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Partial differential equations

Global existence and boundedness of classical solutions in a quasilinear parabolic–elliptic chemotaxis system with logistic source

Existence globale et bornes pour les solutions classiques d'un système quasi linéaire, parabolique–elliptique, de chimiotaxie avec source logistique

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A R T I C L E I N F O

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A B S T R A C T

We consider the quasilinear parabolic–elliptic chemotaxis system

$$\begin{cases} u_t = \nabla \cdot (D(u)\nabla u - \chi u \nabla v) + g(u), & x \in \Omega,\ t > 0, \\ 0 = \Delta v - v + u, & x \in \Omega,\ t > 0, \end{cases}$$

under homogeneous Neumann boundary conditions in a smooth bounded domain $\Omega \subset \mathbb{R}^n, n \geq 1$. We assume that the functions D and g are smooth and satisfy

$$D(s) > 0 \ \text{for}\ s \geq 0,\ D(s) \geq C_D s^{m-1} \ \text{for}\ s > 0,$$

$$g(0) \geq 0,\ g(s) \leq a - bs^{\gamma}, \quad s > 0$$

with some constants $C_D > 0, m \geq 1, a \geq 0, b > 0$ and $\gamma > 2$.
We prove that the classical solutions to the above system are uniformly in-time-bounded without any restrictions on m and b. This result extends one of the recent results by Wang et al. (2014) [16], which assert the boundedness of solutions for $\gamma > 2$ under the condition $b > b^*$ with $b^* = 0$ for $m \geq 2 - \frac{2}{n}$ and $b^* = \frac{(2-m)n-2}{(2-m)n}\chi$ for $m < 2 - \frac{2}{n}$.

R É S U M É

Nous considérons le système quasi linéaire, parabolique–elliptique, de chimiotaxie

$$\begin{cases} u_t = \nabla \cdot (D(u)\nabla u - \chi u \nabla v) + g(u), & x \in \Omega,\ t > 0, \\ 0 = \Delta v - v + u, & x \in \Omega,\ t > 0, \end{cases}$$

avec des conditions au bord homogènes de Neumann, dans un domaine lisse, borné $\Omega \subset \mathbf{R}^n$, $n \geq 1$. Nous supposons que les fonctions D et

$$D(s) > 0 \ \text{pour}\ s \geq 0,\ D(s) \geq C_D s^{m-1} \ \text{pour}\ s > 0,$$

$$g(0) \geq 0,\ g(s) \leq a - bs^{\gamma}, \quad s > 0$$

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pour certaines constantes $C_D > 0$, $m \geq 1$, $a \geq 0$, $b > 0$ et $\gamma > 2$.
Nous démontrons que les solutions classiques du système ci-dessus sont uniformément bornées en temps, sans restriction sur m et b. Ceci étend un résultat récent de Wang et al. (2014) [16], qui borne les solutions pour $\gamma > 2$ sous la condition $b > b^*$, où $b^* = 0$ si $m \geq 2 - \frac{2}{n}$ et $b^* = \frac{(2-m)n-2}{(2-m)n}\chi$ si $m < 2 - \frac{2}{n}$.

1. Introduction

In this Note, we study the following initial boundary value problem:

$$\begin{cases} u_t = \nabla \cdot (D(u)\nabla u - S(u)\nabla v) + g(u), & x \in \Omega,\ t > 0, \\ \tau v_t = \Delta v - v + u, & x \in \Omega,\ t > 0, \\ \dfrac{\partial u}{\partial \nu} = \dfrac{\partial v}{\partial \nu} = 0, & x \in \partial\Omega,\ t > 0, \\ u(x,0) = u_0,\ \ v(x,0) = v_0, & x \in \Omega, \end{cases} \tag{1.1}$$

where $\Omega \subseteq \mathbb{R}^n, n \geq 1$, is a bounded domain with smooth boundary, $\tau \in \{0, 1\}$ and ν denotes the unit outward normal vector to $\partial\Omega$. We assume that $S(u) = \chi u$ with $\chi > 0$ and the function D belongs to $C^2([0,\infty))$ and satisfies

$$D(s) > 0 \ \text{ for } s \geq 0,\ D(s) \geq C_D s^{m-1} \ \text{ for } s > 0 \tag{1.2}$$

with some constants $C_D > 0$ and $m \geq 1$. We also assume that g is a smooth function that satisfies

$$g(0) \geq 0,\ g(s) \leq a - bs^\gamma \ \text{ for } s > 0 \tag{1.3}$$

with constants $a \geq 0, b > 0$ and $\gamma > 2$. Moreover, $u_0 \in C^\alpha(\overline{\Omega})$ and $v_0 \in W^{1,r}(\Omega)$ for some $\alpha > 0$ and $r > n$.

