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Partial differential equations

Boundedness in a three-dimensional chemotaxis–haptotaxis model with nonlinear diffusion

Existence de solution bornée pour les modèles tri-dimensionnels de chimio-haptotaxie avec diffusion non-linéaire

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ABSTRACT

The quasilinear chemotaxis–haptotaxis system

$$\begin{cases} u_t = \nabla \cdot (D(u)\nabla u) - \chi \nabla \cdot (u \nabla v) - \xi \nabla \cdot (u \nabla w) & \\ \qquad + \mu u(1 - u - w), & x \in \Omega,\ t > 0, \\ v_t = \Delta v - v + u, & x \in \Omega,\ t > 0, \\ w_t = -vw, & x \in \Omega,\ t > 0, \end{cases}$$

is considered under homogeneous Neumann boundary conditions in a bounded and smooth domain $\Omega \subset \mathbb{R}^3$. Here $\chi > 0$, $\xi > 0$ and $\mu > 0$, $D(u) \geq c_D u^{m-1}$ for all $u > 0$ with some $c_D > 0$ and $D(u) > 0$ for all $u \geq 0$. It is shown that if the ratio $\frac{\chi}{\mu}$ is sufficiently small, then the system possesses a unique global classical solution that is uniformly bounded. Our result is independent of m.

RÉSUMÉ

Nous considérons le système quasi-linéaire de chimio-haptotaxie

$$\begin{cases} u_t = \nabla \cdot (D(u)\nabla u) - \chi \nabla \cdot (u \nabla v) - \xi \nabla \cdot (u \nabla w) & \\ \qquad + \mu u(1 - u - w), & x \in \Omega,\ t > 0, \\ v_t = \Delta v - v + u, & x \in \Omega,\ t > 0, \\ w_t = -vw, & x \in \Omega,\ t > 0, \end{cases}$$

sous des conditions aux limites de Neumann homogènes, dans un domaine borné et lisse $\Omega \subset \mathbb{R}^3$. Ici $\chi > 0$, $\xi > 0$, $\mu > 0$, $D(u) \geq c_D u^{m-1}$ pour tout $u > 0$ et un $c_D > 0$, et $D(u) > 0$ pour tout $u \geq 0$. Lorsque le quotient χ/μ est assez petit, nous montrons que le système

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possède une unique solution globale classique, qui est uniformément bornée. Notre résultat est sans restriction sur m.

1. Introduction

In this paper, we consider the chemotaxis–haptotaxis system with nonlinear diffusion

$$\begin{cases} u_t = \nabla\cdot(D(u)\nabla u) - \chi\nabla\cdot(u\nabla v) - \xi\nabla\cdot(u\nabla w) + \mu u(1-u-w), & x\in\Omega,\ t>0,\\ v_t = \Delta v - v + u, & x\in\Omega,\ t>0,\\ w_t = -vw, & x\in\Omega,\ t>0,\\ D(u)\frac{\partial u}{\partial \nu} - \chi u\frac{\partial v}{\partial \nu} - \xi u\frac{\partial w}{\partial \nu} = \frac{\partial v}{\partial \nu} = 0, & x\in\partial\Omega,\ t>0,\\ u(x,0)=u_0(x),\, v(x,0)=v_0(x),\, w(x,0)=w_0(x) & x\in\Omega, \end{cases} \tag{1.1}$$

where $\Omega\subset\mathbb{R}^n$ is a bounded domain with smooth boundary $\partial\Omega$ and $\partial/\partial\nu$ denotes the derivative with respect to the outer normal of $\partial\Omega$, and the parameters χ, ξ and μ are assumed to be positive. The origin of the system was proposed by Chaplain and Lolas [3,2] to describe cancer cell invasion into surrounding healthy tissue. Here, $u=u(x,t)$ denotes the density of the cancer cell, $v=v(x,t)$ represents the concentration of enzyme, and $w=w(x,t)$ represents the density of extracellular matrix. In this model, cancer cells bias their movement towards a gradient of a diffusible matrix-degrading enzyme (MDE) secreted by themselves, as well as a gradient of a static tissue, the so-called extracellular matrix (ECM), by detecting matrix molecules such as the macromolecules adhered therein. Here we assume that $D(u)$ is a nonlinear function and satisfies

$$D\in C^2([0,\infty)) \tag{1.2}$$

and there exist some constants $c_D>0$ and $m\in\mathbb{R}$ such that

$$D(u)\ge c_D u^{m-1} \text{ for all } u>0. \tag{1.3}$$

If, in addition to (1.2) and (1.3), $D(u)$ satisfies

$$D(u)>0 \text{ for all } u\ge 0, \tag{1.4}$$

so the diffusion is nondegenerate and the solutions may be considered in the sense of classical.

