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Norm-resolvent convergence for elliptic operators in domain with perforation along curve

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A R T I C L E   I N F O

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A B S T R A C T

We consider an infinite strip perforated along a curve by small holes. In this perforated domain, we consider a scalar second-order elliptic differential operator subject to classical boundary conditions on the holes. Assuming that the perforation is non-periodic, we describe possible homogenized problems and prove the norm-resolvent convergence of the perturbed operator to a homogenized one. We also provide estimates for the rate of the convergence.

R É S U M É

On considère une bande infinie avec une famille de petits trous placés le long d'une courbe. Dans ce domaine perforé, on étudie un opérateur scalaire elliptique du second ordre, avec des conditions aux limites classiques aux bords des trous. En supposant que l'emplacement des trous n'est pas périodique, on décrit les problèmes homogénéisés possibles et on démontre la convergence au sens de la norme de la résolvante des opérateurs perturbés vers les opérateurs homogénéisés. On obtient également des estimées pour le taux de convergence.

1. Introduction

Homogenization theory is a rich field in modern mathematics, see, for instance, [1–11], and references therein. One of the perturbations treated in homogenization theory is the perforation by small holes along a given curve or manifold; see, for instance, [8], further works by these authors and references therein. The main result obtained for a general non-periodic perforation was the description of homogenized problems and the proof of convergence of perturbed solutions to the homogenized ones. Reformulating it in terms of operator theory, strong and/or weak convergence of perturbed operators to

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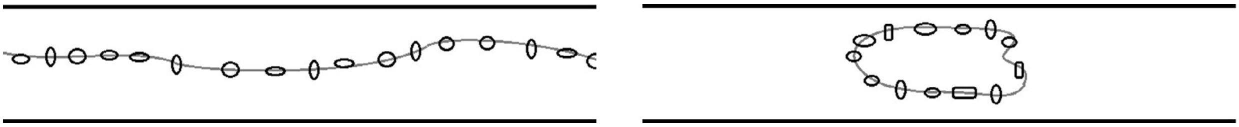

(a) Perforation along an infinite curve (b) Perforation along a closed curve

Fig. 1. Perforated domain.

homogenized ones were proven. Recently results on norm-resolvent convergence for operators with fast periodically oscillating coefficients appeared [2,6,7,9,11]. Moreover, the estimates for the rate of this convergence were established. These results were extended for operators with frequent alternation of boundary conditions [3,4] and with fast oscillating boundary [5]. And in this note we present a similar study for operators with perforation along a curve. Our main results are the proof of norm-resolvent convergence in various operator norms and the estimates for the rate of convergence. This is done for all possible homogenized operators. We succeeded to get these results for a general non-periodic perforation under rather weak assumptions.

2. Formulation of problem and main results

Let $x=(x_1,x_2)$ be the Cartesian coordinates in $\mathbb{R}^2$, $\Omega:=\{x:0<x_2<d\}$ be an infinite strip, and γ be a curve in Ω separated from $\partial\Omega$ by a positive distance. We assume that γ is C^2-smooth, has a bounded curvature and no self-intersections. Curve γ is either an infinite curve or finite and closed. By s we indicate the arc length of γ, $s\in(-s_*,s_*)$, where s_* is either finite or $s_*=+\infty$, and $x=\varrho(s)$ is the equation of γ.

Let $\mathbb{M}^\varepsilon\subseteq\mathbb{Z}$ be a set and $s_k\in(-s_*,s_*)$, $k\in\mathbb{M}^\varepsilon$, be some points obeying $s_k<s_{k+1}$. By ω_k, $k\in\mathbb{M}^\varepsilon$, we indicate bounded domains in $\mathbb{R}^2$ with C^2-boundaries. Denoting by ε a small positive parameter, we let $\theta^\varepsilon:=\theta_0^\varepsilon\cup\theta_1^\varepsilon$, $\theta_i^\varepsilon:=\bigcup_{k\in\mathbb{M}_i}\omega_k^\varepsilon$, $i=0,1$, $\omega_k^\varepsilon:=\{x:\varepsilon^{-1}\eta^{-1}(\varepsilon)(x-y_k^\varepsilon)\in\omega_k\}$, $y_k^\varepsilon:=\varrho(s_k\varepsilon)$, where $\mathbb{M}_i^\varepsilon$ are subsets of $\mathbb{M}^\varepsilon$, $\mathbb{M}_0^\varepsilon\cap\mathbb{M}_1^\varepsilon=\emptyset$, $\mathbb{M}_0^\varepsilon\cup\mathbb{M}_1^\varepsilon=\mathbb{M}^\varepsilon$, and $\eta=\eta(\varepsilon)$ is a certain function obeying $0<\eta(\varepsilon)\leqslant 1$.