Problems of this kind are used in mathematical biology to illustrate the mechanism of chemotaxis, that is, the movement of cells towards the gradient of a substance called chemoattractant produced by the cells themselves. Here, $u = u(x,t)$ denotes the cell density and $v = v(x,t)$ is the concentration of the chemical substance. While the functions D and S are the diffusivity and chemotactic sensitivity, respectively, and g is the growth of u [7,4].

In the absence of a logistic source, when $D \equiv 1$, $S(u) = \chi u$ with $\chi > 0$ and $\tau > 0$, for the one-dimensional case, it is shown that blow-up phenomena cannot occur [11]. For the two-dimensional case, it is proved that if $\|u_0\|_{L^1(\Omega)} < \frac{4\pi}{\chi}$, then all solutions are global and bounded [10]. While for $\|u_0\|_{L^1(\Omega)} > \frac{4\pi}{\chi}$, solutions become unbounded either in finite or infinite time [5]. For higher-dimensional case, it is shown that for each $q > \frac{n}{2}$ and $p > n$, there exists $\epsilon_0 > 0$ such that if $\|u_0\|_{L^q(\Omega)} < \epsilon$ and $\|v_0\|_{L^p(\Omega)} < \epsilon$ for $\epsilon < \epsilon_0$, then the corresponding solutions are global and bounded [18]. Also, when Ω is a ball for $\|u_0\|_{L^1(\Omega)} > 0$, it is proved that there exist solutions blow up in finite time [22]. For the case $D \equiv 1$, $S(s) \leq c(s+1)^q$ for $s \geq 0$ with $c > 0$ and $\tau > 0$, solutions are global and bounded provided that $q < \frac{2}{n}$ and solutions blow up in finite time or infinite time if $S(s) \geq c(s+1)^q$ for $s \geq 0$ with $c > 0$ and $q > \frac{2}{n}$ [6]. When Ω is a bounded convex domain in $\mathbb{R}^n, n \geq 2$, and $\tau > 0$, then solutions are uniformly-in-time bounded provided that $\frac{S(s)}{D(s)} \leq c(s+1)^\alpha$ for $s \geq 0$ with $c > 0$ and $\alpha < \frac{2}{n}$ and other additional conditions are fulfilled [13], whereas for the case where $\frac{S(s)}{D(s)} \geq c(s+1)^\alpha$ for $s \geq 0$ with $c > 0$ and $\alpha > \frac{2}{n}$, there exist solutions blow up either in finite or infinite time [19]. If the second equation is replaced with $0 = \Delta v - M + u$, where M denotes the mean value of initial data u_0, then under the conditions $D(s) \geq c_D s^{-p}$ and $S(s) \leq c_S s^q$ for $s \geq 1$ with $c_D, c_S > 0$, $p \geq 0$ and $q \in \mathbb{R}$, all solutions are global and uniformly bounded provided that $p + q < \frac{2}{n}$, whereas for $0 < D(s) \leq c_D s^{-p}$ and $S(s) \geq c_S s(s+1)^{q-1}$ for $s \geq 0$ with $c_D, c_S > 0, p \geq 0, q > 0$ and $p + q > \frac{2}{n}$, there exist radial solutions that become unbounded in finite time [25].

