Appendix to "Peristaltic transport of fourth grade fluid with heat transfer and induced magnetic field", by T. Hayat, S. Noreen

The purpose here is to present all the values appearing in the expressions of Ψ, $\mathrm{d}p/\mathrm{d}x$ and ϕ. These are

$$
\begin{aligned}
C_1(y) \;=\; & (-1 + FM)\,y\cosh(3My) + (F + h)\cosh(M(h + 2y)) \\
& + y\cosh(M(2h + 3y)) + FMy\cosh(M(2h + 3y)) - F\cosh(M(h + 4y)) \\
& - h\cosh(M(h + 4y)) + (-1 + FM)\,y\sinh(3My) + (F + h)\sinh(M(h + 2y)) \\
& (1 + FM)\,y\sinh(M(2h + 3y)) - (F + h)\sinh(M(h + 4y)),
\end{aligned}
$$

$$
\begin{aligned}
C_2(y) \;=\; & -2My\cosh(3My) - 16My\cosh(3M(2h + y)) + 16My\cosh(M(2h + 3y)) \\
& + 48hM^2 y\cosh(M(4h + 3y)) + 2My\cosh(M(8h + 3y)) - (1 + hM)\times \\
& (\cosh(3Mh) - \sinh(3Mh)) + (1 - hM)(\cosh(5Mh) - \sinh(5Mh)) \\
& - 2My\sinh(3My) - 16My\sinh(3M(h + 2y)),
\end{aligned}
$$

$$
\begin{aligned}
C_3(y) \;=\; & (1 + 3hM)(\cosh(M(h + 2y)) + \sinh(M(h + 2y))) + (1 + hM) \\
& (\cosh(3M(h + 2y)) + \sinh(3M(h + 2y))) + 12M(h^2 M - y - Myh) \\
& \times (\cosh(M(3h + 2y)) + \sinh(M(3h + 2y))) - 12M(h^2 M - y + Myh) \\
& \times (\cosh(M(5h + 2y)) + \sinh(M(5h + 2y))) + (-1 + 3hM) \\
& \times (\cosh(M(7h + 2y)) + \sinh(M(7h + 2y))),
\end{aligned}
$$

$$
\begin{aligned}
C_4(y) \;=\; & 16My\sinh(M(2h + 3y)) + 48hM^2\sinh(M(4h + 3y)) - (-1 + 3hM) \\
& \times (\cosh(M(h + 4y)) + \sinh(M(h + 4y))) + 2My\sinh(M(8h + 3y)) \\
& - 12M(\cosh(M(3h + 4y)) + \sinh(M(3h + 4y)))(h^2 M + y + hMy) \\
& + 12M(\cosh(M(5h + 4y)) + \sinh(M(5h + 4y)))(h^2 M + y - hMy) \\
& + (\cosh(M(7h + 4y)) + \sinh(M(7h + 4y)))(1 - 3hM) + (-1 + hM) \\
& (\cosh(M(5h + 6y)) + \sinh(M(5h + 6y))),
\end{aligned}
$$

$$
C_5(y) = (E - 2B_0 FM)\cosh(Mh) + B_0(F + h)M\cosh(My) - (2B_0 + E)\sinh(Mh),
$$

$$C_6(y) = -12\left(+B_0 + B_0 hM + B_0\left(-1 + hM\right)\left(\cosh\left(2Mh\right) + \sinh\left(2Mh\right)\right)\right)$$
$$\left(\cosh\left(3Mh\right) + \sinh\left(3Mh\right)\right)\left(-1 + \left(\cosh\left(M2y\right) + \sinh\left(M2y\right)\right)\right)^2$$
$$\left(1 + \left(\cosh\left(2My\right) + \sinh\left(2My\right)\right)\right) + 2B_0\left(12hM - 8\sinh\left(2Mh\right) + \sinh\left(4Mh\right)\right)$$
$$\left(\cosh\left(M\left(4h + 3y\right)\right) + \sinh\left(M\left(4h + 3y\right)\right)\right),$$

$$C_7(y) = 2\left(F + h\right)\left(\cosh\left(Mh\right) + \sinh\left(Mh\right)\right) - \left(\cosh\left(My\right) + \sinh\left(My\right)\right)\left(2F + 2h\right.$$
$$\left. + h^2 M + h^2 EM - Fh^2 M^2 + h^3 EM^2 - My^2 - hMy^2 + FM^2 y^2 - hEM^2 y^2\right)$$
$$\left(\cosh\left(M\left(2h + y\right)\right) + \sinh\left(M\left(2h + y\right)\right)\right) + 2\left(F + h\right)$$
$$\left(\cosh\left(M\left(h + 2y\right)\right) + \sinh\left(M\left(h + 2y\right)\right)\right),$$

$$C_8(y) = \left(1 + hM\right)\left(\cosh\left(3Mh\right) + \sinh\left(3Mh\right)\right) + \left(\cosh\left(5Mh\right) + \sinh\left(5Mh\right)\right)$$
$$\left(-1 + hM\right)\left(\cosh\left(3My\right) + \sinh\left(3My\right)\right)\left(2 + 8hM + 3h^2 M^2 - 3M^2 y^2\right)$$
$$+ 8\left(4 - 8hM + 3h^2 M^2 - 3M^2 y^2\right)\left(\cosh\left(3M\left(2h + y\right)\right) + \sinh\left(3M\left(2h + y\right)\right)\right)$$
$$- 3\left(1 + 3hM\right)\left(\cosh\left(M\left(h + 2y\right)\right) + \sinh\left(M\left(h + 2y\right)\right)\right) + \left(1 + hM\right)\times$$
$$\left(\cosh\left(3M\left(h + 2y\right)\right) + \sinh\left(3M\left(h + 2y\right)\right)\right),$$