Since γ has a bounded curvature and is either infinite or closed, it splits domain Ω into two disjoint subdomains. The upper/exterior one is denoted by Ω_+ and the lower/interior one by Ω_-. By ν^0 we denote the normal to γ which is inward for Ω_-. Let τ be the distance from γ to a point measured along ν^0.

Set θ^ε is the union of small holes ω_k^ε and we cut out them in Ω, introducing then a perforated domain $\Omega^\varepsilon:=\Omega\setminus\theta^\varepsilon$, cf. Fig. 1. The main object of our study is the operator $\mathcal{H}^\varepsilon$ in $L_2(\Omega^\varepsilon)$ introduced by the differential expression

$$-\sum_{i,j=1}^{2}\frac{\partial}{\partial x_i}A_{ij}\frac{\partial}{\partial x_j}+\sum_{j=1}^{2}A_j\frac{\partial}{\partial x_j}-\frac{\partial}{\partial x_j}\overline{A_j}+A_0 \tag{1}$$

in Ω^ε, subject to the Dirichlet condition on $\partial\Omega\cup\partial\theta_0^\varepsilon$ and to the Robin condition

$$\left(\frac{\partial}{\partial N^\varepsilon}+a\right)u=0\quad\text{on }\partial\theta_1^\varepsilon,\qquad \frac{\partial}{\partial N^\varepsilon}:=\sum_{i,j=1}^{2}A_{ij}\nu_i^\varepsilon\frac{\partial}{\partial x_j}+\sum_{j=1}^{2}\overline{A}_j\nu_j^\varepsilon,$$

where $\nu^\varepsilon=(\nu_1^\varepsilon,\nu_2^\varepsilon)$ is the inward normal to $\partial\theta_1^\varepsilon$, $a=a(x)$ is a some function defined as $|\tau|<\tau_0$, where τ_0 is a small fixed positive number. It is assumed that $a\in W_\infty^1(\{x:|\tau|<\tau_0\})$. The coefficients of operator $\mathcal{H}^\varepsilon$ are supposed to satisfy the conditions $A_{ij}, A_i\in W_\infty^1(\Omega)$, $A_{ji}=A_{ji}$, $i,j=1,2$, $A_0\in L_\infty(\Omega)$, and A_{ij}, A_0 are real-valued. For coefficients A_{ij}, we also assume the standard uniform ellipticity condition in Ω. Rigorously we introduce $\mathcal{H}^\varepsilon$ as the self-adjoint operator associated with an appropriate symmetric sesquilinear form in $L_2(\Omega^\varepsilon)$; this form can be easily written in view of the above definition of $\mathcal{H}^\varepsilon$ and we do not dwell on it. In what follows, all the operators are introduced in the same way.

A physical motivation for the above operator comes from the waveguide theory. The domains like infinite two-dimensional strips, multi-dimensional tubes or multi-dimensional layers appear in modeling semiconductors or waveguides. An elliptic operator is in fact a Hamiltonian describing the dynamics of a quantum particle living in the considered region. Here we deal with a general operator and as particular cases it involves a usual Schrödinger operator, a magnetic Schrödinger operator with an electric potential. By choosing appropriate coefficients A_{ij}, we can also introduce various metrics in the space.

The Dirichlet boundary condition corresponds to the wall, and here the particle can not pass through. The Neumann condition can be interpreted as a kind of “window”, while the Robin condition corresponds to a “window” influenced by a magnetic field. The perforation by small holes with such boundary conditions should be regarded as micro-defects in our media. The presence of such defects is a very natural assumption, since there are always some inhomogeneities in the reality. And the aim of this paper is to study the influence of such micro-defects on the macroscopic properties of the model. Namely, we show how one can replace approximately the defects along the reference curve by some classical regime on this curve. We also estimate the error made under such replacement.

Mathematical formulation for the aim of the present paper is to study the resolvent convergence of $\mathcal{H}^\varepsilon$ as $\varepsilon \to +0$. In order to do it, we make the following assumptions.

(A1) There exist fixed numbers $0 < R_1 < R_2$, $b > 1$, $L > 0$, and points $x^k \in \mathbb{R}^2$, $k \in \mathbb{M}^\varepsilon$, such that

$$\begin{aligned} &B_{R_1}(x^k) \subset \omega_k \subset B_{R_2}(0), \qquad |\partial\omega_k| \leqslant L \quad \text{for each } k \in \mathbb{M}^\varepsilon,\\ &B_{bR_2\varepsilon}(y_k^\varepsilon) \cap B_{bR_2\varepsilon}(y_i^\varepsilon) = \emptyset \quad \text{for each } i,k \in \mathbb{M}^\varepsilon,\ i \neq k, \end{aligned} \tag{2}$$

and for all sufficiently small ε.