In the presence of logistic source, when $D \equiv 1, S(u) = \chi u$ with $\chi > 0$ and $\tau = 0$, it is proved that problem (1.1) admits at least one global very weak solution if $\gamma > 2 - \frac{1}{n}$ [17]. Also, in this case, it is shown that solutions are global and bounded if $\gamma = 2$ and $b > \frac{n-2}{n}\chi$ [14]. The same result is true for $\tau > 0$ and $\gamma = 2$ provided that $n \leq 2$ or $n \geq 3$ and $b > b_0$ with b_0 sufficiently large [12,20]. Also, in this case, for $n \geq 3$, it is proved that there exists at least one global weak solution for arbitrary $b > 0$ [8]. Moreover, in this case when the ratio $\frac{b}{\chi}$ is sufficiently large, it is shown that for any choice of suitably regular nonnegative initial data, there exists a unique global classical solution (u, v) such that $\|u(.,t) - \frac{1}{b}\|_{L^\infty(\Omega)} \to 0$ and $\|v(.,t) - \frac{1}{b}\|_{L^\infty(\Omega)} \to 0$ as $t \to \infty$ [23]. If the first equation is written as $u_t = -\nabla \cdot (u\nabla v) + au - bu^2$ and $\tau = 0$, then all solutions are global in time for $b \geq 1$, and there exist solutions blow up in finite time if $b < 1$. These results have been obtained by Winkler for the one-dimensional case [24] and Lankeit for higher-dimensional case [9]. When Ω is a ball in $\mathbb{R}^n, n \geq 5$, for the case $D \equiv 1$ and $S(u) = \chi u$ with $\chi > 0$, if the second equation is replaced with $0 = \Delta v + u - \frac{1}{|\Omega|}\int_\Omega u(x,t)dx$ and g satisfies in the condition $g(s) \geq -bs^\gamma$ for $s \geq 0$ with $b \geq 0$ and $\gamma > 1$ and other additional conditions, then radially symmetric solutions blow up in finite time provided that $1 < \gamma < \frac{3}{2} + \frac{1}{2n-2}$ [21]. Also, for $n \geq 5$ in the case where $D(s) \leq s^{-p}$

and $S(s) = s^q$ for $s > 0$ with $p, q \in \mathbb{R}$ and g satisfies in the condition $g(s) \geq a - bs^\gamma$ for $s \geq 0$ with $a, b \geq 0$ and $\gamma > 1$ and other additional conditions, blow-up phenomena occur in finite time if $\frac{2}{n} - 1 < p < 1, q > 1$ with $2p + 3q < 4$ and $1 < \gamma < \frac{(3-p)n-2}{2n-2}$ [26]. Moreover, it is shown that under the conditions $D(s) \geq (s+1)^{-p}$ and $S(s) \leq s^q$ for $s \geq 0$ with $p, q \in \mathbb{R}$, all solutions are global and uniformly bounded provided that $p + q < \frac{2}{n}$ and $\gamma > 1$ or $p + q \geq \frac{2}{n}$, $b > \frac{(p+q)n-2}{(p+q)n}\chi$ and $\gamma \geq q + 1$ with $q \geq 1$ [26]. In the case where $\tau > 0$, under the conditions $D(s) \geq c_1 s^p, c_2 s^q \leq S(s) \leq c_3 s^q$ for $s \geq s_0 > 1$ with $c_i > 0, i = 1, 2, 3$ and $p, q \in \mathbb{R}$ and g is smooth on $[0, \infty)$ satisfying $g(s) \leq as - bs^2$ for $s \geq 0$ with $a \geq 0$ and $b > 0$, it is proved that the solutions are global and bounded provided that $q < 1$. This result is independent of the choice of p [2]. Wang et al. [16] studied problem (1.1) under the conditions (1.2) and (1.3) with $S(u) = \chi u$ and $\tau = 0$, where χ is some positive constant. Their results improve the recent result in [3], which asserts the boundedness of solutions with $\gamma = 2$ under the condition $b > \chi(1 - \frac{2}{n(1-m)_+})$, or, equivalently, $m > 1 - \frac{2\chi}{n(\chi-b)_+}$. In fact, Wang et al. proved that for $\gamma \geq 2$ under the condition $b > b^*$ with $b^* = 0$ for $m \geq 2 - \frac{2}{n}$ and $b^* = \frac{(2-m)n-2}{(2-m)n}\chi$ for $m < 2 - \frac{2}{n}$, problem (1.1) has a unique nonnegative classical solution, which is global and bounded. They also proved that the same result is true provided that $1 < \gamma < 2$ and $m > 2 - \frac{2}{n}$. Besides, for the case $\gamma = 2$, under the conditions $0 < b \leq \frac{(2-m)n-2}{(2-m)n}\chi$ and $m < 2 - \frac{2}{n}$, they proved that problem (1.1) has at least one nonnegative global solution in the weak sense.

