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Mathematical analysis of a nonlinear PDE model for European options with counterparty risk

Analyse mathématique d'un modèle d'EDP non linéaire pour les options européennes avec risque de contrepartie

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ABSTRACT

In this work, we analyze a nonlinear partial differential equation (PDE) model for the total value adjustment on European options in the presence of a counterparty risk. We transform the nonlinear PDE into an equivalent one, involving a sectorial operator, and prove the existence and uniqueness of a solution.

RÉSUMÉ

Dans ce travail, nous analysons un modèle d'équations aux dérivées partielles (EDP) non linéaires pour l'ajustement XVA d'options européennes en présence d'un risque de contrepartie. Nous transformons l'EDP non linéaire en une équation équivalente, impliquant un opérateur sectoriel, et prouvons l'existence et l'unicité de la solution.

1. Introduction

Since the last financial crisis starting in 2007, the credit entities are aware of the default risk of counterparties of a contract. Thus, different adjustments such as Credit Value Adjustment (CVA), Funding Value Adjustment (FVA) or Debit Value Adjustment (DVA) are now included in contracts in order to take into account the possibility of counterparty default. The Total Value Adjustment (XVA) gathers all these adjustments, and its valuation is an important issue for the financial entities.

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Among the different strategies currently used to obtain the XVA, we follow the one based on the solution to partial differential equations (PDEs) [3]. In [1], hedging arguments have been used to deduce one-factor PDE models for the XVA related to European call and put options. Depending on the mark-to-market close-out value, the governing PDE is either linear or nonlinear. We also proposed numerical techniques to compute the solutions to both models. However, the existence and uniqueness of a solution to the nonlinear model is an open problem that we solve in the present paper. Recently, in [2], analogous models depending on two stochastic factors have been posed, analyzed, and numerically solved.

More precisely, in the present paper, we focus on the mathematical analysis of the following nonlinear final value problem:

$$\begin{cases} \partial_t U + \mathcal{A}U - rU = (1-R_{\mathrm{B}})\lambda_{\mathrm{B}}(V+U)^- + (1-R_{\mathrm{C}})\lambda_{\mathrm{C}}(V+U)^+ + s_{\mathrm{F}}(V+U)^+ \\ U(T,S) = 0\,, \end{cases} \tag{1}$$

for $S \in [0,+\infty)$ and $t \in [0,T]$, where t is the time and S is the asset price. The unknown $U = U(t,S)$ is the XVA, operator $\mathcal{A}$ is given by

$$\mathcal{A}U \equiv \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 U}{\partial S^2} + r_{\mathrm{R}} S \frac{\partial U}{\partial S}$$

and V is the given explicit solution to the classical Black–Scholes equation for pricing European options without counterparty risk [6]. At the expiry time of the option, T, the XVA vanishes, as it is assumed in the final condition. Note that the nonlinearities come from the sign functions $f^+ = \max(f,0)$ and $f^- = \min(f,0)$. Concerning the involved constant parameters, r denotes the risk-free interest rate, R_{B} and $R_{\mathrm{C}} \in [0,1]$ represent the recovery rates on the derivatives positions of parties B and C, λ_{B} and λ_{C} are the default intensities of both parties, $s_{\mathrm{F}} = r_{\mathrm{F}} - r$ is the funding cost of the entity, r_{R} is the rate paid for the underlying asset in a repurchase agreement, and σ is the volatility.

Note that the proposed techniques in this article can be applied to models that include a collateral, which are also deduced in [1].