(A2) For b and R_2 in (A1) and $k \in \mathbb{M}^\varepsilon$, there exists a generalized solution $X_k : B_{b_* R_2}(0) \setminus \omega_k \mapsto \mathbb{R}^2$, $b_* := (b+1)/2$, to the boundary value problem

$$\operatorname{div} X_k = 0 \quad \text{in } B_{b_* R_2}(0) \setminus \omega_k, \qquad X_k \cdot \nu = -1 \quad \text{on } \partial\omega_k, \qquad X_k \cdot \nu = \varphi_k \quad \text{on } \partial B_{b_* R_2}(0), \tag{3}$$

belonging to $L_\infty(B_{b_* R_2}(0) \setminus \omega_k)$ and bounded in the sense of this space uniformly in $k \in \mathbb{M}^\varepsilon$. Here ν is the outward normal to $\partial B_{b_* R_2}(0)$ and to $\partial\omega_k$, while φ_k is a some function in $L_\infty(\partial B_{b_* R_2}(0))$ satisfying

$$\int\limits_{\partial B_{b_* R_2}(0)} \varphi_k \, \mathrm{d}s = |\partial\omega_k|. \tag{4}$$

Let us introduce the first homogenized operator. We denote it by $\mathcal{H}_{\mathrm{D}}^0$ and it is the operator in $L_2(\Omega)$ with the differential expression (1) subject to the Dirichlet condition on γ and $\partial\Omega$.

By i we indicate the imaginary unit and the symbol $\|\cdot\|_{X\to Y}$ stands for the norm of an operator acting from a Banach space X to a Banach space Y.

Now we are ready to formulate our first main result.

Theorem 2.1. *Let*

$$\varepsilon \ln \eta(\varepsilon) \to 0, \quad \varepsilon \to +0, \tag{5}$$

suppose (A1), (A2), *and*

(A3) *There exists a constant $R_3 > bR_2$ such that*

$$\{x : |\tau| < \varepsilon b R_2\} \subset \bigcup_{k \in \mathbb{M}_0^\varepsilon} B_{R_3\varepsilon}(y_k^\varepsilon), \qquad \omega_k^\varepsilon \subset B_{R_3\varepsilon}(y_k^\varepsilon) \quad \textit{for all } k \in \mathbb{M}_0^\varepsilon.$$

Then the estimate

$$\big\|(\mathcal{H}^\varepsilon - \mathrm{i})^{-1} - (\mathcal{H}_{\mathrm{D}}^0 - \mathrm{i})^{-1}\big\|_{L_2(\Omega)\to W_2^1(\Omega^\varepsilon)} \leqslant C\varepsilon^{1/2}\big(|\ln \eta(\varepsilon)|^{1/2} + 1\big)$$

holds true, where C is a positive constant independent of ε.

Given a function $\beta \in W_\infty^1(\gamma)$, we introduce the operator $\mathcal{H}_\beta^0$ with the differential expression (1) subject to the boundary conditions

$$[u]_\gamma = 0, \qquad \left[\frac{\partial u}{\partial N^0}\right]_\gamma + \beta u|_\gamma = 0, \qquad \frac{\partial}{\partial N^0} := \sum_{i,j=1}^2 A_{ij} \nu_i^0 \frac{\partial}{\partial x_j}, \tag{6}$$

where $\nu^0 = (\nu_1^0, \nu_2^0)$ and $[u]_\gamma = u|_{\tau=+0} - u|_{\tau=-0}$. Once $\beta = 0$, we shall simply write $\mathcal{H}^0$ instead of $\mathcal{H}_0^0$. In this case, there is no jump of the normal derivative in (6) and it means that the boundary condition on γ disappears.

Our next main result is as follows.