In the present paper, we will study problem (1.1) with $\tau = 0$ under the conditions (1.2) and (1.3). We will prove that for $\gamma > 2$, problem (1.1) has a unique solution, which is uniformly in-time-bounded. We do not know, for the limit case $\gamma = 2$ under the conditions $0 < b \leq \frac{(2-m)n-2}{(2-m)n}\chi$ and $m < 2 - \frac{2}{n}$, whether solutions exist globally or blow up in finite time. So, the global existence or the blowing up of solutions will remain an open question in the case where $m < 2 - \frac{2}{n}$ and $1 < \gamma \leq 2$. In the next section, we will prove the above result.

2. Proof of main result

Here, we state the standard well-posedness and classical solvability result.

Lemma 2.1. *Let functions D and g satisfy (1.2) and (1.3). Moreover, we assume that $u_0 \in C^\alpha(\overline{\Omega})$ and $v_0 \in W^{1,r}(\Omega)$ are nonnegative functions for some $\alpha > 0$ and $r > n$. Then problem (1.1) has a unique non-negative classical solution that can be extended up to its maximal existence time $T_{\max} \in (0, \infty]$. In addition, if $T_{\max} < +\infty$, then*

$$\lim_{t \to T_{\max}} \|u(.,t)\|_{L^\infty(\Omega)} = \infty.$$

For details of the proof, we refer the reader to [25,16].

In order to prove the boundedness of the solution, we need the following lemma, which is given in [15].

Lemma 2.2. *Let y be a positive absolutely continuous function on $(0, \infty)$ that satisfies*

$$\begin{cases} \frac{dy}{dt} + Ay^p \leq B, \\ y(0) = y_0 \end{cases}$$

with some constants $A > 0$, $B \geq 0$ and $p > 1$. Then for $t > 0$, we have

$$y(t) \leq \max\left\{ y_0, \left(\frac{B}{A}\right)^{\frac{1}{p}} \right\}.$$

Although the proof of the following lemma is given in [16], we present it here for completeness.

Lemma 2.3. *Assume that the function g satisfies (1.3). Then for all $t \in (0, T_{\max})$, there exists a constant $C > 0$ such that*

$$\|u(.,t)\|_{L^1(\Omega)} \leq C. \tag{2.1}$$

Proof. Integrating the first equation in (1.1) and using (1.3), we get

$$\frac{d}{dt} \int_\Omega u \, dx = \int_\Omega g(u) \, dx \leq \int_\Omega (a - bu^\gamma) \, dx.$$

Hence,

$$\frac{d}{dt} \int_\Omega u \, dx + b \int_\Omega u^\gamma \, dx \leq a|\Omega|. \tag{2.2}$$

Making use of Hölder's inequality, we obtain

$$\int_\Omega u^\gamma \,dx \ge \Big(\int_\Omega u\,dx\Big)^\gamma |\Omega|^{1-\gamma}.$$

Combining this inequality with (2.2) gives

$$\frac{d}{dt}\int_\Omega u\,dx + b|\Omega|^{1-\gamma}\Big(\int_\Omega u\,dx\Big)^\gamma \le a|\Omega|.$$

Because of $\gamma > 2$, we can apply Lemma 2.2 and obtain the desired result. □

Lemma 2.4. *Assume that the function g satisfies (1.3). Then for all $k > 1$ and $t \in (0, T_{\max})$, there exists a constant $C > 0$ such that*

$$\|u(.,t)\|_{L^k(\Omega)} \le C.$$

Proof. At first, we compute

$$\begin{aligned}\frac{d}{dt}\int_\Omega u^k dx &= k\int_\Omega u^{k-1}u_t\,dx\\ &= k\int_\Omega u^{k-1}\nabla\cdot\Big(D(u)\nabla u - \chi u\nabla v\Big)dx + k\int_\Omega u^{k-1}g(u)\,dx\\ &\le -k(k-1)\int_\Omega u^{k-2}D(u)|\nabla u|^2\,dx + k(k-1)\chi\int_\Omega u^{k-1}\nabla u\cdot\nabla v\,dx\\ &\quad + ak\int_\Omega u^{k-1}\,dx - bk\int_\Omega u^{k+\gamma-1}\,dx. \end{aligned} \tag{2.3}$$

