C. R. Acad. Sci. Paris, Ser. I 355 (2017) 769–773

Contents lists available at ScienceDirect

C. R. Acad. Sci. Paris, Ser. I

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Complex analysis/Analytic geometry

On the Lie group structure of automorphism groups

Sur la structure de groupe de Lie des groupes d'automorphismes

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ARTICLE INFO

Article history:
Received 28 February 2017
Accepted after revision 27 June 2017
Available online 4 July 2017

Presented by Jean-Pierre Demailly

ABSTRACT

We give a sufficient condition for complex manifolds for automorphism groups to become Lie groups. As an application, we see that the automorphism group of any strictly pseudo-convex domain or finite-type pseudoconvex domain has a Lie group structure.

RÉSUMÉ

Nous donnons une condition suffisante pour que le groupe des automorphismes d'une variété complexe possède une structure de groupe de Lie. Comme application, nous obtenons que le groupe des automorphismes de tout domaine strictement pseudo-convexe ou de type pseudo-convexe fini a une structure de groupe de Lie.

1. Introduction

Let M be a complex manifold. We denote by $\mathrm{Aut}(M)$ the group of all holomorphic automorphisms of M, equipped with the compact-open topology. We are interested in the structure of automorphism groups. If Ω is a bounded domain in $\mathbb{C}^n$, then, by H. Cartan, $\mathrm{Aut}(\Omega)$ has a Lie group structure as follows: there exists a invariant Hermitian metric under the automorphism group action on Ω, called the Bergman metric. It is known that the isometry group of any Hermitian metric is a Lie group with respect to the compact-open topology, and $\mathrm{Aut}(\Omega)$ is a closed subgroup of the isometry group of the Bergman metric. Since any closed topological subgroup of a Lie group has a Lie group structure, $\mathrm{Aut}(\Omega)$ is a Lie group. Apart from that, it is known that the automorphism group of a compact complex manifold is a Lie group, cf. [6]. However, in general, $\mathrm{Aut}(M)$ is not a Lie group with respect to the compact-open topology. For example, $\mathrm{Aut}(\mathbb{C}^n)$, $n > 1$, is not a Lie group, since it contains the space of entire holomorphic functions on $\mathbb{C}^{n-1}$ so that $\mathrm{Aut}(\mathbb{C}^n)$ is not finite-dimensional. Furthermore, for a Stein manifold X, if X is a homogeneous space G/H of a complex Lie group G and a closed complex subgroup H, then the automorphism group $\mathrm{Aut}(X)$ is not a Lie group [4]. In this paper, we generalize the argument for bounded domains above to give a sufficient condition for complex manifolds that the automorphism groups become Lie groups.

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http://dx.doi.org/10.1016/j.crma.2017.06.007
1631-073X/

Let us recall the Bergman kernel form and the Bergman pseudometric (see also [5]). Let $A^2(M)$ be the set of holomorphic n-forms f on M such that

$$\mathrm{i}^{n^2}\int_M f\wedge\bar{f}<\infty.$$

With an inner product

$$(f,g)=\mathrm{i}^{n^2}\int_M f\wedge\bar{g},$$

$A^2(M)$ becomes a separable complex Hilbert space. Let $\{f_j\}$ be a complete orthonormal basis for $A^2(M)$. Then the Bergman kernel form K on M is defined by

$$K(z)=\sum_j f_j(z)\wedge\overline{f_j(z)},$$

which is an (n,n)-form. It is known that K is invariant under the automorphism group of M. Thus the zero loci of K,

$$Z:=\{z\in M : K(z)=0\},$$

is preserved by the automorphisms. For a local coordinate $z=(z_1,\dots,z_n)$, let

$$K(z)=K^*(z)\mathrm{d}z_1\wedge\cdots\wedge\mathrm{d}z_n\wedge\mathrm{d}\bar{z}_1\wedge\cdots\wedge\mathrm{d}\bar{z}_n.$$

Then the Bergman pseudometric $\mathrm{d}s^2_{M\setminus Z}$ on $M\setminus Z$ is defined by

$$\mathrm{d}s^2_{M\setminus Z}:=\sum\frac{\partial^2\log K^*(z)}{\partial z^\alpha\partial\bar{z}^\beta}\mathrm{d}z^\alpha\mathrm{d}\bar{z}^\beta.$$