2. Mathematical analysis: existence and uniqueness of a solution

In order to obtain the existence and uniqueness of a solution to problem (1), we transform it into an equivalent one governed by a sectorial operator. For this purpose, the changes of variable $x = \ln(S/K)$ and $\tau = (\sigma^2/2)(T-t)$, and the change of unknown

$$u(\tau,x) = \frac{1}{K}\mathrm{e}^{(\gamma+\alpha)x+\beta\tau}U(t,S)$$

in terms of a parameter $\gamma \in \mathbb{R}$, with $\alpha = -\frac{1}{2}\left(1-\frac{2r_{\mathrm{R}}}{\sigma^2}\right)$ and $\beta = \left(1-\frac{2r_{\mathrm{R}}}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}$, are introduced in (1) to obtain the equivalent problem:

$$\begin{cases} \dfrac{\partial u}{\partial \tau} - \dfrac{\partial^2 u}{\partial x^2} = J(\tau,u), \qquad x \in \mathbb{R},\ \tau \in \left(0, \dfrac{\sigma^2 T}{2}\right] \\ u(0,x) = 0\,. \end{cases} \tag{2}$$

The functional $J : \left[0, \frac{\sigma^2}{2}T\right] \times H^1(\mathbb{R}) \to L^2(\mathbb{R})$ in (2) is defined as follows:

$$J(\tau,\varphi)(x) = \gamma^2\varphi(x) - 2\gamma\frac{\partial\varphi}{\partial x}(x) + \mathrm{e}^{\gamma x}h(\tau, \mathrm{e}^{-\gamma x}\varphi(x)),\ x \in \mathbb{R}, \tag{3}$$

for all $\tau \in \left[0, \frac{\sigma^2 T}{2}\right]$, $\varphi \in H^1(\mathbb{R})$, with $h : [0, \frac{\sigma^2}{2}T] \times H^1(\mathbb{R}) \to L^2(\mathbb{R})$ given by

$$h(\tau,\varphi)(x) = -\,\mathrm{e}^{\alpha x+\beta\tau}\frac{2}{K\sigma^2}\Big[C_{\mathrm{B}}\big(\mathcal{G}(\tau,\varphi)(x)\big)^- + C_{\mathrm{C}}\big(\mathcal{G}(\tau,\varphi)(x)\big)^+\Big],$$

where

$$\mathcal{G}(\tau,\varphi)(x) = V(\tau, K\mathrm{e}^x) + K\varphi(x)\mathrm{e}^{-\alpha x-\beta\tau},\ C_{\mathrm{B}} = (1-R_{\mathrm{B}})\lambda_{\mathrm{B}},\ C_{\mathrm{C}} = (1-R_{\mathrm{C}})\lambda_{\mathrm{C}} + s_{\mathrm{F}}.$$

Next, we recall the definition of a sectorial operator (see [4], for example).

Definition 2.1. A linear operator $\mathcal{B}$ in a Banach space X is a sectorial operator if it is a closed densely defined operator such that, for some $\phi \in (0,\pi/2)$, $M_0 \geq 1$ and $a \in \mathbb{R}$, the sector $S_{a,\phi} = \{\lambda \mid \phi \leq |arg(\lambda - a)| \ \leq \pi, \lambda \neq a\}$ is in the resolvent set of $\mathcal{B}$, and

$$\left\|(\lambda - \mathcal{B})^{-1}\right\| \leq \frac{M_0}{|\lambda - a|}, \quad \text{for any } \lambda \in S_{a,\phi}\,.$$

Note that, associated with a sectorial operator $\mathcal{B}$, we can introduce a scale of fractional power spaces $X^\alpha = \mathrm{Rang}(\mathcal{B}^{-\alpha})$ for $\alpha > 0$, such that $X = X^0$ and $X^1 = \mathrm{Dom}(\mathcal{B})$, equipped with the norm $\|y\| = \|\mathcal{B}^\alpha y\|$ for all $y \in X^\alpha$, where $\mathcal{B}^\alpha$ is a fractional power of $\mathcal{B}$.

Theorem 2.1 *([4]). Assume that $\mathcal{B}$ is a sectorial operator in a Hilbert space X, $0 \le \alpha < 1$ and $f : \mathcal{U} \to X$, with $\mathcal{U}$ an open subset of $\mathbb{R} \times X^\alpha$ and $f(\tau, y)$ a locally Hölder continuous function in τ and locally Lipschitzian in y. Then, for any $(\tau_0, y_0) \in \mathcal{U}$, there exists $T_0 = T_0(\tau_0, y_0) > 0$ such that the initial-value nonlinear PDE problem:*

$$\begin{cases} \dfrac{\mathrm{d}y}{\mathrm{d}\tau} + \mathcal{B}y = f(\tau, y), & \tau > \tau_0, \\ y(\tau_0) = y_0, \end{cases} \tag{4}$$

has a unique solution y on $(\tau_0, \tau_0 + T_0)$.

