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Mécanique

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Volume 348, issue 8-9 (2020), p. 705-727

Published online: 14 December 2020

<https://doi.org/10.5802/crmeca.57>



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Les Comptes Rendus. Mécanique sont membres du
Centre Mersenne pour l'édition scientifique ouverte
www.centre-mersenne.org
e-ISSN : 1873-7234



Historical article / *Article historique*

Correspondence between de Saint-Venant and Boussinesq. 4: The role of Frédéric Reech

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Abstract. Frédéric Reech was a notable French naval engineer, whose name is particularly known for his role in the theory of mechanical similitude. Based on a submission of a paper to the Academy of Sciences, Paris, de Saint-Venant (dSV) had to deal with it, given his role as president of the Mechanics Section. Part 2 of this work deals with the exchange of letters between dSV and Boussinesq on the theory of permanent waves, the topic addressed by Reech in his submission. Given the relevance of Reech in hydraulic modeling, and the poor knowledge on both his career and his biography, part 1 deals with a hardly known publication of Moritz Weber, the namesake of the Froude number in 1919. He changed this later into the Reech number, based on detailed researches on Reech's career as a naval engineer, professor at the Marine School of Lorient, and his writings. This paper therefore both accounts for the Correspondence and the excellent work of Weber, whose name is kept in the Weber number for effects of capillarity.

Keywords. Academy of Sciences, Biography, History of hydraulics, Periodic waves, Shallow water equations.

Manuscript received 19th October 2020, accepted 21st October 2020.

Introduction

Frédéric Reech's name is well known in thermodynamics, steam machinery, naval engineering, hydromechanics, and mechanical similitude. He may be considered a somewhat forgotten great engineer and physicist of the 19th century, given his role mainly as a teacher at and director of the School of Maritime Engineering, first at the harbor town of Lorient in Brittany, later then in Paris. A hardly noticed paper written by the German Moritz Weber, namesake of the Weber number, first proposed the Froude number accounting for the ratio between inertia and gravity forces [1]. However, Weber realized only in 1942 that Reech had made this proposal by more than 20 years ahead of the great British naval engineer Froude. Weber [2] also identified the true merits of these two scientists. The first portion of this work mainly deals with the 1942 paper of Weber in which the various merits and ideas of Reech (Figure 1) are presented.

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Figure 1. Photograph of Frédéric Reech [2].

The second portion of this work deals with letters exchanged between Adhémar Barré de Saint-Venant (dSV) and Joseph Boussinesq (JB) between 1868 and 1885, that is, shortly before the death of dSV. As stated by [3–5], these letters are kept at the Library of the *Institut de France, Académie des Sciences*, Paris. It is accessible only after having obtained the entrance letter of the Secretary perpetual of the Academy; the letters may only be photographed. This was done in the above three mentioned works, after having transcribed the contents of the letters on a WORD file, from where the translation into English language was obtained. The original letters may neither be shared with others, nor be published. The final manuscript was then submitted to the Directrice of the Library for approval. The letters between dSV and JB deal with steady free surface waves, a topic studied by Reech, yet overlooked by Weber in the discussion of his works. Note that JB also exchanged few letters with Reech, yet these are not available at the Library of the Academy. However, there are several letters in which dSV asks JB for details in the mathematical developments, and in questions of scientific priority in the establishment of the results. These issues shall be considered here, based on the letters available in the library.

The professional career and the life of Frédéric Reech

Introduction

Weber [2] gives a detailed account on the issues addressed in the above title. His paper deals (1) with the main law of similitude in the study of naval engineering, (2) with the role of Reech as

a marine engineer, and (3) his biography based mainly on the account of [6]. Given the excessive length of this paper, only excerpts shall be included in the following, allowing for a detailed insight in the career and the biography of Reech. Note: The publications of Mr. Reech are separately cited at the end of our paper.