It is known that $\mathrm{d}s^2_{M\setminus Z}$ is invariant under the automorphism group of M. Thus the degenerate loci of $\mathrm{d}s^2_{M\setminus Z}$,

$$D:=\{z\in M\setminus Z : \det\left(\frac{\partial^2\log K^*(z)}{\partial z^\alpha\partial\bar{z}^\beta}\right)=0\},$$

is also invariant under the automorphism group. Consequently, we see that any automorphism of M preserves $M\setminus(Z\cup D)$. If $M\setminus(Z\cup D)\neq\emptyset$, we have an injective continuous homomorphism

$$\mathrm{Aut}(M)\hookrightarrow\mathrm{Aut}(M\setminus(Z\cup D)),\quad f\mapsto f|_{M\setminus(Z\cup D)}.$$

We state the main theorems.

Theorem 1.1. *Let Ω be a domain in $\mathbb{C}^n$. If $\Omega\neq Z\cup D$, then* Aut(Ω) *has a Lie group structure.*

Theorem 1.2. *Let M be a Stein manifold. If $M\neq Z\cup D$, then* Aut(M) *has a Lie group structure.*

From the proofs of Theorems 1.1, 1.2 in the next section, we can generalize Theorem 1.1 for a domain in a Stein manifold.

Corollary 1.3. *Let Ω be a domain in a Stein manifold. If $\Omega\neq Z\cup D$, then* Aut(Ω) *has a Lie group structure.*

We also give a sufficient condition for $M\neq Z\cup D$.

Theorem 1.4. *Let M be a Stein manifold. If there exists a bounded-from-above plurisubharmonic function ϕ on M such that ϕ is strictly plurisubharmonic on a non-empty open set $U\subset M$ and the Lelong number of ϕ is zero on U, then $M\neq Z\cup D$.*

For pseudoconvex domains in $\mathbb{C}^n$, Theorem 1.4 is explained in [3]. Using their results and Theorem 1.4, we can restate Theorem 1.2 in terms of the core $\mathfrak{c}'(\Omega)$.

Corollary 1.5. *Let Ω be a Stein domain in a complex manifold. If $\Omega\neq\mathfrak{c}'(\Omega)$, then* Aut($\Omega$) *is a Lie group. In particular, if $\partial\Omega$ has a (smooth or non-smooth) strictly pseudoconvex point or a C^∞-smooth boundary point of finite type in the sense of D'Angelo, then* Aut(Ω) *is a Lie group.*

For the definition of the core $\mathfrak{c}'(\Omega)\subseteq\Omega$, see Gallagher–Harz–Herbort [3]. In fact, we can see that $Z\cup D\subset\mathfrak{c}'(\Omega)$, and $\Omega\neq\mathfrak{c}'(\Omega)$ if $\partial\Omega$ has a strictly pseudoconvex point or a point of finite type (see Theorem 2 in [3]).

2. Proofs of the theorems

Before going to prove the theorems, we give some remarks. First of all, it is obvious that Z is an analytic subset in M. We can see that $\mathrm{d}s^2_{M\setminus Z}$ is positive semidefinite and D is an analytic subset in $M \setminus Z$ as follows: for any point $p \in M \setminus Z$, there exists j such that $f_j(p) \neq 0$. Therefore, there exists a holomorphic mapping $\mu : z \mapsto [f_1(z) : f_2(z) : \cdots]$ from $M \setminus Z$ to a (finite or infinite dimensional) complex projective space. Then the Bergman pseudometric can be written as $\mathrm{d}s^2_{M\setminus Z} = \mu^*\omega_{FS}$, where ω_{FS} is the Fubini–Study metric on the projective space, which is positive definite. Thus $\mathrm{d}s^2_{M\setminus Z}$ is positive semidefinite and D is an analytic subset in $M \setminus Z$ since D coincides with the set of singular points of the holomorphic mapping μ. Finally, the Bergman pseudometric $\mathrm{d}s^2_{M\setminus(Z\cup D)}$ on $M \setminus (Z \cup D)$ coincides with the restriction of $\mathrm{d}s^2_{M\setminus Z}$ to $M \setminus (Z \cup D)$, by Riemann's removable singularity theorem. Thus $\mathrm{d}s^2_{M\setminus(Z\cup D)}$ is a positive definite Kähler metric, and therefore, $\mathrm{Aut}(M \setminus (Z \cup D))$ has a Lie group structure, as we explained in Section 1.