In order to apply Theorem 2.1, we consider $X = L^2(\mathbb{R})$, $X^\alpha = H^1(\mathbb{R})$ with $\alpha = 1/2$ and $\mathcal{U} = \left(0, \frac{\sigma^2 T}{2}\right) \times H^1(\mathbb{R})$. From [4], the operator $-\partial^2/\partial x^2$ is sectorial in $L^2(\mathbb{R})$ and we can prove the following result.

Proposition 2.2. *For $\gamma < -\dfrac{1}{2} - \dfrac{r_R}{\sigma^2}$ in the case of a call option and for $\gamma > \dfrac{1}{2} - \dfrac{r_R}{\sigma^2}$ in the case of a put option, the function $J : \mathcal{U} \to X$ given by (3) is well defined, locally Hölder continuous in τ, and locally Lipschitzian in φ.*

Proof. First, the Black–Scholes formula for a European call option implies that

$$V(\tau, x) = K \exp(x) \exp\left(-D_0 \frac{2}{\sigma^2}\tau\right) N(d_1^*) - K \exp\left(-r\frac{2}{\sigma^2}\tau\right) N(d_2^*),$$

while, for a put option, we have

$$V(\tau, x) = K \exp\left(-r\frac{2}{\sigma^2}\tau\right) N(-d_2^*) - K \exp(x) \exp\left(-D_0 \frac{2}{\sigma^2}\tau\right) N(-d_1^*),$$

where

$$d_1^* = \frac{x + (r - D_0 + \sigma^2/2)\frac{2}{\sigma^2}\tau}{\sqrt{2\tau}}, \qquad d_2^* = \frac{x + (r - D_0 - \sigma^2/2)\frac{2}{\sigma^2}\tau}{\sqrt{2\tau}}, \tag{5}$$

with $D_0 = r - r_\mathrm{R}$ and $N(x)$ represents the distribution function of the standard $\mathcal{N}(0, 1)$ random variable.

To prove that $J(\tau, \varphi) \in L^2(\mathbb{R})$, we write $J(\tau, \varphi) = J_1(\tau, \varphi) + J_2(\tau, \varphi)$, where

$$J_1(\tau, \varphi)(x) = \gamma^2 \varphi(x) - 2\gamma \frac{\partial \varphi}{\partial x}(x) \quad \text{and} \quad J_2(\tau, \varphi)(x) = \mathrm{e}^{\gamma x} h(\tau, \mathrm{e}^{-\gamma x}\varphi(x)).$$

First, as $\varphi \in H^1(\mathbb{R})$ then $J_1(\tau, \varphi) \in L^2(\mathbb{R})$. Secondly, we write

$$J_2(\tau, \varphi)(x) = -\frac{2}{K\sigma^2}\Big[C_\mathrm{B}\,(\mathcal{F}(\tau, \varphi)(x))^- + C_\mathrm{C}\,(\mathcal{F}(\tau, \varphi)(x))^+\Big]$$

with

$$\mathcal{F}(\tau, \varphi) = \mathrm{e}^{\Theta_1 x + \Theta_2 \tau} V(\tau, K\mathrm{e}^x) + K\varphi(x),$$

$$\Theta_1 = \gamma - \frac{1}{2} + \frac{r_\mathrm{R}}{\sigma^2} \qquad \text{and} \qquad \Theta_2 = \left(1 - \frac{2r_\mathrm{R}}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}.$$

Next, to prove that $J_2(\tau, \varphi) \in L^2(\mathbb{R})$, we consider

$$J_3(\tau, \varphi)(x) = \exp\Big(\Theta_1 x + \Theta_2 \tau\Big) V(\tau, K\mathrm{e}^x)$$

and prove that $J_3(\tau, \varphi) \in L^2(\mathbb{R})$. For this purpose, we study the limits of $J_3(\tau, \varphi)$ when $x \to \pm\infty$.