We make use of integrating by parts and use the second equation in (1.1) to obtain

$$\begin{aligned}k(k-1)\chi\int_\Omega u^{k-1}\nabla u\cdot\nabla v\,dx &= (k-1)\chi\int_\Omega \nabla u^k\cdot\nabla v\,dx\\ &= -(k-1)\chi\int_\Omega u^k\Delta v\,dx\\ &= -(k-1)\chi\int_\Omega u^k(v-u)\,dx\\ &\le (k-1)\chi\int_\Omega u^{k+1}\,dx.\end{aligned}$$

Substituting this inequality into (2.3), we get:

$$\begin{aligned}&\frac{d}{dt}\int_\Omega u^k dx + k(k-1)\int_\Omega u^{k-2}D(u)|\nabla u|^2\,dx\\ &\le (k-1)\chi\int_\Omega u^{k+1}\,dx + ak\int_\Omega u^{k-1}\,dx - bk\int_\Omega u^{k+\gamma-1}\,dx.\end{aligned} \tag{2.4}$$

Because of $\gamma > 2$, we can use the Young inequality with exponents $s = \frac{k+\gamma-1}{k+1}$ and $s' = \frac{k+\gamma-1}{\gamma-2}$. Thus, we obtain

$$\int_\Omega u^{k+1}\,dx \le \frac{b}{\chi}\int_\Omega u^{k+\gamma-1}\,dx + C_1,$$

where $C_1 = (\frac{b}{\chi}s)^{-\frac{s'}{s}}(s')^{-1}|\Omega|$ is a positive constant. Hence, we can write

$$-bk\int_\Omega u^{k+\gamma-1}\,dx \le -\chi k\int_\Omega u^{k+1}\,dx + C_2$$

with $C_2 = \chi k C_1$. Combining the last inequality with (2.4) gives

$$\frac{d}{dt}\int_\Omega u^k dx + k(k-1)\int_\Omega u^{k-2}D(u)|\nabla u|^2\,dx \le -\chi\int_\Omega u^{k+1}\,dx + ak\int_\Omega u^{k-1}\,dx + C_2. \tag{2.5}$$

We now make use of Young's inequality to the second term on the right-hand side of (2.5) to get

$$ak\int_\Omega u^{k-1}\,dx \le \frac{\chi}{2}\int_\Omega u^{k+1}\,dx + C_3,$$

where $C_3 = \frac{2}{k+1}(ak)^{\frac{k+1}{2}}\left(\frac{2(k-1)}{\chi(k+1)}\right)^{\frac{k-1}{2}}|\Omega|$ is a positive constant. Substituting this inequality into (2.5) yields

$$\frac{d}{dt}\int_\Omega u^k dx + \frac{\chi}{2}\int_\Omega u^{k+1}\,dx \le C_4 \tag{2.6}$$

with $C_4 = C_2 + C_3$. We now make use of Hölder's inequality to obtain

$$\int_\Omega u^{k+1}\,dx \ge \left(\int_\Omega u^k\,dx\right)^{\frac{k+1}{k}}|\Omega|^{-\frac{1}{k}}.$$

This inequality along with (2.6) yields

$$\frac{d}{dt}\int_\Omega u^k dx + \frac{\chi}{2}|\Omega|^{-\frac{1}{k}}\left(\int_\Omega u^k\,dx\right)^{\frac{k+1}{k}} \le C_4.$$

By applying Lemma 2.2, the last inequality yields $\|u\|_{L^k(\Omega)}$, which is bounded for all $t \in (0, T_{\max})$. This completes the proof. □

By using Lemma 2.4 and Alikakos' iterative technique [1] (see also [13, Lemma A.1]), we can infer our main result.

Theorem 2.5. *Let $u_0 \in C^\alpha(\overline{\Omega})$ and $v_0 \in W^{1,r}(\Omega)$ be nonnegative functions for some $\alpha > 0$ and $r > n$. Moreover, we assume that functions D and g satisfy (1.2) and (1.3). Then problem (1.1) admits a unique global classical solution which is uniformly in-time-bounded.*

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