomorphism.

Let $dt : TX_1 \rightarrow t^*TX_0$ be the differential of *t*. We get a map:

$$\theta := dt \circ (d_{\mathscr{H}} s)^{-1} : s^* T X_0 \to t^* T X_0.$$
⁽¹⁾

For any $y \in X_1$, let:

$$\theta_{\mathbf{y}} := \theta|_{(\mathbf{s}^* T X_0)_{\mathbf{y}}} : T_{\mathbf{s}(\mathbf{y})} X_0 \to T_{t(\mathbf{y})} X_0 \tag{2}$$

be the restriction of θ to the fiber $(s^*TX_0)_v = T_{s(v)}X_0$. We now define:

Definition 3.1. A *connection* on a groupoid $\mathbb{X} = [X_1 \rightrightarrows X_0]$ is a distribution $\mathcal{H} \subset TX_1$ such that:

(i) for every $x \in X_0$, the image of the differential $de(x) : T_x X_0 \to T_{e(x)} X_1$ coincides with the subspace $\mathscr{H}_{e(x)} \subset T_{e(x)} X_1$, and (ii) for every $y, z \in X_1$ with t(y) = s(z), the homomorphism:

 $\theta_{m(z,y)}: T_{s(y)}X_0 \to T_{t(z)}X_0$

coincides with the composition $\theta_z \circ \theta_y$.

A connection on \mathbb{X} is said to be *flat* if the distribution \mathcal{H} is integrable.

Remark 3.2. The above definition of (flat) connections on a groupoid is equivalent to the one given by Behrend in [2] and the one given by Laurent-Gengoux, Tu and Xu [14] in the case of étale groupoids. Flat connections in the differentiable setting were also introduced independently as étalifications by Tang [17]. In fact, a connection as defined above gives a subgroupoid of the tangent groupoid TX, which in the differentiable category is equivalent to the horizontal paths forming a subgroupoid of the path groupoid of X. This definition is also used by Laurent-Gengoux, Stiénon and Xu in [13] to define connections for the general framework of non-Abelian differentiable gerbes via Ehresmann connections on Lie groupoid extensions.

Let \mathscr{H} be a connection on a groupoid $\mathbb{X} = [X_1 \rightrightarrows X_0]$ and $\tau : TX_0 \rightarrow X_0$ be the natural projection. Consider the fiber product $Y_1 := X_1 \times_{s,X_0,\tau} TX_0$. Let $s' : Y_1 \rightarrow TX_0$ be the projection to the second factor and let $t' : Y_1 \rightarrow TX_0$ be the morphism defined by $t'(x, v) = \theta_x(v) \in T_{t(x)}X_0$, where θ_x is defined as in (2). Furthermore, let $p : Y_1 = X_1 \times_{s,X_0,\tau} TX_0 \rightarrow X_1$ be the projection to the first factor. For any $z, y \in Y_1$ with t'(y) = s'(z), define m'(z, y) := (m(p(z), p(y)), s'(y)). Let $e' : TX_0 \rightarrow Y_1$ be the morphism defined by $v \mapsto (e(f(v)), v)$ and finally let $i' : Y_1 \rightarrow Y_1$ be the involution defined by $(z, v) \mapsto (i(z), \theta_z(v))$. We note that $([Y_1 \rightrightarrows TX_0], s', t', m', e', i')$ is a groupoid. In other words, TX_0 becomes a vector bundle over the groupoid \mathbb{X} .

(Flat) connections on groupoids as defined above behave well with respect to pullbacks of Cartesian morphisms and Morita equivalences of groupoids, and therefore we can speak of (flat) connections on stacks. A (flat) connection on a groupoid X descends to the stack $\mathfrak{B}X$ of X-torsors [5].

However, even in the differentiable category, these connections might not always exist. But if X is an étale groupoid, there is always a connection on X, because then we have $\mathcal{H} = TX_1$ and so get actually a unique flat connection (see also [2]). Therefore, connections on a Deligne–Mumford stack \mathfrak{X} always exist and can be defined using an étale groupoid presentation.

4. Connections on principal bundles over groupoids and stacks

Let *G* be a Lie group and let $\alpha : E_G \to X_0$ be a principal *G*-bundle over X_0 . The adjoint vector bundle for E_G will be denoted by $ad(E_G)$. Let $At(E_G) := (\alpha_* T E_G)^G$ be the Atiyah bundle for E_G [1]. We have the exact sequence of Atiyah [1]:

$$0 \to \mathrm{ad}(E_G) \to \mathrm{At}(E_G) \to TX_0 \to 0. \tag{3}$$

The projection $At(E_G) \rightarrow TX_0$ is given by the differential $d\alpha$. We recall that a connection on E_G is a splitting of the exact sequence (3) (see [1]).