Model (1.1) and its analogue have been extensively studied up to now [1,8,12–18,20–22,33]. For instance, when $D(u)$ satisfies (1.2)–(1.4), Tao and Winkler [15] showed that model (1.1) has global solutions provided that $m>\frac{2n^2+4n-4}{n(n+4)}$ $(n\le 8)$ or $m>\frac{2n^2+3n+2-\sqrt{8n(n+1)}}{n(n+2)}$ $(n\ge 9)$. If $n\ge 2$, the global boundedness of solutions to (1.1) has been constructed for $m>2-\frac{2}{n}$ [8,20]. Recently, the range of global and bounded to (1.1) is extended to $m>2-\frac{4}{n+2}$ by Wang [21]. In particular, if $n=2$, Zheng et al. [33] showed that (1.1) has global and bounded solutions, but for $n\ge 3$, the authors gave an open questions whether there exist global and bounded solutions to (1.1) under the similar assumption.

When $w\equiv 0$, this system has been widely studied by many authors, where the main issue of the investigation was whether the solutions to the models are bounded or blow-up [4,6,7,9–11,19–32]. In particular, it is known that arbitrarily small $\mu>0$ guarantee the boundedness of solutions when $n=2$ [11], for sufficiently large μ, the solutions are global and bounded when $n\ge 3$ [26]; however, solutions to this system with $\mu=0$ may blow up in finite time [4,28]. On the other hand, the solutions to the parabolic–elliptic system may blow up in finite time provided growth restrictions weaker than quadratic [27].

The main goal of this paper is to give an affirmative answer to the question of global boundedness proposed in [33] in the three-dimensional. As to the initial data (u_0,v_0,w_0), we suppose that for some $\alpha\in(0,1)$

$$\begin{cases} u_0\in C(\overline{\Omega}), & u_0\ge 0 \text{ in } \Omega,\\ v_0\in W^{1,\infty}(\Omega), & v_0\ge 0 \text{ in } \Omega,\\ w_0\in C^{2+\alpha}(\overline{\Omega}), & w_0\ge 0 \text{ in } \overline{\Omega}, \quad \frac{\partial w_0}{\partial\nu}=0 \text{ on } \partial\Omega. \end{cases} \tag{1.5}$$

Our main results in this paper are stated as follows.

Theorem 1.1. *Let $\Omega\subset\mathbb{R}^3$ be a bounded domain with smooth boundary, and (u_0,v_0,w_0) satisfy (1.5) and D satisfy (1.2)–(1.4). Then there exists $\vartheta_0>0$ such that for $\chi>0$, $\xi>0$ and $\mu>0$ if $\frac{\chi}{\mu}<\vartheta_0$, model (1.1) has a unique nonnegative global and bounded solution.*

Remark 1.1. (i) Our result is independent of m, which is improve previous results [15,20,21].
(ii) If $D(u) \equiv 1$, then Theorem 1.1 is consistent with the result of Cao [1].
(iii) If the effects of the ECM are ignored (i.e. $w \equiv 0$) and $D(u) \equiv 1$, then Theorem 1.1 is consistent with the result of Winkler [26].

2. Preliminaries

Let us state the local existence result, which has been established in [15].