Proof of Theorem 1.1. To show that $\mathrm{Aut}(\Omega)$ has a Lie group structure, it suffices to show that $\mathrm{Aut}(\Omega) \subset \mathrm{Aut}(\Omega \setminus (Z \cup D))$ is a closed embedding.

Take a sequence $\{f_k\} \subset \mathrm{Aut}(\Omega)$ such that f_k converges to $f \in \mathrm{Aut}(\Omega \setminus (Z \cup D))$ as $k \to \infty$. Take a point $p \in Z \cup D$. Note that Z and D are analytic sets. Therefore, changing the coordinate of $\mathbb{C}^n$, we may assume that p is the origin in $\mathbb{C}^n$ and the connected component of $(Z \cup D) \cap \{(z_1, 0, \dots, 0) \in \mathbb{C}^n\}$ including the point p is $\{p\}$. We take a simple closed curve γ in $\mathbb{C}$ around the origin such that $\gamma \times \{(0, \dots, 0) \in \mathbb{C}^{n-1}\} \subset \Omega \setminus (Z \cup D)$. Then, by the Cauchy integral formula, we have

$$f_k(z_1, \dots, z_n) = \frac{1}{2\pi \mathrm{i}} \int_\gamma \frac{f_k(w, z_2, \dots, z_n)}{w - z_1} \mathrm{d}w,$$

for $z' = (z_2, \dots, z_n)$ sufficiently small $|z'|$. Here we understand that f_k is a tuple of holomorphic functions and the integral formula applies to each component. Since f_k converges to f uniformly on compact sets in $\Omega \setminus (Z \cup D)$, we have

$$f(z_1, \dots, z_n) = \frac{1}{2\pi \mathrm{i}} \int_\gamma \frac{f(w, z_2, \dots, z_n)}{w - z_1} \mathrm{d}w.$$

Thus we see that f is holomorphic at $p = (0, \dots, 0)$, and therefore f is a holomorphic mapping from Ω to $\overline{\Omega}$. By Hurwitz's theorem, $\mathrm{Jac}_{\mathbb{C}} f(z) \neq 0$ for any $z \in \Omega$, and therefore f is an open mapping. We use the following proposition to conclude the proof.

Proposition 2.1. *[8, Prop. 5 of Chap. 5] Let $\{f_k\}$ be a sequence of continuous open mappings of $\Omega \subset \mathbb{C}^n$ into $\mathbb{C}^n$. Suppose that f_k converges uniformly on compact sets in Ω to a map $f : \Omega \to \mathbb{C}^n$. If, for $p \in \Omega$, p is an isolated point of $f^{-1}(f(p))$, then for any neighborhood U of p, there exists k_0 such that $f(p) \in f_k(U)$ for $k \geq k_0$.*

Thus we see that f maps Ω into Ω. Applying the same argument to $\{f_k^{-1}\}$ and f^{-1}, we see that $f \in \mathrm{Aut}(\Omega)$. This completes the proof. □

Proof of Theorem 1.2. Let M be a Stein manifold of dimension n. Then, by Bishop [1] and Narasimhan [7], there exists a proper holomorphic embedding $F : M \to \mathbb{C}^{2n+1}$, from M into the $(2n + 1)$-dimensional Euclidean space. Thus, we may consider M as a closed submanifold of $\mathbb{C}^{2n+1}$. Take a sequence $\{f_k\} \subset \mathrm{Aut}(M)$ such that f_k converges to $f \in \mathrm{Aut}(M \setminus (Z \cup D))$ as $k \to \infty$. Any f_k is a tuple of $2n + 1$ holomorphic functions on M. Thus, as in the proof of Theorem 1.1, we can see that f is a tuple of $2n + 1$ holomorphic functions on M, and since $M \subset \mathbb{C}^{2n+1}$ is closed, for any $p \in Z \cup D$, $f_k(p)$ converges to a point in M. Therefore f is a holomorphic self-mapping of M. Applying the same argument to f^{-1}, we see that $f \in \mathrm{Aut}(M)$. This completes the proof. □

We now prove Theorem 1.4. To prove the theorem, we need solutions to $\bar{\partial}$ with L^2 estimates. According to Propositions 1.1, 1.4 and their proofs in [9], we can prove the following.