Fix a connection \mathscr{H} on the groupoid $\mathbb{X} = [X_1 \Rightarrow X_0]$ and let $\mathbb{E}_G := ([s^* E_G \Rightarrow E_G], \widetilde{s}, \widetilde{t}, \widetilde{m}, \widetilde{e}, \widetilde{i})$ be a principal *G*-bundle over \mathbb{X} . We will show that $\mathbb{E}_G = [s^* E_G \Rightarrow E_G]$ has a connection \mathscr{H} induced by \mathscr{H} . Let:

$$\phi_{X_1}: s^*E_G \to X_1$$

be the natural projection and let $d\phi_{X_1}$: $Ts^*E_G \rightarrow TX_1$ be its differential. Define:

$$\widetilde{\mathscr{H}} := (\mathrm{d}\phi_{X_1})^{-1}(\mathscr{H}) \subset Ts^* E_G.$$

It is straightforward to check that $\widetilde{\mathcal{H}}$ is a connection on the groupoid $\mathbb{E}_G = [s^* E_G \rightrightarrows E_G]$.

As noted above, using \mathscr{H} , the tangent bundle TE_G is turned into a vector bundle over \mathbb{X} . We recall that $At(E_G) = (\alpha_* TE_G)^G$. Using the action of G, the vector bundle TE_G over $\mathbb{E}_G = [s^*E_G \rightrightarrows E_G]$ descends to make $At(E_G)$ a vector bundle over \mathbb{X} . All the morphisms of the vector bundle $At(E_G)$ over \mathbb{X} are given by descent of the corresponding morphisms of the vector bundle TE_G over $\mathbb{E}_G = [s^*E_G \rightrightarrows E_G]$.

Consequently, the exact sequence (3) of vector bundles over X_0 is an exact sequence of vector bundles over the groupoid X.

Definition 4.1. A connection on a principal *G*-bundle $\mathbb{E}_G = [s^*E_G \Rightarrow E_G]$ over a groupoid $\mathbb{X} = [X_1 \Rightarrow X_0]$ is a splitting of the Atiyah sequence of vector bundles over \mathbb{X} given by (3). It is said to be *flat* if it is integrable.

(Flat) connections behave again well with respect to pullbacks of Cartesian morphisms and Morita equivalences of groupoids. So we have, as observed before by Laurent-Gengoux, Stiénon and Xu [13] as well as by Tang [17] (see also [5]):

Proposition 4.1. If X and Y are Morita equivalent groupoids, then there is an equivalence between the respective categories of principal *G*-bundles with (flat) connections over X and Y.

Therefore we can speak of (flat) connections for principal *G*-bundles on a stack \mathfrak{X} (see [5]). In general these might not exist, but for principal *G*-bundles on orbifolds, i.e. proper étale Deligne–Mumford stacks, connections always exist (see also [14]).

A stack \mathfrak{X} determines a collection of Morita equivalent groupoids associated with the presentations of the stack and any principal *G*-bundle $\mathfrak{E}_G \to X_0$ over each atlas $X_0 \to \mathfrak{X}$ satisfying appropriate gluing conditions (see for example [3]). We have the following [5]:

Theorem 4.2. Giving a (flat) connection for a principal *G*-bundle \mathfrak{E}_G over a stack \mathfrak{X} is equivalent to giving (flat) connections for the above principal *G*-bundles on the associated groupoids, which are compatible with pullbacks along Cartesian morphisms. Giving a (flat) connection for a principal *G*-bundle \mathbb{E}_G on a groupoid \mathbb{X} gives a (flat) connection for the principal *G*-bundle \mathfrak{B}_G of \mathbb{E}_G -torsors over the classifying stack $\mathfrak{B}\mathbb{X}$.

5. Curvature and characteristic differential forms

Let \mathscr{H} be a connection on the groupoid $\mathbb{X} = [X_1 \Rightarrow X_0]$. Using the canonical decomposition of the tangent space:

$$TX_1 = \mathscr{H} \oplus \text{kernel}(\text{d}s),$$

we get a projection:

$$\wedge^{j}TX_{1} \to \wedge^{j}\mathscr{H} \hookrightarrow \wedge^{j}TX_{1}.$$

The composition $\wedge^j T X_1 \rightarrow \wedge^j T X_1$ gives by duality an endomorphism of the space of *j*-forms on X_1 .

For a differential form ω on X_1 , the differential form on X_1 induced by the above endomorphism will be denoted by $H(\omega)$.

Definition 5.1. A differential *j*-form on the groupoid $[X_1 \rightrightarrows X_0]$ is a differential *j*-form ω on X_0 such that $H(s^*\omega) = H(t^*\omega)$.

We have the following basic property:

Lemma 5.2. Assume that the distribution $\mathscr{H} \subset TX_1$ is integrable. Let ω be a differential form on X_0 such that $H(s^*\omega) = H(t^*\omega)$. Then $H(s^* d\omega) = H(t^* d\omega)$.

The space of connections on a principal *G*-bundle E_G over the groupoid X is an affine space for the space of all $ad(E_G)$ -valued 1-forms on the groupoid [5].