Lemma 2.1. *Let $\chi > 0$, $\xi > 0$ and $\mu > 0$, and assume that (u_0, v_0, w_0) satisfies (1.5) and D satisfies (1.2)–(1.4). Then (1.1) possesses a unique classical solution*

$$\begin{cases} u \in C^0(\overline{\Omega} \times [0, T_{\max})) \cap C^{2,1}(\overline{\Omega} \times (0, T_{\max})), \\ v \in C^0(\overline{\Omega} \times [0, T_{\max})) \cap C^{2,1}(\overline{\Omega} \times (0, T_{\max})), \\ w \in C^{2,1}(\overline{\Omega} \times (0, T_{\max})) \end{cases} \tag{2.1}$$

with $u \geq 0$, $v \geq 0$ and $0 \leq w \leq \|w_0\|_{L^\infty(\Omega)}$, where $T_{\max} \in (0, \infty]$ denotes the maximal existence time. Finally, if $T_{\max} < +\infty$, then

$$\limsup_{t \nearrow T_{\max}} \|u(\cdot, t)\|_{L^\infty(\Omega)} = \infty. \tag{2.2}$$

Next let us give some important estimates for v.

Lemma 2.2. *Let $T \in (0, \infty)$. We consider the following heat equations:*

$$\begin{cases} v_t = \Delta v - v + u, & \Omega \times (0, T), \\ \frac{\partial v}{\partial \nu} = 0, & \partial\Omega \times (0, T), \\ v(x, 0) = v_0(x), & x \in \Omega, \end{cases} \tag{2.3}$$

where $v_0 \in W^{2,\theta}(\Omega)$ ($\theta > 1$) and $u \in L^\theta(0, T; L^\theta(\Omega))$ are nonnegative functions. Then for $C_\theta > 0$, there exists a unique solution

$$v \in W^{1,\theta}(0, T; L^\theta(\Omega)) \cap L^\theta(0, T; W^{2,\theta}(\Omega)), \tag{2.4}$$

and if for $s \in (0, T)$, $v(\cdot, s) \in W^{2,\theta}(\Omega)$ ($\theta > 1$) with $\frac{\partial v(\cdot, s)}{\partial \nu} = 0$ on $\partial\Omega$, then

$$\int_s^T \int_\Omega e^{\theta t} |\Delta v|^\theta \mathrm{d}x\,\mathrm{d}t \leq C_\theta \int_s^T \int_\Omega e^{\theta t} u^\theta \mathrm{d}x\,\mathrm{d}t + C_\theta \int_\Omega v^\theta(\cdot, s)\mathrm{d}x + C_\theta \int_\Omega |\Delta v(\cdot, s)|^\theta \mathrm{d}x. \tag{2.5}$$

Let $K > 0$. If $\|u(\cdot, t)\|_{L^1(\Omega)} \leq K$ for all $t \in (0, T)$, then for $r \in \left[1, \frac{n}{n-2}\right)$ there exists $C = C(r, K) > 0$ such that

$$\|v(\cdot, t)\|_{L^r(\Omega)} \leq C \quad \textit{for all } t \in (0, T). \tag{2.6}$$

If $\|u(\cdot, t)\|_{L^\theta(\Omega)} \leq K$ for all $t \in (0, T)$, then for $\theta > \frac{n}{2}$ there exists $C = C(\theta, K) > 0$ such that

$$\|v(\cdot, t)\|_{L^\infty(\Omega)} \leq C \quad \textit{for all } t \in (0, T). \tag{2.7}$$

Proof. The estimates (2.4) and (2.5) are referred to as a variation of Maximal Sobolev Regularity, which was proposed in Theorem 3.1 in [5] (see also Lemma 2.2 in [31]). The estimates (2.6) and (2.7) are from Lemma 2.6 in [8]. □

The following estimates result from integration of the first and the second equation in (1.1).

Lemma 2.3. *Let (u, v, w) be the solution to (1.1), then there exists $C > 0$ such that*

$$\|u(\cdot, t)\|_{L^1(\Omega)} \leq C \quad \textit{and} \quad \|v(\cdot, t)\|_{L^1(\Omega)} \leq C \quad \textit{for all } t \in (0, T_{\max}). \tag{2.8}$$