$$C_9(y) = -3\left(1 + 3hM\right)\left(\cosh\left(M\left(h + 4y\right)\right) + \sinh\left(M\left(h + 4y\right)\right)\right) - 36\left(-1 - hM\right.$$
$$\left. + h^2 M^2 - My - hM^2 y\right)\left(\cosh\left(M\left(3h + 2y\right)\right) + \sinh\left(M\left(3h + 2y\right)\right)\right) + 36$$
$$\left(-1 - hM + h^2 M^2 - My - hM^2 y\right)\left(\cosh\left(M\left(5h + 2y\right)\right) + \sinh\left(M\left(5h + 2y\right)\right)\right)$$
$$+ \left(3 - 9hM\right)\left(\cosh\left(M\left(7h + 2y\right)\right) + \sinh\left(M\left(7h + 2y\right)\right)\right) - 8\left(4 + 8hM\right.$$
$$\left. + 3h^2 M^2 - 3M^2 y^2\right)\left(\cosh\left(M\left(2h + 3y\right)\right) + \sinh\left(M\left(72h + 3y\right)\right)\right) - 72hM^3$$
$$\left(h^2 - y^2\right)\left(\cosh\left(M\left(4h + 3y\right)\right) + \sinh\left(M\left(4h + 3y\right)\right)\right),$$

$$C_{10}(y) = \left(\cosh\left(M\left(8h + 3y\right)\right) + \sinh\left(M\left(8h + 3y\right)\right)\right)\left(-2 + 8hM - 3h^2 M^2 + 3M^2 y^2\right)$$
$$- 36\left(-hM + h^2 M^2 + My + hM^2 y\right)\left(\cosh\left(M\left(8h + 3y\right)\right) + \sinh\left(M\left(8h + 3y\right)\right)\right)$$
$$+ 36\left(\cosh\left(M\left(3h + 4y\right)\right) + \sinh\left(M\left(3h + 4y\right)\right)\right)\left(-hM + h^2 M^2 + My + hM^2 y\right)$$
$$\left(\cosh\left(M\left(5h + 4y\right)\right) + \sinh\left(M\left(5h + 4y\right)\right)\right)\left(hM + h^2 M^2 + My - hM^2 y\right)$$
$$\left(3 - 9hM\right)\left(\cosh\left(M\left(5h + 6y\right)\right) + \sinh\left(M\left(5h + 6y\right)\right)\right),$$

$$C_{11}(y) = 4\left(2 + h^2 M^2\left(-2 + Br\left(F + h\right)^2 M^2\right) - Br\left(F + h\right)^2 M^4 y^2\right)$$
$$\left(\cosh\left(2hM\right) + \sinh\left(2hM\right)\right) + \left(4 - 8hM + \left(4h^2 + Br\left(F + h\right)^2\right)M^2\right)$$
$$Br\left(F + h\right)^2 M^2\left(\cosh\left(2M\left(h + y\right)\right) + \sinh\left(2M\left(h + y\right)\right)\right),$$

$$C_{12}(y) = (1+hM)(\cosh(4hM) + \sinh(4hM)) + 5(-1+hM)(\cosh(6hM) + \sinh(6hM))$$
$$(\cosh(4yM) + \sinh(4yM)(-1+7hM)) - 4(\cosh(2M(h+y)) + \sinh(2M(h+y)))$$
$$\times (1+3hM) - 16(\cosh(4M(h+y)) + \sinh(4M(h+y)))\left(1 - 5hM + 18h^2M^2\right.$$
$$\left. +12h^3M^3 + 12h^4M^4 - 12M^2y^2 - 12hM^3y^2 - 12h^2M^4y^2\right),$$

$$C_{13}(y) = 4\left(5 - 5hM + 12h^2M^2 + 12My - 12hM^2y\right)$$
$$(\cosh(6M(h+y)) + \sinh(6M(h+y))) - (\cosh(2M(2h+y)) + \sinh(2M(2h+y)))$$
$$\times 4\left(5 + 5hM + 12h^2M^2 + 12My - 12hM^2y\right) + \left(25 + 33hM - 16h^2M^2 + 48h^3M^3\right.$$
$$\left. +16M^2y^2 - 48hM^3y^2\right)(\cosh(4M(2h+y)) + \sinh(4M(2h+y))) + (5 - 5hM+$$
$$+12h^2M^2 - 12My + 12hM^2y) + (\cosh(2M(4h+y)) + \sinh(2M(4h+y)))$$
$$\times (4 - 12hM) + \left(25 - 33hM - 16h^2M^2 - 48h^3M^3 + 16M^2y^2 + 48hM^3y^2\right),$$

$$C_{14}(y) = 5(\cosh(4M(h+2y)) + \sinh(4M(h+2y)))(1+hM) - 4(1+3hM)$$
$$(\cosh(2M(h+3y)) + \sinh(2M(h+3y))) + 16\left(1 + 5hM + 18h^2M^2 - 12h^3M^3\right.$$
$$\left. +12h^4M^4 - 12M^2y^2 + 12hM^3y^2 + 12h^2M^4y^2\right) \times (\cosh(M(6h+4y))$$
$$+ \sinh(M(6h+4y))) + (1 + 7hM)(\cosh(M(10h+4y)) + \sinh(M(10h+4y)))$$
$$+4\left(5 + 5hM + 12h^2M^2 + 12My - 12hM^2y\right),$$