First, in view of the expressions (5) for d_1^* and d_2^*, we obtain for $i = 1, 2$:

$$\lim_{x\to+\infty} d_i^* = +\infty \text{ implies that } \lim_{x\to+\infty} N(d_i^*) = 1, \ \lim_{x\to+\infty} N(-d_i^*) = 0,$$
$$\lim_{x\to-\infty} d_i^* = -\infty \text{ implies that } \lim_{x\to-\infty} N(d_i^*) = 0, \ \lim_{x\to-\infty} N(-d_i^*) = 1.$$

Also note that $N(-d_i^*) \to 0$ faster than $e^x \to \infty$ when $x \to +\infty$, while $N(d_i^*) \to 0$ faster than $e^{-x} \to \infty$ when $x \to -\infty$.

For a call option, the expression of J_3 is given by

$$J_3(\tau,\varphi)(x) = K e^{(\Theta_1+1)x+\left(\Theta_2 - D_0 \frac{2}{\sigma^2}\right)\tau} N(d_1^*) - K e^{\Theta_1 x+\left(\Theta_2 - r \frac{2}{\sigma^2}\right)\tau} N(d_2^*)\,, \tag{6}$$

so that $J_3(\tau,\varphi)(x) \to 0$ when $x \to -\infty$ for all $\gamma \in \mathbb{R}$. If we impose that $\Theta_1 + 1 < 0$, then $e^{(\Theta_1+1)x}$ and $e^{\Theta_1 x}$ tend to zero when $x \to +\infty$. Therefore, we obtain that $J_3(\tau,\varphi) \in L^2(\mathbb{R})$ for $\gamma < -\frac{1}{2} - \frac{r_R}{\sigma^2}$.

For a put option, the expression of J_3 is given by

$$J_3(\tau,\varphi)(x) = K e^{\Theta_1 x+\left(\Theta_2 - r \frac{2}{\sigma^2}\right)\tau} N(-d_2^*) - K e^{(\Theta_1+1)x+\left(\Theta_2 - D_0 \frac{2}{\sigma^2}\right)\tau} N(-d_1^*)\,. \tag{7}$$

In this case, $J_3(\tau,\varphi)(x) \to 0$ when $x \to +\infty$ for all $\gamma \in \mathbb{R}$. If we choose $\Theta_1 > 0$, we get $e^{(\Theta_1+1)x} \to 0$ and $e^{\Theta_1 x} \to 0$ when $x \to -\infty$, so that $J_3(\tau,\varphi)(x) \to 0$. Therefore, $J_3(\tau,\varphi) \in L^2(\mathbb{R})$ for $\gamma > \frac{1}{2} - \frac{r_R}{\sigma^2}$.

Hence, $J_2(\tau,\varphi) \in L^2(\mathbb{R})$ if $\gamma < -\frac{1}{2} - \frac{r_R}{\sigma^2}$ for a European call option and if $\gamma > \frac{1}{2} - \frac{r_R}{\sigma^2}$ for a European put option. Therefore, under these assumptions on γ, $J(\tau,\cdot) : H^1(\mathbb{R}) \to L^2(\mathbb{R})$ is well defined.

Next, we prove that J is locally Lipschitz in φ, i.e.

$$\| J(\tau,\varphi_1) - J(\tau,\varphi_2)\|_{L^2(\mathbb{R})} \le L_J \, \|\varphi_1 - \varphi_2\|_{H^1(\mathbb{R})}\,, \quad \text{for all } \varphi_1, \varphi_2 \in H^1(\mathbb{R})\,.$$