3. Proof of Theorem 1.1

Motivated by the methods in [1] (see also [31]), we drive a bound for u in the space $L^\infty(0, T_{\max}; L^r(\Omega))$ for all $r > 1$. Let us now pick any $s \in (0, T_{\max})$ and $s \leq 1$, then by the regularity principle asserted by Lemma 2.1, we have $(u(\cdot, s), v(\cdot, s), w(\cdot, s)) \in (C^2(\overline{\Omega}))^3$ with $\frac{\partial v(\cdot, s)}{\partial \nu} = 0$ on $\partial\Omega$. In particular, we can pick M_0 such that

$$\sup_{0 \leq \tau \leq s} \|u(\cdot, \tau)\|_{L^\infty(\Omega)} \leq M_0, \quad \sup_{0 \leq \tau \leq s} \|v(\cdot, \tau)\|_{L^\infty(\Omega)} \leq M_0, \quad \|\Delta v(\cdot, s)\|_{L^\infty(\Omega)} \leq M_0. \tag{3.1}$$

Lemma 3.1. *Assume that D satisfies (1.2)–(1.4) and (u_0, v_0, w_0) satisfies (1.5). Let $\chi > 0$, $\xi > 0$, $\mu > 0$ and $r > 1$. There exists $\vartheta_r > 0$ and $C > 0$ such that if $\vartheta := \frac{\chi}{\mu}$ satisfies $\vartheta \le \vartheta_r$ and*

$$\|v(\cdot,t)\|_{L^{r+1}(\Omega)} \le C \quad \text{for all} \quad t \in (0, T_{\max}), \tag{3.2}$$

then

$$\|u(\cdot,t)\|_{L^{r}(\Omega)} \le C \quad \text{for all} \quad t \in (0, T_{\max}). \tag{3.3}$$

Proof. Multiplying the first equation of (1.1) by u^{r-1} and integrating, we obtain from (1.4)

$$\begin{aligned}
\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}t}\int_\Omega u^r &= \int_\Omega u^{r-1}\nabla\cdot(D(u)\nabla u - \chi u\nabla v - \xi u \nabla w) + \mu\int_\Omega u^r(1-u-w) \\
&= -(r-1)\int_\Omega D(u)u^{r-2}|\nabla u|^2 + \chi(r-1)\int_\Omega u^{r-1}\nabla u\cdot\nabla v \\
&\quad + \xi(r-1)\int_\Omega u^{r-1}\nabla u\cdot\nabla w + \mu\int_\Omega u^r(1-u-w) \\
&\le -\frac{r+1}{r}\int_\Omega u^r + \frac{\chi(r-1)}{r}\int_\Omega \nabla u^r\cdot\nabla v + \frac{\xi(r-1)}{r}\int_\Omega \nabla u^r\cdot\nabla w \\
&\quad + \left(\mu + \frac{r+1}{r}\right)\int_\Omega u^r - \mu\int_\Omega u^{r+1}.
\end{aligned} \tag{3.4}$$

Once more integrating by parts and by Young's inequality, we obtain

$$\begin{aligned}
\frac{\chi(r-1)}{r}\int_\Omega \nabla u^r\cdot\nabla v &= -\frac{\chi(r-1)}{r}\int_\Omega u^r\Delta v \le \chi\int_\Omega u^r|\Delta v| \\
&\le \frac{\mu}{8}\int_\Omega u^{r+1} + c_1\chi^{r+1}\left(\frac{\mu}{8}\right)^{-r}\int_\Omega |\Delta v|^{r+1},
\end{aligned} \tag{3.5}$$

where $c_1 = \sup\limits_{r>1}\frac{1}{r}\left(1+\frac{1}{r}\right)^{-(r+1)} < \infty$. Due to $\Delta w(x,t) \ge \Delta w_0(x)\mathrm{e}^{-\int_0^t v(x,s)\mathrm{d}s} + 2\nabla \mathrm{e}^{-\int_0^t v(x,s)\mathrm{d}s}\cdot\nabla w_0(x) - \frac{w_0(x)}{e} - w_0(x) \times v(x,t)\mathrm{e}^{-\int_0^t v(x,s)\mathrm{d}s}$ (see Lemma 2.2 in [13]), we have