$$C_{15}(y) = 2\left(-162(F+h)M^{13}\left(h^2M + y + hMy\right) + 18Gr(F+h)^2M^7\left(-8 - 8hM + 4h^2M^2\right.\right.$$
$$\left. +12h^3M^3 + 9h^5M^5 + 9h^3M^4y + 9h^4M^5y\right)\gamma - 3Gr^2(F+h)M^3\left(-41h^3M^2 + 40\right.$$
$$\times h^4M^3 + 79h^5M^4 + 30h^6M^5 + 18h^8M^7 + 9(1+hM)\left(9 - 9h^2M^2 + 2h^6M^6\right)y$$
$$\left. -54M^2(-1+hM)(1+hM)^2y^3 + 81M(-1+hM)(y+hMy)^2\right)\gamma^2 + Gr^3h^3$$
$$\left(-648\left(1 + hM - h^2M^2\right) + 623h^3M^3 + 56h^4M^4 + 71h^5M^5 + 6h^6M^6 + 6h^8M^8\right.$$
$$+81My + 81hM^2y - 81h^2M^3y - 81h^3M^4y + 6h^6M^7y + 6h^7M^8y - 81M^2y^2$$
$$-81hM^3y^2 + 81h^2M^4y^2 + 81h^3M^5y^2 + 54M^3y^3 + 54hM^4y^3 - 54h^2M^5y^3$$
$$\left.\left. -54h^3M^6y^3\right)\gamma^3 (\cosh(M(3h+4y)) + \sinh(M(3h+4y)))\right),$$

$$C_{16}(y) = -2\left(-162(F+h)^3 M^{13}\left(h^2 M + y - hMy\right) + 18Gr(F+h)^2\left(8 - 8hM - 4h^2 M^2\right.\right.$$
$$\left. + 12h^3 M^3 + 9h^5 M^5 + 9h^3 M^4 y - 9h^4 M^5 y\right)\gamma M^7 + 3Gr^2(F+h)M^3\left(41h^3 M^2\right.$$
$$+ 40h^4 M^3 - 79h^5 M^4 + 30h^6 M^5 + 18h^8 M^7 - 9(-1+hM)\left(9 - 9h^2 M^2 + 2h^6\right)$$
$$M^6\right)y - 81M(-1+hM)^2(1+hM)y^2 + 54M^2\left(-1+hM)^2 + (1+hM)y^3\right)\gamma^2$$
$$+ Gr^3 h^3\left(-648 + 648hM + 648h^2 M^2 - 623h^3 M^3 + 56h^4 M^4 - 71h^5 M^5 + 6h^6\right.$$
$$M^6 + 6h^8 M^8 + 81My - 81hM^2 y - 81h^2 M^3 y + 81h^3 M^4 y + 6h^6 M^7 y - 6h^7 M^8 y,$$
$$-81M^2 y^2 + 81hM^3 y^2 + 81h^2 M^4 y^2 - 81h^3 M^5 y^2 + 54M^3 y^2 - 54hM^4 y^3 - 54h^2$$
$$\left.\left. M^5 y^3 + 54h^3 M^6 y^3\right)\gamma^3\right)\left(\cosh\left(M(5h+4y)\right) + \sinh\left(M(5h+4y)\right)\right),$$

$$C_{17}(y) = \left(-27(F+h)^3 M^{12}(-1+3hM) + 9Gr(F+h)^2 M^7\left(32 - 96hM + 112h^2 M^2 - \right.\right.$$
$$51h^3 M^3 + 9h^4 M^4\right)\gamma - 3Gr^2(F+h)M^3\left(82h^3 M^2 - 192h^4 M^3 + 170h^5 M^4\right.$$
$$-63h^6 M^5 + 9h^7 M^6 - 54(-1+hM)^3 y + 54M(-1+hM)^3 y^2 - 36(-1+hM)^3 y^3$$
$$M^2\right)\gamma^2 + Gr^3 h^3\left(-432 + M\left(1296h - 1296h^2 M + 482h^3 M^2 - 96h^4 M^3 + 58\right.\right.$$
$$h^5 M^4 - 13h^6 M^5 + 3h^7 M^6 - 54(1+hM)^3 y + 54M(-1+hM)^3 y^2 - 36M^2$$
$$\left.\left.\left.(-1+hM)^3 y^3\right)\right)\gamma^3\right)\left(\cosh\left(M(7h+4y)\right) + \sinh\left(M(7h+4y)\right)\right)$$

$$B_0 = \frac{1}{1 + hM - \cosh(2hM) + hM\cosh(2hM) - \sinh(2hM) + hM\sinh(2hM)},$$

$$B_1(y) = \left(\cosh My + \sinh My + \cosh M(h+2y) + \sinh M(h+2y)\right)\left(-3FM^2 - 3hM^2 + Grh^3\beta_1\right)$$
$$\times 8 + \left(\cosh My + \sinh My\right)\left(24(F+h)M^2 + 12h^2 M^3(1+L) - 12M^3 y^2(1+L) - 12\right.$$
$$(F - hL)h^2 M^4 + 12y^2 M^4(F - hL) - 8Grh^3\beta_1 - 5Grh^4 M\beta_1 - Grh^5 M^2\beta_1 + Grh^2 y^2$$
$$\beta_1 M(6 + 2hM) - GrMy^4\beta_1(1+hM)\right) + \left(\cosh M(2h+y) + \sinh M(2h+y)\right)\left((F+h)\right.$$
$$24M^2 - 12h^2 M^3(1 + L + FM - LM) + 12M^3 y^2(1 + L + FM - hLM) - 8Grh^3\beta_1 +$$
$$\left.\left. 5Grh^4 M\beta_1 - Grh^5 M^2\beta_1 - Grh^2 My^2\beta_1\left(6 - 2hM + GrMy^4\gamma(1-hM)\right)\right)\right),$$