Theorem 2.2. *Suppose* (A1), (A2), *let*

$$\frac{1}{\varepsilon \ln \eta(\varepsilon)} \to -\rho, \quad \varepsilon \to +0, \tag{7}$$

and set $\mathbb{M}_0^\varepsilon$ be non-empty. For b and R_2 in (A1) *and $s \in \mathbb{R}$ we denote*

$$\alpha^\varepsilon(s) := \begin{cases} \frac{\pi}{bR_2}, & |s - \varepsilon s_k| < bR_2\varepsilon,\ k \in \mathbb{M}_0^\varepsilon,\\ 0, & \textit{otherwise}. \end{cases}$$

Assume also that

(A4) *there exists a function* $\alpha\in W^1_\infty(\gamma)$ *and a function* $\varkappa=\varkappa(\varepsilon)$, $\varkappa(\varepsilon)\to+0$, $\varepsilon\to+0$, *such that for all sufficiently small* ε *the estimate*

$$\sum_{q\in\mathbb{Z}}\frac{1}{|q|+1}\left|\int\limits_n^{n+\ell}\big(\alpha^\varepsilon(s)-\alpha(s)\big)\mathrm{e}^{-\frac{\mathrm{i}q}{2\pi\ell}(s-n)}\,\mathrm{d}s\right|^2\leqslant\varkappa^2(\varepsilon)\tag{8}$$

is valid, where $n=-s_*$, $\ell=2s_*$, *if* γ *is a finite curve, and* $n\in\mathbb{Z}$, $\ell=1$, *if* γ *is an infinite curve. In the latter case, estimate (8) is supposed to hold uniformly in n.*

Denote

$$\beta:=-\alpha\frac{(\rho+\mu)}{A_{11}A_{22}-A_{12}^2},\qquad\beta_0:=-\alpha\frac{\rho}{A_{11}A_{22}-A_{12}^2},\qquad\mu(\varepsilon):=-\frac{1}{\varepsilon\ln\eta(\varepsilon)}-\rho.$$

Then the estimates

$$\big\|\big(\mathcal{H}^\varepsilon-\mathrm{i}\big)^{-1}f-\big(\mathcal{H}^0_\beta-\mathrm{i}\big)^{-1}f\big\|_{L_2(\Omega)\to L_2(\Omega^\varepsilon)}\leqslant C\big(\varepsilon^{1/2}+\varkappa(\varepsilon)\big),\tag{9}$$

$$\big\|\big(\mathcal{H}^\varepsilon-\mathrm{i}\big)^{-1}-\big(\mathcal{H}^0_{\beta_0}-\mathrm{i}\big)^{-1}\big\|_{L_2(\Omega)\to L_2(\Omega^\varepsilon)}\leqslant C\big(\varepsilon^{1/2}+\varkappa(\varepsilon)+\mu(\varepsilon)\big)\tag{10}$$

hold true, where C is a positive constant independent of ε. *There exists an explicit function* W^ε *such that the estimate*

$$\big\|\big(\mathcal{H}^\varepsilon-\mathrm{i}\big)^{-1}-\big(1-W^\varepsilon\big)\big(\mathcal{H}^0_\beta-\mathrm{i}\big)^{-1}\big\|_{L_2(\Omega)\to W_2^1(\Omega^\varepsilon)}\leqslant C\big(\varepsilon^{1/2}+\varkappa(\varepsilon)\big)\tag{11}$$

is valid, where C is a positive constant independent of ε. *If* $\rho=0$, *then the estimate*

$$\big\|\big(\mathcal{H}^\varepsilon-\mathrm{i}\big)^{-1}-\big(\mathcal{H}^0_{\beta_0}-\mathrm{i}\big)^{-1}\big\|_{L_2(\Omega)\to W_2^1(\Omega^\varepsilon)}\leqslant C\big(\varepsilon^{1/2}+\mu^{1/2}(\varepsilon)\big)\tag{12}$$

holds true, where C is a positive constant independent of ε.

In the next two theorems we study the case when $\mathbb{M}^\varepsilon_0$ is empty and there are no holes with Dirichlet condition.

Theorem 2.3. *Suppose* (A1), (A2), *let set* $\mathbb{M}^\varepsilon_0$ *be empty and either* $a\equiv0$ *or* $\eta(\varepsilon)\to0$, $\varepsilon\to+0$. *Then the estimates*

$$\big\|\big(\mathcal{H}^\varepsilon-\mathrm{i}\big)^{-1}-\big(\mathcal{H}^0-\mathrm{i}\big)^{-1}\big\|_{L_2(\Omega)\to W_2^1(\Omega^\varepsilon)}\leqslant C\eta(\varepsilon)\big(\big|\ln\eta(\varepsilon)\big|+1\big),\quad a\not\equiv0,$$

$$\big\|\big(\mathcal{H}^\varepsilon-\mathrm{i}\big)^{-1}f-\big(\mathcal{H}^0-\mathrm{i}\big)^{-1}f\big\|_{L_2(\Omega)\to W_2^1(\Omega^\varepsilon)}\leqslant C\varepsilon^{1/2}\eta(\varepsilon)\big(\big|\ln\eta(\varepsilon)\big|^{1/2}+1\big),\quad a\equiv0,$$

holds true, where C is a positive constant independent of ε.