$$\begin{aligned}
\frac{\xi(r-1)}{r}\int_\Omega \nabla u^r\cdot\nabla w &= -\frac{\xi(r-1)}{r}\int_\Omega u^r\Delta w \\
&\le \xi\|\Delta w_0\|_{L^\infty(\Omega)}\int_\Omega u^r v + M\xi\int_\Omega u^r \\
&\le \frac{\mu}{8}\int_\Omega u^{r+1} + c_1\xi^{r+1}\|\Delta w_0\|^{r+1}_{L^\infty(\Omega)}\left(\frac{\mu}{8}\right)^{-r}\int_\Omega v^{r+1} + M\xi\int_\Omega u^r,
\end{aligned} \tag{3.6}$$

where $M := \|\Delta w_0\|_{L^\infty(\Omega)} + 4\|\nabla\sqrt{w_0}\|^2_{L^\infty(\Omega)} + \frac{\|w_0\|_{L^\infty(\Omega)}}{e}$. Collecting (3.4)–(3.6), we obtain

$$\begin{aligned}
\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}t}\int_\Omega u^r + \frac{r+1}{r}\int_\Omega u^r &\le -\frac{3\mu}{4}\int_\Omega u^{r+1} + c_1\chi^{r+1}\left(\frac{\mu}{8}\right)^{-r}\int_\Omega|\Delta v|^{r+1} \\
&\quad + c_1\xi^{r+1}\|\Delta w_0\|^{r+1}_{L^\infty(\Omega)}\left(\frac{\mu}{8}\right)^{-r}\int_\Omega v^{r+1} + \left(\mu + \frac{r+1}{r} + M\xi\right)\int_\Omega u^r.
\end{aligned} \tag{3.7}$$

By Young's inequality, we have

$$\left(\mu + \frac{r+1}{r} + M\xi\right)\int_\Omega u^r \le \frac{\mu}{4}\int_\Omega u^{r+1} + c_1\left(\mu + \frac{r+1}{r} + M\xi\right)^{r+1}\left(\frac{\mu}{4}\right)^{-r}|\Omega|. \tag{3.8}$$

Combining (3.7), (3.8) and (3.2), we obtain

$$\begin{aligned}\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}t}\int_\Omega u^r+\frac{r+1}{r}\int_\Omega u^r &\le -\frac{\mu}{2}\int_\Omega u^{r+1}+c_1\xi^{r+1}\|\Delta w_0\|^{r+1}_{L^\infty(\Omega)}\left(\frac{\mu}{8}\right)^{-r}\int_\Omega v^{r+1}\\ &\quad+c_1\chi^{r+1}\left(\frac{\mu}{8}\right)^{-r}\int_\Omega|\Delta v|^{r+1}+c_1\left(\mu+\frac{r+1}{r}+M\xi\right)^{r+1}\left(\frac{\mu}{4}\right)^{-r}|\Omega|\\ &\le -\frac{\mu}{2}\int_\Omega u^{r+1}+c_1\chi^{r+1}\left(\frac{\mu}{8}\right)^{-r}\int_\Omega|\Delta v|^{r+1}+c_2,\end{aligned}\tag{3.9}$$

where $c_2=c_1C^{r+1}\xi^{r+1}\|\Delta w_0\|^{r+1}_{L^\infty(\Omega)}\left(\frac{\mu}{8}\right)^{-r}+c_1\left(\mu+\frac{r+1}{r}+M\xi\right)^{r+1}\left(\frac{\mu}{4}\right)^{-r}|\Omega|$. Hence, using the variation-of-constants formula to the above inequality and (3.1) we obtain

$$\begin{aligned}\frac{1}{r}\int_\Omega u^r &\le \frac{\mathrm{e}^{-(r+1)(t-s)}}{r}\int_\Omega u^r(\cdot,s)+c_2\int_s^t \mathrm{e}^{-(r+1)(t-\tau)}\\ &\quad-\frac{\mu}{2}\int_s^t \mathrm{e}^{-(r+1)(t-\tau)}\int_\Omega u^{r+1}+c_1\chi^{r+1}\left(\frac{\mu}{8}\right)^{-r}\int_s^t \mathrm{e}^{-(r+1)(t-\tau)}\int_\Omega|\Delta v|^{r+1}\\ &\le \frac{\mathrm{e}^{(r+1)s}M_0^r|\Omega|}{r}+\frac{c_2}{r+1}-\frac{\mu}{2}\int_s^t\int_\Omega \mathrm{e}^{-(r+1)(t-\tau)}u^{r+1}\\ &\quad+c_1\chi^{r+1}\left(\frac{\mu}{8}\right)^{-r}\int_s^t\int_\Omega \mathrm{e}^{-(r+1)(t-\tau)}|\Delta v|^{r+1}.\end{aligned}\tag{3.10}$$