Proposition 2.2. *Let (X, ω) be a complex manifold with a smooth complete Kähler metric, and ψ be a smooth plurisubharmonic function on X such that ψ is strictly plurisubharmonic on an open subset $U \subset X$ and $\omega \leq \mathrm{i}\partial\bar{\partial}\psi$ on U. Then for any $\bar{\partial}$-closed $(n, 1)$-form f on X with $\mathrm{supp} f \subset U$ and $\int_U |f|^2_\omega \mathrm{e}^{-\psi} \mathrm{d}V_\omega < \infty$, there exists an $(n, 0)$-form α such that $\bar{\partial}\alpha = f$ and*

$$\int_X \alpha \wedge \bar{\alpha} \mathrm{e}^{-\psi} \leq \int_U |f|^2_\omega \mathrm{e}^{-\psi} \mathrm{d}V_\omega.$$

Here $|\cdot|_\omega$ is the pointwise norm and $\mathrm{d}V_\omega$ the volume with respect to the metric ω.

To explain this result, we fix notations. Put

$$P(u)^2 = \int_X |u|^2_{\omega+i\partial\bar\partial\psi} e^{-\psi} dV_{\omega+i\partial\bar\partial\psi},$$

for $u \in C_0^{p,q}(X)$, and denote by $L_1^{p,q}(X)$ the completion of $C_0^{p,q}(X)$ with respect to the norm P. Then $L_1^{p,q}(X)$ is a separable complex Hilbert space with the inner product $(u, v) = \int_X \langle u, v\rangle_{\omega+i\partial\bar\partial\psi} e^{-\psi} dV_{\omega+i\partial\bar\partial\psi}$. We denote the adjoint of $\bar\partial$ by $\bar\partial^*$. Furthermore, we put

$$L_2^{p,q}(X) = \{f \in L_1^{p,q}(X) : Q_\psi(f)^2 = \int_U |f|^2_{\partial\bar\partial\psi} e^{-\psi} dV_\psi < \infty\}.$$

Note that if a measurable (n,q)-form f on U satisfies $Q_\psi(f) < \infty$, then the zero extension of f to $X \setminus U$ is in $L_1^{n,q}(X)$ (see Proposition 1.1 in [9]). Then, as in the proof of Proposition 1.4 in [9], we can prove the inequality

$$Q_\psi(f)^2\{P(\bar\partial u)^2 + P(\bar\partial^* u)^2\} \geq |(\chi_U f, u)|^2. \tag{1}$$

Here χ_U is the characteristic function of U, $q \geq 1$, $u \in \mathrm{Dom}(\bar\partial) \cap \mathrm{Dom}(\bar\partial^*) \subset L_1^{n,q}(X)$, and $f \in L_2^{n,q}(X)$. Then it follows from this inequality that, for any $\bar\partial$-closed $(n,1)$-form $f \in L_2^{n,1}(X)$ with $\mathrm{supp} f \subset U$, a linear functional A defined by $A(\bar\partial^* u) = (u, f)$ on $\mathrm{Im}(\mathrm{Dom}(\bar\partial^*)) \subset L_1^{n,0}(X)$ is well defined, since $\mathrm{Ker}\bar\partial^*_{(n,1)} = \overline{\mathrm{Im}\bar\partial^*_{(n,2)}} \oplus (\mathrm{Ker}\bar\partial_{(n,1)} \cap \mathrm{Ker}\bar\partial^*_{(n,1)})$, and that the operator norm of A is bounded by $Q_\psi(f)$. By the Hahn–Banach theorem and the Riesz representation theorem, there exists an element $\alpha \in L_1^{n,0}(X)$ such that $(\bar\partial^* u, \alpha) = (u, f)$ for any $u \in \mathrm{Dom}(\bar\partial^*)$ and $P(\alpha) = ||A||_{\mathrm{op}} \leq Q_\psi(f)$. Thus $\bar\partial\alpha = f$. Note that $P(\alpha) = \int_M \alpha \wedge \bar\alpha e^{-\psi}$ for an $(n,0)$-form α. As in the proof of Proposition 1.1 in [9], we have $Q_\psi(f) \leq \int_U |f|^2_\omega e^{-\psi} dV_\omega$, so Proposition 2.2 holds.