Theorem 2.4. *Suppose* (A1), (A2), *let* $\eta=\mathrm{const}$, *set* $\mathbb{M}^\varepsilon_0$ *be empty, and for b and* R_2 *in* (A1) *we denote*

$$\alpha^\varepsilon(s):=\begin{cases}\frac{|\partial\omega_k|}{2bR_2}, & |s-\varepsilon s_k|<bR_2\varepsilon\eta,\ k\in\mathbb{M}^\varepsilon,\\ 0, & \textit{otherwise}.\end{cases}$$

Assume that

(A5) *there exists a function* α *in* $W^1_\infty(\gamma)$ *and a function* $\varkappa=\varkappa(\varepsilon)$, $\varkappa(\varepsilon)\to+0$, $\varepsilon\to+0$, *such that for all sufficiently small* ε *the estimates (8) are valid.*

then the estimate

$$\big\|\big(\mathcal{H}^\varepsilon-\mathrm{i}\big)^{-1}-\big(\mathcal{H}^0_{\alpha a}-\mathrm{i}\big)^{-1}\big\|_{L_2(\Omega)\to W_2^1(\Omega^\varepsilon)}\leqslant C\big(\varepsilon^{1/2}+\varkappa^{1/2}(\varepsilon)\big)$$

holds true, where C is a positive constant independent of ε.

Let us discuss briefly the main results. We first dwell on the assumptions. Set $\mathbb{M}^\varepsilon$ can be arbitrary and even finite. It means that there is a lot of freedom in distributing the holes along the curve. For instance, the distances between them can be small, finite or even large, once we choose $\mathbb{M}^\varepsilon$ in an appropriate way. Then the distribution of holes with different types of boundary conditions is also arbitrary. The main assumptions are (A1), (A2). The former is very natural and it just says that all the holes are approximately of the same linear size $\sim\varepsilon\eta$, while the minimal distances between the holes is $\sim\varepsilon$. The second assumption is more gentle. It is clear that for each fixed hole boundary value problem (3) is solvable, since function φ_k is not fixed and the only condition (4) for this function is in fact the solvability condition of (3). What we in fact assume here is that solutions for (3) are bounded in $L_\infty(B_{b*R_2}(0)\setminus\omega_k)$ uniformly in k.

Theorem 2.1 describes the case when the homogenization leads to the Dirichlet condition on γ. Here we need (5) providing the relation between the sizes of the holes and the distances between them and also assumption (A3). The latter means that there should be infinitely many holes with Dirichlet condition and they should be distributed rather uniformly along the whole curve.

Theorem 2.2 deals with the case when the sizes of the holes with Dirichlet condition are exponentially small, cf. (7). Function α^ε describes the distribution of the holes with Dirichlet condition, and additional assumption (A4) says that this function converges in the sense of (8). We observe that the left-hand side in (8) is the norm in $W_2^{\frac{1}{2}}(n,n+\ell)$. Under the above assumptions, the homogenized operator involves condition (6), which can be interpreted as a delta-interaction. We also note that condition (A4) is satisfied for periodic distribution of the holes with Dirichlet condition as well as for various non-periodic cases. For instance, one can take periodically distributed holes and change arbitrarily some of them so that the amount of deformed holes is relatively small with respect to unchanged ones. Another example is a perforation by a fixed number of holes. We stress that in this theorem there is no special assumptions for the holes with Robin condition, except (A1), (A2). We also observe that in this theorem there are several estimates stating the uniform resolvent convergence; they are formulated either for different operator norm or for different operators that can serve as a homogenized one. Namely, the worst estimate (10) is formulated for the norm $\|\cdot\|_{L_2(\Omega)\to L_2(\Omega^\varepsilon)}$ and the right-hand side depends on ε, $\varkappa$, and μ. Once we replace β_0 by β (the latter depends on μ), we can get rid of μ in the estimate. And by employing a special corrector W^ε we can improve the operator norm to $\|\cdot\|_{L_2(\Omega)\to W_2^1(\Omega^\varepsilon)}$. And once ρ vanishes, there is a special estimate (12).

In the other theorems, it is assumed that there are no holes with Dirichlet condition. In this case the homogenized operator either has no condition on γ (Theorem 2.3) or involves boundary condition (6) (Theorem 2.4). In the latter case, we need additional assumption (A5), which is the restriction for the distribution of the perimeters and the positions of the holes.

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