$$
\begin{aligned}
B_2(y) =\ & -M^6\left(3(F+h)M^2 - Grh^3\beta_1\right)^3(\cosh 3Mh + \sinh 3Mh) + (\cosh 5Mh + \sinh 5Mh) \\
& (1+hM)M^6(-1+hM)\left(-3(F+h)M^2 + Grh^3\beta_1\right)^3 + (\cosh 3My + \sinh 3My) \\
& \left(-27M^{12}(F+h)^3\left(2 + 8hM + 3M^2(h^2 - y^2)\right)\right) + 27GrM^7\beta_1(F+h)^2(10+ \\
& 40hM + 70h^2M^2 + 54h^3M^3 + 20h^4M^4 + 3h^5M^5 - 4M^2y^2 - 16hM^3y^2 - \\
& 12h^2M^4y^2 - 3h^3M^5y^2) + 9Gr^2M^2(F+h)\left(-378 - 1512hM - 2457h^2M^2\right. \\
& -2180h^3M^3 - 1241h^4M^4 - 518h^5M^5 - 160h^6M^6 - 32h^7M^7 - 3h^8M^8 + 27 \\
& M^2y^2 + 108hM^3y^2 + 189h^2M^4y^2 + 170h^3M^5y^2 + 86h^4M^6y^2 + 24h^5M^7y^2 \\
& \left.+3h^6M^8y^2\right)\beta_1^2 + Gr^3\beta_1^3\left(2430h^3 + 9234h^4M + 13851h^5M^2 + 10722h^6\right. \\
& \times M^3 + 4899h^7M^4 + 1506h^8M^5 + 320h^9M^6 + 44h^{10}M^7 + 3h^{11}M^8 - h^3y^2 \\
& 972h^2My^2 + 3321M^2 - 4212h^4M^3y^2 - 2511h^5M^4y^2 - 822h^6M^5y^2 - \\
& 210h^7M^6y^2 - 36h^8M^7y^2 - 3h^9M^8y^2 + 162My^4 + 648hM^2y^4 + 972h^2M^3y^4 \\
& \left.\left.+648h^3M^4y^4 + 162h^4M^5y^4\right)\right),
\end{aligned}
$$

$$
\begin{aligned}
B_3(y) =\ & (11 + 6My)\left(Grh^3\beta_1 - 3(F+h)M^2\right)^2(\cosh M(2h+y) + \sinh M(2h+y)) \\
& 6GrM^3\beta_1 - 2\left(108M^{12}(F+h)^3\left(4 - 8hM + 3(h^2 - y^2)M^2\right) - 54GrM^7(F+h)^2\right. \\
& \left(11(1 - 2hM) + 11h^2M^2 + 10h^3M^3 - 22h^4M^4 + 6h^5M^5 + 10y^2M^2 - 20y^2hM^3\right) \\
& \left.+6y^2h^2M^4 - 6y^2h^3M^5\right)\beta_1 + 9M^2Gr^2(F+h)\left(378 - 756hM + 189h^2M^2 + 674h^3\right. \\
& M^3 - 889h^4M^4 + 494h^5M^5 - 102h^6M^6 - 20h^7M^7 + 12h^8M^8 - 27M^2y^2 + 54hM^3 \\
& \left.\times y^2 - 27h^2M^4y^2 - 50h^3M^5y^2 + 46h^4y^2M^6 - 12h^5M^7y^2 - 12h^6M^8y^2\right)\beta_1^2 + Gr^3 \\
& \left(-2430h^3 + 3888h^4M + 729h^5M^2 - 4548h^6M^3 + 3831h^7M^4 - 1740h^8M^5 + 350\right. \\
& \times h^9M^6 - 40h^{10}M^7 - 12h^{11}M^8 - 1944h^2My^2 + h^33321M^2y^2 - 162h^4M^3y^2 - \\
& 1863h^5M^4y^2 + 858h^6M^5y^2 - 258h^7M^6y^2 + 72h^8M^7y^2 + 12h^9M^8y^2 + 324My^4 \\
& \left.\left.-648hM^2y^4 + 648h^3M^4y^4 + 324h^4M^5y^4\right)\gamma^3\right)(\cosh 3M(2h+y) \\
& +\sinh 3M(2h+y)),
\end{aligned}
$$

$$
\begin{aligned}
B_4\left(y\right) =\ & 12M^3\beta_1\left(h^2M^2-1\right)\left(Grh^3\beta_1-3\left(F+h\right)M^2\right)^2\left(\cosh M\left(4h+y\right)+\sinh M\left(4h+y\right)\right)\\
& \left(11+6My\right)\left(1+hM\right)^2Gr+6\left(hM-1\right)^2\left(\cosh M\left(6h+y\right)+\sinh M\left(6h+y\right)\right)\\
& \left(Grh^3\beta_1-3\left(F+h\right)M^2\right)^2\left(11+6hM\right)GrM^3\beta_1+3\left(27M^{12}\left(F+h\right)^3\left(1+3hM\right)\right.\\
& -9GrM^7\beta_1\left(F+h\right)^2\left(32+96hM+112h^2M^2+51h^3M^3+9h^4M^4\right)+3Gr^2M^2\\
& \left(F+h\right)\left(378+M\left(1134h+1134h^2M+460h^3M^2+192h^4M^3+170h^5M^4+\right.\right.\\
& \left.63h^6M^5+9h^7M^6+378y\left(1+hM\right)^3+162My^2\left(1+hM\right)^3+36M^2\left(y+hMy\right)^3\right)\right)\\
& \beta_1^2-Gr^3h^3\left(810+M\left(2430h+2430h^2M+860h^3M^2+96h^4M^3+58h^5M^4\right.\right.\\
& \left.\left.\left.+13h^6M^5+3h^7M^6+y\left(378+162My+36M^2y^2\right)\left(1+hM\right)^3\right)\right)\beta_1^3\right)\\
& \left(\cosh M\left(h+2y\right)+\sinh M\left(h+2y\right)\right),
\end{aligned}
$$