Proof of Theorem 1.4. According to [5], it suffices to show the following: (1) for $p \in U$, there exists an $(n,0)$-form $f \in A^2(M)$ such that $f(p) \neq 0$; (2) for each $V \in T_p M$, there exists a $g \in A^2(M)$ such that $g(p) = 0$ and $Vg(p) \neq 0$. After coordinate change, we may assume that $V = \partial/\partial z_1$ for some local coordinate $(\tilde U, (z_1, \ldots, z_n))$ centered at p.

We assume that $U (= \tilde U) \subset\subset M$ is a local coordinate with a coordinate map z. Let $\chi : M \to [0,1]$ be a C^∞-smooth function such that $\mathrm{supp}\chi \subset U$ and $\chi = 1$ near p. We put a weight function

$$\psi(z) = K\phi(z) + (2n+1)\chi(z)\log|z - p|,$$

where $K > 0$ is taken so that ψ is plurisubharmonic on M and strictly plurisubharmonic on U. Let ω be a Kähler form on M such that $3\omega \leq i\partial\bar\partial\psi$ on U; let η be a smooth strictly plurisubharmonic exhaustion function on M, and put $\Omega_k = \{z \in M : \eta(z) < k\}$ for $k \in \mathbb{R}$; take a complete Kähler form ω_k on Ω_k such that $2(\omega + \omega_k) \leq i\partial\bar\partial\psi$ on U. Since M is a Stein manifold, for each k, there exists a family of smooth strictly plurisubharmonic functions $\{\psi_{\epsilon,k}\}_{\epsilon>0}$ on Ω_{k+1} such that $\psi_{\epsilon,k}(z) \searrow \psi(z)$ as $\epsilon \to 0$ for each $z \in \Omega_k$ and $i\partial\bar\partial\psi_{\epsilon,k} \geq \omega + \omega_k$ on U for sufficiently small ϵ. Put $f_1 = \bar\partial\chi dz_1 \wedge \ldots \wedge dz_n$, and $f_2 = z_1\bar\partial\chi dz_1 \wedge \ldots \wedge dz_n$. Then, they are $\bar\partial$-closed, and therefore, we can solve the equation $\bar\partial\alpha^{(i)}_{\epsilon,k} = f_i$ on Ω_k for each small ϵ with the estimate

$$\int_{\Omega_k} \alpha^{(i)}_{\epsilon,k} \wedge \bar\alpha^{(i)}_{\epsilon,k} e^{-\psi_{\epsilon,k}} \leq \int_U |f_i|_{\omega+\omega_k} e^{-\psi_{\epsilon,k}} dV_{\omega+\omega_k} \leq \int_U |f_i|_\omega e^{-\psi_{\epsilon,k}} dV_\omega.$$

Here, the first inequality follows from Proposition 2.2, and the second inequality follows from the proof of Proposition 1.1 in [9]. Note that Lebesgue's monotone convergence theorem implies

$$\int_U |f_i|_\omega e^{-\psi_{\epsilon,k}} dV_\omega \to \int_U |f_i|_\omega e^{-\psi} dV_\omega < \infty.$$

For the finiteness of the limit, we used the condition that the Lelong number of ϕ is zero on U. Since $\psi_{\epsilon,k}$ is bounded on Ω_k, we have $\alpha^{(i)}_{\epsilon,k} \in L^2_{n,0}(\Omega_k)$. Thus, letting $\epsilon \to 0$, we can take a weak limit $\alpha^{(i)}_k$ of $\{\alpha^{(i)}_{\epsilon,k}\}$ in $L^2_{n,0}(\Omega_k)$. Since

$$\int_{\Omega_k} \alpha^{(i)}_k \wedge \bar\alpha^{(i)}_k e^{-\psi_{\epsilon',k}} \leq \varliminf_{\epsilon\to 0} \int_U \alpha^{(i)}_{\epsilon,k} \wedge \bar\alpha^{(i)}_{\epsilon,k} e^{-\psi_{\epsilon',k}},$$

for any small $\epsilon' > 0$, we have

$$\int_{\Omega_k} \alpha^{(i)}_k \wedge \bar\alpha^{(i)}_k e^{-\psi} \leq \int_U |f_i|_\omega e^{-\psi} dV_\omega < \infty.$$