Denoting the last term on the right-hand side of (3.10) by I, by (2.5) yields

$$\begin{aligned}I&=c_1\chi^{r+1}\left(\frac{\mu}{8}\right)^{-r}\mathrm{e}^{-(r+1)t}\int_s^t\int_\Omega \mathrm{e}^{(r+1)\tau}|\Delta v|^{r+1}\\ &\le c_1C_{r+1}\chi^{r+1}\left(\frac{\mu}{8}\right)^{-r}\mathrm{e}^{-(r+1)t}\int_s^t\int_\Omega \mathrm{e}^{(r+1)\tau}u^{r+1}+c_1C_{r+1}\chi^{r+1}\left(\frac{\mu}{8}\right)^{-r}\mathrm{e}^{-(r+1)t}\|v(\cdot,s)\|^{r+1}_{W^{2,r+1}(\Omega)}\\ &\le c_1C_{r+1}\chi^{r+1}\left(\frac{\mu}{8}\right)^{-r}\int_s^t\int_\Omega \mathrm{e}^{-(r+1)(t-\tau)}u^{r+1}+c_1C_{r+1}\chi^{r+1}\left(\frac{\mu}{8}\right)^{-r}\|v(\cdot,s)\|^{r+1}_{W^{2,r+1}(\Omega)}.\end{aligned}\tag{3.11}$$

Inserting (3.11) into (3.10), we have

$$\begin{aligned}\frac{1}{r}\int_\Omega u^r&\le-\mu\left(\frac{1}{2}-\frac{c_1C_{r+1}}{8^r}\left(\frac{\chi}{\mu}\right)^{r+1}\right)\int_s^t\int_\Omega \mathrm{e}^{-(r+1)(t-\tau)}u^{r+1}+\frac{\mathrm{e}^{(r+1)s}M_0^r|\Omega|}{r}+\frac{c_2}{r+1}\\ &\quad+c_1C_{r+1}\chi^{r+1}\left(\frac{\mu}{8}\right)^{-r}\|v(\cdot,s)\|^{r+1}_{W^{2,r+1}(\Omega)}.\end{aligned}\tag{3.12}$$

Choosing $\vartheta_r>0$ small enough such that

$$\frac{1}{2}-\frac{c_1C_{r+1}}{8^r}\vartheta_r^{r+1}\ge 0,$$

due to $\frac{\chi}{\mu}=\vartheta\le\vartheta_r$, together (3.1) and (3.12) imply (3.3). $\square$

We can now easily prove our main result.

Proof of Theorem 1.1. Let $C > 0$, according to (2.6) and the first inequality in (2.8), we obtain $\|v(\cdot,t)\|_{L^{r_1+1}(\Omega)} \le C$ for all $t \in (0, T_{\max})$ and $r_1 < 2$, then choose $\vartheta \le \vartheta_{r_1}$, we can use Lemma 3.1 to obtain $\|u(\cdot,t)\|_{L^{r_1}(\Omega)} \le C$ for all $t \in (0, T_{\max})$. Hence, we can choose $r_1 \in (\frac{3}{2}, 2)$ and use (2.7), we have $\|v(\cdot,t)\|_{L^\infty(\Omega)} \le C$ for all $t \in (0, T_{\max})$. Now let $r_2 > 3$ and $\vartheta < \vartheta_0 := \min\{\vartheta_{r_1}, \vartheta_{r_2}\}$, one more use Lemma 3.1 to obtain $\|u(\cdot,t)\|_{L^{r_2}(\Omega)} \le C$ for all $t \in (0, T_{\max})$, then together Lemma 4.1 in [7], we obtain $\|v(\cdot,t)\|_{W^{1,\infty}(\Omega)} \le C$ for all $t \in (0, T_{\max})$. Hence, the boundedness of u can be proved as done in [8,20]. The proof of Theorem 1.1 is complete together with Lemma 2.1. □

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