For this purpose, we consider that

$$\begin{aligned}\Big| J(\tau,\varphi_1)(x) - J(\tau,\varphi_2)(x)\Big| &\le \gamma^2 \, |\varphi_1(x) - \varphi_2(x)| + 2\gamma \left| \frac{\partial \varphi_1}{\partial x}(x) - \frac{\partial \varphi_2}{\partial x}(x)\right| \\ &\quad + e^{\gamma x} L_h \left| e^{-\gamma x}\varphi_1(x) - e^{-\gamma x}\varphi_2(x)\right| \\ &\le (\gamma^2 + L_h)\, |\varphi_1(x) - \varphi_2(x)| + 2\gamma \left| \frac{\partial \varphi_1(x)}{\partial x} - \frac{\partial \varphi_2(x)}{\partial x}\right|\,,\end{aligned}$$

where we have used the fact that $\left|\chi_1^+ - \chi_2^+\right| \le |\chi_1 - \chi_2|$ and $\left|\chi_1^- - \chi_2^-\right| \le |\chi_1 - \chi_2|$, with

$$\chi_i = V(\tau,\cdot) + K e^{(-(\alpha+\gamma)x - \beta\tau)} \varphi_i\,.$$

Moreover, we have introduced the constant

$$L_h = \frac{2}{\sigma^2}\Big(|(1-R_B)\lambda_B| + |(1-R_C)\lambda_C + s_F| \Big)\,.$$

Next, by integrating, we get

$$\begin{aligned}&\int_{\mathbb{R}} |J(\tau,\varphi_1)(x) - J(\tau,\varphi_2)(x)|^2 \, dx \\ &\qquad \le 2(\gamma^2 + L_h)^2 \int_{\mathbb{R}} |\varphi_1(x) - \varphi_2(x)|^2 \, dx + 2(2\gamma)^2 \int_{\mathbb{R}} \left| \frac{\partial \varphi_1}{\partial x}(x) - \frac{\partial \varphi_2}{\partial x}(x)\right|^2 dx\,,\end{aligned} \tag{8}$$

which is equivalent to

$$\| J(\tau,\varphi_1) - J(\tau,\varphi_2)\|^2_{L^2(\mathbb{R})} \le L_J^2 \, \|\varphi_1 - \varphi_2\|^2_{H^1(\mathbb{R})}\,,$$

with $L_J = 2\max\{\gamma^2 + L_h, 2\gamma\}$, so that J is locally Lipschitz in the variable φ.

Next, we prove that J is Hölder continuous in τ by proving it is locally Lipschitz continuous in τ. First, for $\tau_1, \tau_2 \in \left[0, \frac{\sigma^2}{2} T\right]$, we obtain

$$\begin{aligned}\Big| J(\tau_1,\varphi)(x) - J(\tau_2,\varphi)(x)\Big| &= \left| e^{\gamma x}(h(\tau_1, e^{-\gamma x}\varphi)(x) - h(\tau_2, e^{-\gamma x}\varphi)(x))\right| \\ &= \left| - e^{(\gamma+\alpha)x} \frac{2}{K\sigma^2} \left[C_B \left(\tilde{\mathcal{G}}(\tau_1, x)^- - \tilde{\mathcal{G}}(\tau_2, x)^-\right) + C_C \left(\tilde{\mathcal{G}}(\tau_1, x)^+ - \tilde{\mathcal{G}}(\tau_2, x)^+\right)\right]\right| \\ &\le \mathcal{M} \left| V(\tau_1,\cdot)e^{\beta\tau_1} - V(\tau_2,\cdot)e^{\beta\tau_2}\right|\,,\end{aligned}$$

where $\tilde{\mathcal{G}}(\tau,\varphi) = V(\tau,\cdot)e^{\beta\tau} + K e^{(-\alpha-\gamma)x}\varphi$ and

$$\mathcal{M} = \left| -\mathrm{e}^{(\gamma+\alpha)x} \frac{2}{K\sigma^2} \right| \Big(|(1-R_{\mathrm{B}})\lambda_{\mathrm{B}}| + |(1-R_{\mathrm{C}})\lambda_{\mathrm{C}} + s_{\mathrm{F}}| \Big) .$$