$$
\begin{aligned}
B_5\left(y\right) =\ & -M^6\left(1+hM\right)\left(3\left(F+h\right)M^2-Grh^3\beta_1\right)^3\left(\cosh 3M\left(h+2y\right)+\sinh 3M\left(h+2y\right)\right)\\
& \times\left(h^2M^2-1\right)+3\left(27M^{12}\left(F+h\right)^3\left(1+3hM\right)-9Gr\beta_1M^7\left(F+h\right)^2\left(32+96hM\right.\right.\\
& \left.+112h^2M^2+51h^3M^3+h^4M^4\right)+3Gr^2\beta_1^2M^2\left(F+h\right)\left(378+M\left(1134h+1134\right.\right.\\
& h^2M+460h^3M^2+192h^4M^3+170h^5M^4+63h^6M^5+h^7M^6-y\left(378-162My\right.\\
& \left.\left.+36M^2y^2\right)\left(1+hM\right)^3\right)\right)-\gamma^3Gr^3h^3\left(810+M\left(2430h+2430h^2M+860h^3\right.\right.\\
& M^2+96h^4M^3+58h^5M^4+13h^6M^5+3h^7M^6-y\left(378-162My+36M^2y^2\right)\\
& \left.\left.\left.\left(1+hM\right)^3\right)\right)\right)\left(\cosh M\left(h+4y\right)+\sinh M\left(h+4y\right)\right),
\end{aligned}
$$

$$
\begin{aligned}
B_6\left(y\right) =\ & -6\left(-162M^{12}\left(F+h\right)^3\left(-1-hM+h^2M^2-My-hyM^2\right)+18GrM^7\beta_1\left(F+h\right)^2\right.\\
& \left(-8-8hM+4h^2M^2+3h^3M^3-9h^4M^4+9h^5M^5-9h^3M^4y\left(1+hM\right)\right)+3Gr^2\\
& M^2\left(-F-h\right)\gamma^2\left(-567-567hM+567h^2M^2+526h^3M^3+40h^4M^4+79h^5M^5\right.\\
& +12h^6M^6-18h^7M^7+18h^8M^8-567My\left(1+Mh-h^2M^2-h^3M^3\right)-18h^6M^7y\\
& \left(1+hM\right)-243y^2\left(1+hM-h^2M^2-h^3M^3\right)M^2-54\left(1+hM-h^2M^2-h^3M^3\right)\right)\\
& M^3y^3+Gr^3h^3\left(-1215+M\left(-1215h\left(1-hM\right)+h^31190M^2+56h^4M^3+71h^5\right.\right.\\
& M^4-6h^7M^6+6M^7h^8-3\left(189-189h^2M^2+2h^6M^6\right)y\left(1+hM\right)+54\left(hM-1\right)\\
& \left.\left.\left.\times M^2y^3\left(1+hM\right)^2+243M\left(hM-1\right)\left(y+yhM\right)^2\right)\right)\beta_1^3\right)\\
& \left(\cosh M\left(3h+2y\right)+\sinh M\left(3h+2y\right)\right),
\end{aligned}
$$

$$B_7(y) = 6\left(-162M^{12}(F+h)^3\left(-1+hM+h^2M^2-My+hyM^2\right)+18GrM^7\beta_1(F+h)^2\right.$$
$$\left(\left(8-8hM-4h^2M^2+3h^3M^3+9h^4M^4+9h^5M^5-9h^3M^4y(1-hM)\right)+3Gr^2M^2\right.$$
$$(-F-h)\beta_1^2\left(-567+567hM+567h^2M^2-526h^3M^3+40h^4M^4-79h^5M^5+12h^6\right.$$
$$M^6+18h^7M^7+18h^8M^8-567My\left(1-Mh-h^2M^2+h^3M^3\right)-18h^6M^7y(1-hM)$$
$$-243\left(1-hM-h^2M^2+h^3M^3\right)y^2M^2-54M^3y^3\left(1-hM-h^2M^2+h^3M^3\right)\right)+Gr^3$$
$$h^3\left(-1215\left(1-hM-h^2M^2\right)-1190\,h^3M^3+56h^4M^4-71h^5M^5+6h^7M^7+6h^8\right.$$
$$M^8-567My\left(1-Mh-h^2M^2+h^3M^3\right)-\left(1-hM-h^2M^2+h^3M^3\right)-6(1-hM)$$
$$\left.\left.h^6My+243M^2y^2-54\left(1-hM-h^2M^2+h^3M^3\right)M^3y^3\right)\beta_1^3\right)$$
$$(\cosh M(5h+2y)+\sinh M(5h+2y)),$$

$$B_8(y) = -3\left(-27M^{12}(F+h)^3(3hM-1)+9GrM^7\beta_1(F+h)^2\left(32-96hM+112h^2M^2-51\right.\right.$$
$$\left.h^3M^3+9h^4M^4\right)-3Gr^2M^2(F+h)\beta_1^2\left(-378+M\left(1134h(1-hM)+460h^3M^2-\right.\right.$$
$$\left.\left.192h^4M^3+170h^5M^4-63h^6M^5+9h^7M^6+y\left(378+162My+36M^2y^2\right)\right)\right)$$
$$(hM-1)^3+Gr^3h^3\times\left(-810+M\left(2430h(1-hM)+860h^3M^2-96h^4M^3+58h^5\right.\right.$$
$$\left.\left.\left.M^4-13h^6M^5+3h^7M^6+y(hM-1)^3\left(378+162My+36M^2y^2\right)\right)\right)\beta_1^3\right)$$
$$(\cosh M(7h+2y)+\sinh M(7h+2y)),$$

