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Corrigendum to “The critical exponent functions”

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Erratum / Erratum
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Corrigendum to “The critical exponent functions”

Corrigendum de « Les fonctions de l'exposant critique »

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Abstract. We give a corrigendum to our paper [1] entitled “The critical exponent functions”.

Résumé. Nous donnons un corrigendum à notre article [1] intitulé « Les fonctions de l'exposant critique ».

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In [1], a family of interval maps based on the word-combinatorial concept of critical exponent was introduced, hence referred to as “critical exponent functions”. A purely combinatorial result, namely [1, Theorem 7], was presented and employed in that paper to prove some analytical and dynamical properties of such interval maps. However, [1, Theorem 7] does not hold. Denoting by $\{0, 1\}^*$ and $\{0, 1\}^\omega$ the sets of finite and infinite binary words, respectively, and using $E(w)$ to represent the critical exponent of a word, [1, Theorem 7] was formulated as follows: “Let $w \in \{0, 1\}^*$. For every $\alpha > \max\{2, E(w)\}$, there exists $y \in \{0, 1\}^\omega$ such that $E(wy) = \alpha$ ”. This claim is false, as shown for instance by the counterexample $w = 00100100$. Indeed, in this case we have $E(w) = 8/3$, while both $w0$ and $w1$ are 3-powers, so that $E(wy) \geq 3 > 8/3$ for every $y \in \{0, 1\}^\omega$.

However, most of the properties obtained through that combinatorial result remain valid with minor adjustments. Indeed, after reviewing all the properties of κ_2 , we find that all of them hold, except for the following ones:

- The property of being left- or right-Darboux at every point, as stated in [1, Proposition 19]; it remains unclear if this property holds or not.
- The characterization of the almost fixed points of κ_2 , specifically in [1, Corollary 24], and the subsequent uncountability of them (cf. [1, Corollary 25]).
- The existence of the fixed point can be granted only on any left-neighbourhood of x_τ , not in a neighbourhood of any almost fixed point (cf. [1, Corollary 26]); however, this suffices to prove the existence of countably many fixed points, so that [1, Corollary 27] holds.

Analogously, most of the properties of κ_n hold. The main results that fail are:

- (1) being left- or right-Darboux at every point (cf. [1, Corollary 42]);
- (2) the characterization of the almost-fixed points.

All the properties presented in [1] for κ hold.

The interested reader can find more details in [2].

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Declaration of interests

The authors do not work for, advise, own shares in, or receive funds from any organization that could benefit from this article, and have declared no affiliations other than their research organizations.

References

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