Henceforth, we will assume that the distribution $\mathcal{H} \subset TX_1$ is integrable, i.e., a *flat* connection.

Consider the Atiyah sequence in (3). The Lie bracket of vector fields defines a Lie algebra structure on the sheaves of sections of all three vector bundles. The Lie algebra structure on the sheaf of sections of $ad(E_G)$ is linear with respect to the multiplication by functions on X_0 , or in other words, the fibers of $ad(E_G)$ are Lie algebras.

The Lie algebra of *G* will be denoted by \mathfrak{g} . Recall that $\operatorname{ad}(E_G) = (E_G \times \mathfrak{g})/G$ is given by the adjoint action of *G* on \mathfrak{g} . Since this adjoint action of *G* preserves the Lie algebra structure of \mathfrak{g} , it follows that the fibers of $\operatorname{ad}(E_G)$ are Lie algebras identified with \mathfrak{g} up to conjugations.

Given any splitting of the Atiyah sequence (3):

$$D: TX_0 \rightarrow At(E_G),$$

the obstruction for D to be compatible with the Lie algebra structure is given by a section:

 $K(D) \in H^0(X_0, \operatorname{ad}(E_G) \otimes \Omega^2_{X_0}),$

which is an $ad(E_G)$ -valued 2-form on the groupoid X. The section K(D) is called the *curvature* of the connection D on the principal G-bundle $\mathbb{E}_G = [s^*E_G \Rightarrow E_G]$ over the groupoid $\mathbb{X} = [X_1 \Rightarrow X_0]$. The following result allows us to use a general Chern–Weil theory for constructing characteristic classes [5], which in the case of étale groupoids was developed before in [9] and [14], but where the characteristic classes are living in different cohomological spaces tailored directly for leaf spaces of foliations and étale groupoids.

Theorem 5.3. For any invariant form $\nu \in (\text{Sym}^k(\mathfrak{g}))^G$, the closed 2k form $\nu(K(D))$ on X_0 is a form on the groupoid.

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References

- [1] M.F. Atiyah, Complex analytic connections in fibre bundles, Trans. Amer. Math. Soc. 85 (1957) 181-207.
- [2] K. Behrend, On the de Rham cohomology of differential and algebraic stacks, Adv. Math. 198 (2005) 583-622.
- [3] K. Behrend, P. Xu, Differentiable stacks and gerbes, J. Symplectic Geom. 9 (2011) 285-341.
- [4] I. Biswas, S. Majumder, M.L. Wong, Root stacks, principal bundles and connections, Bull. Sci. Math. 136 (2012) 369–398.
- [5] I. Biswas, F. Neumann, Atiyah sequences, connections and Chern-Weil theory for algebraic and differentiable stacks, in preparation.
- [6] R. Bott, H. Shulman, J. Stasheff, On the de Rham theory of certain classifying spaces, Adv. Math. 20 (1976) 43–56.

^[7] J. Cheeger, J. Simons, Differential characters and geometric invariants, in: Geometry and Topology, College Park, MD, 1983/84, in: Lect. Notes Math., vol. 1167, Springer-Verlag, Berlin-New York, 1985, pp. 50–80.

^[8] M. Crainic, R.J. Fernandes, Secondary characteristic classes of Lie algebroids, in: Quantum Field Theory and Noncommutative Geometry, in: Lect. Notes Phys., vol. 662, Springer, Berlin, 2005, pp. 157–176.

- [9] M. Crainic, I. Moerdijk, Čech-De Rham theory for leaf spaces of foliations, Math. Ann. 328 (2004) 59-85.
- [10] H. Esnault, Algebraic differential characters, in: Regulators in Analysis, Geometry and Number Theory, in: Prog. Math., vol. 171, Birkhäuser Boston, 2000, pp. 89-115.
- [11] M. Felisatti, F. Neumann, Secondary theories for étale groupoids, Contemp. Math. 571 (2012) 135–151.
- [12] J. Heinloth, Notes on differentiable stacks, in: Y. Tschinkel (Ed.), Mathematisches Institut Seminars, Georg-August Universität Göttingen, 2004–2005, pp. 1–32.
- [13] C. Laurent-Gengoux, P. Stiénon, P. Xu, Non-Abelian differentiable gerbes, Adv. Math. 220 (2009) 1357–1427.
- [14] C. Laurent-Gengoux, J.-L. Tu, P. Xu, Chern-Weil map for principal bundles over groupoids, Math. Z. 255 (2007) 451-491.
- [15] E. Lerman, A. Malkin, Hamiltonian group actions on symplectic Deligne-Mumford stacks and toric orbifolds, Adv. Math. 229 (2012) 984-1000.
- [16] M.A. Salazar Pinzón, Pfaffian groupoids, PhD thesis, Utrecht, 2013, arXiv:1306.1164v2 [math.DG].
- [17] X. Tang, Deformation quantization of pseudo-symplectic (Poisson) groupoids, Geom. Funct. Anal. 16 (2006) 731-766.