$$B_9(y) = -2\left(-108M^{12}(F+h)^3\left(4+8hM+3h^2M^2-3M^2y^2\right)+54GrM^7\beta_1(F+h)^2\right.$$
$$\left(-11-22hM-11h^2M^2+10h^3M^3+22h^4M^4+6h^5M^5-10M^2y^2-20\right.$$
$$\left.hM^3y^2-6h^2M^4y^2(1+hM)\right)+9Gr^2M^2(F+h)\beta_1^2\left(-378-756hM-\right.$$
$$189h^2M^2+674h^3M^3+889h^4M^4+494h^5M^5+102h^6M^6-20h^7M^7$$
$$-12h^8M^8+27M^2y^2+54hM^3y^2+27h^2M^4y^2-50h^3M^5y^2-46y^2M^6$$
$$h^4-12h^5M^7y^2+12h^6M^8y^2\right)+Gr^3\beta_1^3\left(2430h^3+3888h^4M-729h^5\right.$$
$$M^2-4548h^6M^3-3831h^7M^4-1740h^8M^5-350h^9M^6-40h^{10}M^7+$$
$$12h^{11}M^8-1944My^2h^2-M^23321h^3y^2-162h^4M^3y^2+1863h^5M^4y^2$$
$$+858h^6M^5y^2+258h^7M^6y^2+72h^8M^7y^2-12h^9M^8y^2+324My^4\right))$$
$$\left.\left.+\left(1+2hM-2h^3M^3-h^4M^4\right)\right)\right)(\cosh M(2h+3y)+\sinh M(2h+3y)),$$

$$
\begin{aligned}
B_{10}\left(y\right) =\ & -36M\left(-54hM^{14}\left(F+h\right)^{3}\left(h^{2}-y^{2}\right)+27GrM^{6}\beta_{1}\left(F+h\right)^{2}\left(3-3h^{2}M^{2}+2h^{4}\right.\right. \\
& \left(1+h^{2}M^{2}\right)M^{4}+2y^{2}M^{2}-2h^{2}y^{2}M^{4}-2h^{4}M^{6}y^{2}\right)-\left(28-27h^{2}M^{2}+2h^{6}M^{6}\right. \\
& \times-3M^{2}y^{2}-2h^{6}M^{4}y^{2}\right)9Gr^{2}h^{3}M^{4}\beta_{1}^{2}\left(F+h\right)+Gr^{3}\beta_{1}^{3}\left(81h^{4}+21h^{6}M^{2}-99\right. \\
& h^{8}M^{4}-6h^{10}M^{6}+2h^{12}M^{8}+162h^{2}y^{2}-216h^{4}M^{2}y^{2}+39y^{2}h^{6}M^{4}+6h^{8}M^{6} \\
& \left.\left.y^{2}-2h^{10}M^{8}y^{2}-27y^{4}+54h^{2}M^{2}y^{4}-27\times y^{4}M^{4}h^{4}\right)\right) \\
& \left(\cosh M\left(4h+3y\right)+\sinh M\left(4h+3y\right)\right),
\end{aligned}
$$

$$
\begin{aligned}
B_{11}\left(y\right) =\ & \left(27M^{12}\left(F+h\right)^{3}\left(2-8hM+3h^{2}M^{2}-3M^{2}y^{2}\right)+27GrM^{7}\beta_{1}\left(F+h\right)^{2}\left(10\right.\right. \\
& -40hM+70h^{2}M^{2}-54h^{3}M^{3}+20h^{4}M^{4}-3h^{5}M^{5}-4M^{2}y^{2}+16h-12h^{2} \\
& M^{4}y^{2}+3h^{3}M^{5}y^{2}\right)+9Gr^{2}M^{2}\left(F+h\right)\gamma^{2}\left(378-1512hM+2457h^{2}M^{2}\right. \\
& -2180h^{3}M^{3}+1241h^{4}M^{4}-518h^{5}M^{5}+160h^{6}M^{6}-32h^{7}M^{7}+3h^{8}M^{8} \\
& -27M^{2}y^{2}+108hM^{3}y^{2}-189h^{2}M^{4}y^{2}+170h^{3}M^{5}y^{2}-86y^{2}M^{6}h^{4}+24 \\
& h^{5}M^{7}y^{2}-3h^{6}M^{8}y^{2}\right)+Gr^{3}\beta_{1}^{3}-2430h^{3}+9234h^{4}M-13851h^{5}M^{2}+ \\
& 10722h^{6}M^{3}-4899h^{7}M^{4}+1506h^{8}M^{5}-320h^{9}M^{6}+44h^{10}M^{7}-3h^{11} \\
& M^{8}-972My^{2}h^{2}+3321h^{3}M^{2}y^{2}-4212h^{4}M^{3}y^{2}+2511h^{5}M^{4}y^{2}-822 \\
& h^{6}M^{5}y^{2}+210h^{7}M^{6}y^{2}-36h^{8}M^{7}y^{2}+3h^{9}M^{8}y^{2}+162My^{4}-648hM^{2} \\
& \left.\left.y^{4}+972h^{2}M^{3}y^{4}-648h^{3}y^{4}M^{4}+162h^{4}M^{5}y^{4}\right)\right)\times \\
& \left(\cosh M\left(8h+3y\right)+\sinh M\left(8h+3y\right)\right),
\end{aligned}
$$