Take a weak limit $\alpha^{(i)}$ of $\alpha_k^{(i)}$ as $k \to \infty$ in $L^2_{n,0}(M, e^{-\psi})$. It satisfies $\bar{\partial}\alpha^{(i)} = f_i$ on M, and therefore $\alpha^{(i)}$ is smooth. Then, by the singularity of ψ, we see that $\alpha^{(i)}(p) = 0$ and $\partial\alpha^{(i)}/\partial z_1(p) = 0$. Since ψ is bounded from above, $f := \chi dz_1 \wedge \ldots \wedge dz_n - \alpha^{(1)}$ and $g := z_i \chi dz_1 \wedge \ldots \wedge dz_n - \alpha^{(2)}$ are in $A^2(M)$. Clearly, $f(p) \neq 0$, $g(p) = 0$ and $\partial g/\partial z_1(p) \neq 0$. This completes the proof. □

3. Remarks

1) For the condition $M \neq Z \cup D$, we can use, the so-called pluricomplex Green function on M. By B.Y. Chen and J.H. Zhang [2], it gives us a slightly different condition from that of Theorem 1.4. The pluricomplex Green function g_M with a logarithmic pole p on M is given by

$$g_M(z, p) = \sup\{u(z)\},$$

where the supremum is taken over all negative plurisubharmonic functions u such that $u(z) - \log|z - p|$ is bounded from above in a sufficiently small neighborhood of p. The following statement is given by the proof of Theorem 1 in [2].

Proposition 3.1. *Let M be a Stein manifold. If there exists a point $p \in M$ and a positive number $a > 0$ such that $\{z \in M : g_M(z, p) < -a\}$ is relatively compact, then $M \neq Z \cup D$.*

2) We would be able to use other invariant metrics to prove the existence of the Lie group structure of automorphism groups. For instance, if the Kobayashi metric exists on a complex manifold M, then the automorphism group is a Lie group. Moreover, if the set D_K of degenerating points of the Kobayashi pseudometric satisfies the condition $D_K \neq M$, then we have a continuous inclusion

$$\mathrm{Aut}(M) \hookrightarrow \mathrm{Aut}(M \setminus D_K), \quad f \mapsto f|_{M \setminus D_K},$$

as in the Bergman pseudometric case. However, it seems that the properties of D_K are not well understood. If we could show that the set D_K is thin, then we could use the argument in Section 2.

3) If the boundary $\partial\Omega$ of a domain $\Omega \subset \mathbb{C}^n$ is complicated, then it is almost sure that $\mathrm{Aut}(\Omega) = \{\mathrm{id}\}$. One may think further that, if the boundary $\partial\Omega$ of a domain $\Omega \subset \mathbb{C}^n$ is very large, then $\mathrm{Aut}(\Omega)$ should be a Lie group. However, if we consider the class of domains with codimension 1 boundary, there exists a domain where the automorphism group is a huge topological group so that it is not a Lie group: consider the domain

$$\Omega = \{(z, w) \in \mathbb{C}^2 : |zw| < 1\}.$$

Then, for any $f \in \mathcal{O}(\mathbb{C})$, the mapping

$$(z, w) \to (ze^{f(zw)}, we^{-f(zw)})$$

is a biholomorphic self mapping of Ω. Therefore, $\mathrm{Aut}(\Omega)$ contains the space of entire holomorphic functions on $\mathbb{C}$ so that $\mathrm{Aut}(\Omega)$ is not finite-dimensional, even though this domain is not homogeneous.

For unbounded pseudoconvex domains Ω with $\Omega = \mathfrak{c}'(\Omega)$, we need other view points for the existence of Lie group structures of automorphism groups. The existence of the Bergman pseudometric is neither non-trivial for such domains. We refer the reader to [10], where the existence of the Bergman metric for some special cases is investigated.

Acknowledgements

The author is grateful to Professor Akio Kodama and Professor Satoru Shimizu, who read the manuscript and helped to improve the theorems. The author is also grateful to his colleagues Masanori Adachi, Tobias Harz and Ngoc Cuong Nguyen for valuable comments. This research is supported by NRF grant 2011-0030044 (SRC-GAIA) funded by the government of Korea.

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