Main law of similitude in naval engineering

“The shape or wave resistance of ships depends on the shape and size of the vessel, its speed and the fluid characteristics; by these circumstances, the geometric shape of the waves produced, including the detachment and vorticity phenomena, as well as the wetted surface and all normal pressings of the water on the ship are essentially determined. For example, if the ship resistance W is written in the form of $W = \alpha \rho F V^2$, in which ρ is the fluid density, F the surface of the main frame [Sic.: the largest surface below water of the boat perpendicular to its longitudinal axis] and V its velocity, then the coefficient α represents no constant. In turn, it depends in a complicated manner for a ship of a certain shape and dimension on its velocity and other factors. It has caused naval engineers a great deal of difficulties to determine both the total resistance of a ship consisting of the shape [Sic.: form resistance] and the frictional [Sic.: skin friction] resistances. These are formed, respectively, by normal tensions and tangential stresses. Both, experimental and mathematical methods have been unsuccessful to determine these questions to a certain degree of accuracy.” Both the French and the British Navies were highly interested in the prediction of naval resistance, to optimize their large fleets. Note here that Great Britain was with Queen Victoria, whose reign started in 1837, at the step of world power, culminating during World War I (WWI) with the large part conquered globally. Similarly, the French Empire started with the conquer of Algeria in 1830, also the initiation of its colonial power. Both of these countries were therefore heavily interested in the questions of ship optimizations and naval improvements to satisfy the needs of transport from their countries to the vast number of colonial possessions.

Weber [2] continues: “In a period that lasted about 200 years, many attempts were made to predict the resistance of a particular ship from analytical formulas with empirically determined numerical coefficients. However, this procedure did not produce any satisfactory results, despite repeated research efforts. It was not until about 1870, when the English scientist William Froude tested his famous method of using the shape resistance of a particular ship based on mechanical similitude. This approach was applied after having conducted series of model tests. The frictional resistance was determined from special, empirically obtained formulas in terms of ship speed, so that it became possible to determine the total resistance of a ship of any shape. Particular model tests for each driving speed increased the accuracy, resulting in satisfactory success.” Froude was here indeed a master in experimentation, realizing that the two portions of naval resistance followed different laws of similitude, as for example, discussed by [7]. Accordingly, *skin friction* mainly depends on viscous forces, expressed today by a Reynolds number dependence, whereas *form resistance* calls for gravity forces, currently described by the concept of the Froude number.

Weber argues as follows: “The foundations of Froude’s theory of mechanical similitude were created ingeniously by Isaac Newton in 1687 in his *Philosophiae naturalis principia mathematica*. Newton proposed complete mechanical similitude for two dynamically comparable processes (1) and (2) if the two corresponding forces K_1 , K_2 satisfy the equation $K_1 : K_2 = \rho_1 F_1 V_1^2 \div \rho_2 F_2 V_2^2$. In Newton’s similarity considerations, however, there is no suggestion that the theory of mechanical similitude may be an approach—and indeed a very promising one—to solve a difficult mechanical problem. More closely, the dynamic processes of the prototype ship (H) are solved by using a geometrically similar model (M). In addition, Newton provides no information on the fact that—with regard to the action of the gravity forces in the formation of waves at

the water surface—a gravity model law for corresponding speeds as $V : v = (gL)^{1/2} : (gl)^{1/2}$. For identical gravity, the simplified equation reads $V : v = L^{1/2} : l^{1/2}$; it must necessarily be satisfied, provided that the two processes are dynamically similar.”

“In order to find the resistance of a ship, Froude used the model law based on the effect of gravity for corresponding models in his classical experiments with small models similar to the large design. The speeds followed the length scale as $V : v = L^{1/2} : l^{1/2}$. However, to my [Sic.: Weber’s] knowledge—no doubt unaware of the facts—he does not state that the French naval engineer Reech ... suggested that the ship resistance should be based on tests with geometrically similarly reduced models. The demand that this special law of similitude must be satisfied in model experiments is published for the first time by Reech (1844b). However, the report was completed as early as 1843, since it bears the signature on page 119, *Lorient, le 19 mai 1843, l’ingénieur de la marine E. Reech*. On pages 27 to 29, we find the Gravity Model Law in the form $\xi = \lambda^{1/2}$ (with ξ as the ratio of respective speeds $V : v$ of prototype and model and λ as the ratio of respective lengths $L^{1/2} : l^{1/2}$.)” Reech further states that this law was established during his lectures 12 years ago, that is, in 1832, or 40 years ahead of Froude. Reech therefore realized shortly after his arrival at Lorient in 1831 the relevance of model experimentation. He in addition proposed the governing model law provided that gravity effects are the leading forces to be considered. He further gives an exact proof of his law in the book *Cours de Mécanique* (Reech 1852a).