$$
\begin{aligned}
B_{12}\left(y\right) =\ & -6\left(-162M^{12}\left(F+h\right)^{3}\left(-1-hM+h^{2}M^{2}+My+hM^{2}y\right)+18GrM^{7}\beta_{1}\left(F+h\right)^{2}\right. \\
& \left(-8-8hM+4h^{2}M^{2}+3h^{3}M^{3}-9h^{4}M^{4}+9h^{5}M^{5}+9h^{4}M^{5}y\right)+3Gr^{2}M^{2} \\
& \left(-F-h\right)\beta_{1}^{2}\left(-567-567hM+567h^{2}M^{2}+526h^{3}M^{3}+40h^{4}M^{4}+79h^{5}M^{5}\right. \\
& +12h^{6}M^{6}-18h^{7}M^{7}+18h^{8}M^{8}+567My\times\left(1+hM-h^{2}M^{2}-h^{3}M^{3}\right)+ \\
& 18h^{6}M^{7}y\left(1+hM\right)-243M^{2}y^{2}\left(1+hM-h^{2}M^{2}-h^{3}M^{3}\right)+54M^{3}y^{3} \\
& \left(1+hM-h^{2}M^{2}-h^{3}M^{3}\right)\right)+Gr^{3}h^{3}\beta_{1}^{3}-1215+M-1215h\left(1-hM\right)+ \\
& 1190h^{3}M^{2}+56h^{4}M^{3}+71h^{5}M^{4}-6h^{7}M^{6}+6h^{8}M^{7}+3\left(189-189h^{2}M^{2}+2h^{6}M^{6}\right) \\
& \left.\left.\left(1+hM\right)y-54M^{2}\left(hM-1\right)\left(1+hM\right)^{2}y^{3}+243M\left(hM-1\right)\left(y+yhM\right)^{2}\right)\right)\times \\
& \left(\cosh M\left(3h+4y\right)+\sinh M\left(3h+4y\right)\right),
\end{aligned}
$$

$$
\begin{aligned}
B_{13}(y) &= 6\left(-162M^{12}(F+h)^3\left(-1+hM+h^2M^2+My-hM^2y\right)+18GrM^7\beta_1(F+h)^2\right. \\
&\quad \left(8-8hM-4h^2M^2+3h^3M^3+9h^4M^4+9h^5M^5+9h^3M^4y-9h^4M^5\beta_1\right)+3 \\
&\quad Gr^2M^2(-F-h)\beta_1^2\left(-567+567hM(1+hM)-526h^3M^3+40h^4M^4-79h^5\right. \\
&\quad M^5+12h^6M^6+18h^7M^7+18h^8M^8+M567y\left(1-hM-h^2M^2+h^3M^3\right)+18h^6 \\
&\quad M^7y(1-hM)-243M^2y^2\left(1-hM-h^2M^2+h^3M^3\right)+\left(1-hM-h^2M^2+h^3M^3\right)) \\
&\quad 54M^3y^3+Gr^3h^3\beta_1^3\left(-1215\left(1-hM-h^2M^2\right)-1190h^3M^3+56h^4M^4-71h^5M^5\right. \\
&\quad +6h^7M^7+6h^8M^8+567My\left(1-hM-h^2M^2+h^3M^3\right)+6h^6M^7y- \\
&\quad 6h^7M^8y-243M^2y^2\left(1-hM-h^2M^2+h^3M^3\right)+54M^3y^3-\left(1+hM-h^2M^2\right))) \\
&\quad 54hM^4y^3\left(\cosh M(5h+4y)+\sinh M(5h+4y)\right),
\end{aligned}
$$

$$
\begin{aligned}
B_{14}(y) &= -3\left(-27M^{12}(F+h)^3(-1+3hM)+9GrM^7\beta_1(F+h)^2\left(32-96hM+112h^2M^2\right.\right. \\
&\quad \left.-51h^3M^3+9h^4M^4\right)+3Gr^2M^2(F+h)\beta_1^2\left(378+M\left(-1134h(1-hM)-460h^3\right.\right. \\
&\quad M^2+192h^4M^3-h^5+170M^4+63h^6M^5-9h^7M^6+\left(378-162My+36M^2y^2\right)))) \\
&\quad y(hM-1)^3+Gr^3h^3\beta_1^3\times\left(-810+M\left(2430h(1-hM)+860h^3M^2-96h^4M^3+\right.\right. \\
&\quad 58h^5M^4-13h^6M^5+3h^7M^6-y\times(hM-1)^3\left(378-162yM-36y^2M^2\right)))) \\
&\quad \left(\cosh M(7h+4y)+\sinh M(7h+4y)\right),
\end{aligned}
$$

$$
\begin{aligned}
B_{15}(y) &= -6Gr(1+Mh)^2\left(Grh^3\beta_1-3M^2(F+h)\right)^2\left(\cosh M(2h+5y)+\sinh M(2h+5y)\right) \\
&\quad +\left(\cosh M(4h+5y)+\sinh M(4h+5y)\right)(-11+6hM)M^3\beta_1-12\left(Grh^3\beta_1-3M^2(F+h)\right)^2 \\
&\quad GrM^3\beta_1\left(M^2h^2-1\right)(-11+6yM)-6GrM^3\beta_1\left(Grh^3\gamma-3M^2(F+h)\right)^2 \\
&\quad \times\left(\cosh M(6h+5y)+\sinh M(6h+5y)\right)(-1+Mh)^2(-11+6yM) \\
&\quad +\left(Grh^3\beta_1-3M^2(F+h)\right)^2\left(\cosh M(5h+6y)+\sinh M(5h+6y)\right) \\
&\quad \times M^6(-1+Mh),
\end{aligned}
$$

$$
\begin{aligned}
L_1(y) &= (1+hM)^3\left(3GrM^2+3GrhM^3+3M^4+3EM^4-3FM^5+3hEM^5-3Gr\beta_1-Gr\right. \\
&\quad \times3\beta_1-3GrhM\beta_1+Grh^3M^3\beta_1\left(\cosh(3My)-\sinh(3My)\right)+2(-1+hM)^2(-6Gr \\
&\quad M^2-6M^4+6GrM^4h^2-6EM^4-3FM^5-3hM^5-6FhM^6+6h^2EM^6+6Gr\beta_1- \\
&\quad 6Grh^2M^2\beta_1+Grh^3M^3\beta_1+2GrM^4h^4\beta_1\right)\left(\cosh(3M(2h+y))-\sinh(3M(2h+y))\right),
\end{aligned}
$$