Moreover, the function $\mathrm{e}^{\beta\tau}$ is Lipschitz continuous in τ in $\left[0, \dfrac{\sigma^2}{2}T\right]$. Then, using that $V \in \mathcal{C}((0, \frac{\sigma^2}{2}T), X)$, we can apply that V is also Lipschitz continuous in τ. Therefore, in terms of the norm, we get:

$$\| J(\tau_1, \varphi) - J(\tau_2, \varphi)\|^2_{L^2(\mathbb{R})} = \int_{\mathbb{R}} |J(\tau_1, \varphi)(x) - J(\tau_2, \varphi)(x)|^2 \,\mathrm{d}x \le C\,|\tau_1 - \tau_2| \,,$$

where $C > 0$ is the Hölder constant associated with the function $V(\tau, x)e^{\beta\tau}$. Finally, $J(\tau, \varphi)$ is Hölder continuous in the τ variable. □

Corollary 2.3. *For any initial condition $u_0 \in H^1(\mathbb{R})$, there exists $T_0 = T_0(0, u_0) > 0$, such that the initial value problem (2) has a unique solution in $(0, T_0)$.*

Corollary 2.3 follows from Proposition 2.2 and Theorem 2.1, and provides the existence and uniqueness of a local solution, as $T_0 = T_0(0, u_0)$ is a local time. In order to extend it to $(0, T)$ for any given $T > 0$, we apply Corollary 3.3.5 in [4].

Proposition 2.4. *Under the hypotheses of Proposition 1, the following inequality holds:*

$$\| J(\tau, \varphi)\|_{L^2(\mathbb{R})} \le \mathcal{K}(\tau)\Big(1 + \|\varphi\|_{H^1(\mathbb{R})}\Big), \qquad \text{for all} \quad (\tau, \varphi) \in (0, \infty) \times H^1(\mathbb{R})\,, \tag{9}$$

where $\mathcal{K}$ is continuous in $(0, \infty)$. Therefore, there exists a unique solution to the problem (2) defined on the whole time interval $\left(0, \frac{\sigma^2}{2}T\right]$.

Proof. First, we note that the Lipschitz continuity properties also hold for $\tau \in (0, \infty)$. Next, if we consider the function $K(\tau) = L_J + \| J(\tau, 0)\|_{L^2(\mathbb{R})}$, which is continuous in τ on $(0, \infty)$, then, for any $(\tau, \varphi) \in (0, \infty) \times H^1(\mathbb{R})$, we have

$$\begin{aligned} \| J(\tau, \varphi)\|_{L^2(\mathbb{R})} &\le \| J(\tau, \varphi) - J(\tau, 0)\|_{L^2(\mathbb{R})} + \| J(\tau, 0)\|_{L^2(\mathbb{R})} \\ &\le L_J\, \|\varphi - 0\|_{H^1(\mathbb{R})} + \| J(\tau, 0)\|_{L^2(\mathbb{R})} = \big(L_J + \| J(\tau, 0)\|_{L^2(\mathbb{R})}\big) \big(\|\varphi\|_{H^1(\mathbb{R})} + 1\big) \,, \end{aligned}$$

where L_J is the Lipschitz constant for J. Thus, inequality (9) is obtained. Next, we can apply Corollary 3.3.5 in [4]. Thus, we consider $u(\tau, \cdot)$ as the unique solution to (2) at time $\tau_0 = T_0/2$ obtained from Corollary 2.3, so that, from Corollary 3.3.5 in [4], the unique solution to (2) through $(\tau_0, u(\tau_0, \cdot))$ exists for all $\tau \ge \tau_0$. Therefore, we obtain the existence and uniqueness of a solution to (2) in $\left(0, \frac{\sigma^2}{2}T\right]$. □

Corollary 2.5. *There exists a unique solution to (1).*

Proof. It follows from the existence and uniqueness of a solution to the equivalent problem (2) for the appropriate choice of parameter γ. □

Further details can be found in [5].

3. Conclusions

Following the theory of sectorial operators, we prove the existence and uniqueness of a solution to an initial value problem governed by a nonlinear PDE, which has been recently introduced to model the total value adjustment for European options in the presence of counterparty risk. The same methodology can be extended to one-factor models that incorporate collateralization of the contracts, which is also used in the financial sector.

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