$$
\begin{aligned}
L_2\left(y\right) =\ & \left(\cosh\left(M\left(2h+3y\right)\right)-\sinh\left(M\left(2h+3y\right)\right)\right)\left(-6GrM^2-6M^4+6GrM^4h^2-6EM^4+3h\right.\\
& \left.M^5+3FM^5+6FhM^6+6h^2EM^6+6Gr\beta_1-6Grh^2M^2\beta_1+Grh^3M^3\beta_1+2GrM^4h^4\beta_1\right)\\
& 2\left(-1+hM\right)^2+6\left(\cosh\left(M\left(4h+3y\right)\right)-\sinh\left(M\left(4h+3y\right)\right)\right)\left(-1+h^2M^2\right)\left(-3GrM^2-\right.\\
& \left.3M^4+3GrM^4h^2-3EM^4-3FhM^6+3h^2EM^6+3Gr\beta_1-3Grh^2M^2\beta_1+Grh^4M^4\beta_1\right)\\
& +\left(-1+hM\right)^3\left(-3GrM^2+3GrhM^3-3M^4-3EM^4-3FM^5+3hEM^5+3Gr\beta_1-Gr\right.\\
& \left.3hM\beta_1+Grh^3M^3\beta_1\right)\left(\cosh\left(M\left(8h+3y\right)\right)-\sinh\left(M\left(8h+3y\right)\right)\right),
\end{aligned}
$$

$$
\begin{aligned}
L_3\left(y\right) =\ & -27\left(F+h\right)^3M^{15}+9Gr\left(F+h\right)^2M^{10}\left(4+16hM+12h^2M^2+3h^3M^3\right)\beta_1-3Gr^2M^5\\
& \left(F+h\right)\left(27+108hM+189h^2M^2+170h^3M^3+86h^4M^4+24h^5M^5+3h^6M^6\right)\beta_1^2\\
& +Gr^3\left(648+2592hM+4212h^2M^2+3699h^3M^3+2052h^4M^4+837h^5M^5+274\right.\\
& \left.\times h^6M^6+70h^7M^7+12h^8M^8+h^9M^9\right)\beta_1^3\right)\left(\cosh\left(3My\right)+\sinh\left(3My\right)\right)-2\left(108\right.\\
& \left(F+h\right)^3M^{15}-36Gr\left(F+h\right)^2M^{10}\left(-14+28hM-12h^2M^2+3h^3M^3\right)\beta_1+3Gr^2M^5\\
& \left(F+h\right)\left(27-54hM+27h^2M^2-22h^3M^3+98h^4M^4-60h^5M^5+12h^6M^6\right)\beta_1^2,
\end{aligned}
$$

$$
\begin{aligned}
L_4\left(y\right) =\ & \left(+1296-2592hM+648h^2M^2+1485h^3M^3-1242h^4M^4+621h^5M^5-250h^6\right.\\
& \left.M^6+14h^7M^7-12h^8M^8+4h^9M^9\right)\beta_1^3\right)\left(\cosh\left(3M\left(2h+y\right)\right)+\sinh\left(3M\left(2h+y\right)\right)\right)\\
& Gr^3-2\left(-108\left(F+h\right)^3M^{15}+36Gr\left(F+h\right)^2M^{10}\left(14+28hM+12h^2M^2+3h^3M^3\right)\right.\\
& \times\beta_1+3Gr^2M^5\left(27+54hM+27h^2M^2+22h^3M^3+98h^4M^4+60h^5M^5+12h^6M^6\right)\\
& \beta_1^2\left(F+h\right)+Gr^3\left(\cosh\left(M\left(2h+3y\right)\right)+\sinh\left(M\left(2h+3y\right)\right)\right)\left(1296+2592hM\right.\\
& +648h^2M^2-1485h^3M^3-1242h^4M^4-621h^5M^5-250h^6M^6-144h^7M^7+12\\
& \left.h^8M^8+4h^9M^9\right)\beta_1^3,
\end{aligned}
$$

$$
\begin{aligned}
L_5\left(y\right) = &\; -12\left(-54h\left(F+h\right)^3 M^{16} + 54Gr\left(F+h\right)^2 M^{10}\left(-3+3h^2 M^2+h^4 M^4\right)\beta_1 - 9\left(F+h\right)\right. \\
&\; Gr^2 h^3 M^8\left(-5+8h^2 M^2+2h^4 M^4\right)\beta_1^2 + Gr^3\left(-324+486h^2 M^2-108h^4 M^4-51\right. \\
&\; \left.\left. h^6 M^6+6h^8 M^8+2h^{10} M^{10}\right)\beta_1^2\right)\left(\cosh\left(M\left(4h+3y\right)\right)+\sinh\left(M\left(4h+3y\right)\right)\right) \\
&\; +\left(27\left(F+h\right)^3 M^{15} - 9Gr\left(F+h\right)^2 M^{10}\left(-4+16hM-12h^2 M^2+3h^3 M^3\right)\beta_1\right. \\
&\; +3Gr^2 M^5\left(27-108hM+189h^2 M^2-170h^3 M^3+86h^4 M^4-24h^5 M^5+3h^6 M^6\right) \\
&\; \beta_1^2\left(F+h\right) - Gr^3\left(\cosh\left(M\left(4h+3y\right)\right)+\sinh\left(M\left(4h+3y\right)\right)\right)\left(-648+2592hM\right. \\
&\; -4212h^2 M^2+3699h^3 M^3-2052h^4 M^4+837h^5 M^5-274h^6 M^6+70h^7 M^7 \\
&\; \left.\left. -12h^8 M^8+h^9 M^9\right)\beta_1^3\right).
\end{aligned}
$$