Weber [2] draws the conclusion: “On the basis of these findings, there can be no doubt, according to our current knowledge of the facts: Reech is the creator of the fundamental law for model tests to determine ship resistance. Therefore, the model law of speeds corresponding to the gravity effect should not be named after Froude, whose proposals for the conduct of ship model tests began only around 1867, but must be justifiably called Reech’s Model Law. Similarly, $V/(gL)^{1/2}$ is the Reech number, to be abbreviated as R in contrast to the Reynolds number $Re = VL/v$. The other gravity related ratios such as time, force, or output [Sic.: power] follow the relations $T : t = (L/l)^{1/2}$, $K : k = (L/l)^3$ and $E : e = (L/l)^{7/2}$.” Weber continues: “The outstanding merits of Froude relating to the practical solution of the naval resistance problem, is in no way diminished by Reech’s Law: they lie in overcoming the great difficulties due to the frictional effects of the moving ship in the practical implementation of model experiments. It is not known whether Reech has actually conducted model experiments on the basis of his similarity law at Lorient, although this is the port in which Thévenard’s trials were conducted in 1769 and 1770 to explore the naval resistance.”

Weber [2] summarizes his findings as follows: “This is the great merit of Froude: he recognized the difficulties associated with friction and overcame them by the approximation bearing his name, the foundations of which are based on the following ideas:

1. According to Froude, the ship’s resistance can be broken down into two main parts: (1) the *form resistance*, which depends on the shape of the ship and is substantially subject to gravity as a result of the wave formation, resulting of all normal wetted surface effects; and (2) the *frictional resistance* corresponding to all resulting forces acting tangentially to the wetted surface.
2. According to Froude, the form resistance is treated independently of the friction processes and is practically found from model experiments, provided that one can use appropriate measurements of the speed of the geometrically similar model to the prototype. It ensures that the system of surface waves of the model is maintained dynamically similar to that of the prototype. This last requirement led Froude to apply the model rule due to the acceleration effect of gravity $V : v = (L/l)^{1/2}$, whose fulfilment had been demanded by Reech decades before Froude.
3. According to Froude’s knowledge, the frictional processes of the prototype cannot be dynamically similar in the model at the same time as the gravity processes in the model

system; therefore, they are separately deduced for the prototype and the model from special empirical formulas. He obtained the numerical data from extensive trials of plate-shaped bodies of different roughness, which he had made specifically for this purpose. From these formulas, he determined the frictional resistance for each driving speed.

4. After deduction of the model-friction resistance from the overall model resistance determined by the experiment, according to Froude, the remainder of the form resistance of the model according to the Newtonian general law of similarity, both the form and the friction resistances, i.e. the total resistance, is known.”

Weber here ably highlights the eminent role of Froude in the practical determination of ship resistance. He has thus finally opened the way to develop the knowledge in this difficult problem, and, more importantly, set the way to start hydraulic modeling by a scientific approach. This was launched from the start of the 20th century mainly in Europe, with dozens of Hydraulic Laboratories active until the outbreak of WWI. It should be stated here, however, that neither Reech nor Froude should be names in connection with the gravity model law, because both were essentially naval engineers, not having at all worked in the field of free surface open channel flows. The true namesake should be dSV, as proposed by [7].

Reech as marine engineer

Weber [2] writes: “Reech was an outstanding designer of the French navy, who, with excellent knowledge of physical phenomena and theoretical bases around the development of the steam engine, earned great merits during its introduction to the navy. As a young naval engineer, Reech was initially entrusted with various designs in Lorient: in 1833, he designed a hydraulic press, a chain-testing machine and a braiding machine for flag ropes [6]. The French engineers also see him as the inventor of the circular slider diagram, which was named after Gustav Zeuner [Sic.: 1828–1907, German engineer] in Germany only much later; Reech conceived this diagram as early as 1833, long before Zeuner’s time, and together with Fauveau, devised the exact egg-shaped slider diagram. Several treatises written by Reech show that he was one of the first scientists dealing with the steam engine and the theory of heat.”

The most important writings of Reech are according to Weber:

Reech (1844a): “*Memoir on steam engines and their application to navigation*”. This memorandum is preceded by the following interesting preliminary remark: “This work was written in the winter of 1837 to 1838 and presented before May 1, 1838, to the competition opened by a Royal Ordinance of November 13, 1834. Yet, it was withdrawn in June 1839, before the judgement of the competition, and deposited in the Ministry of the Navy, where it has remained unpublished to this day. According to [6], the memoir was first published in the *Annales maritimes* of July 1837 (Reech 1837) and was not published until 1844 as an independent book.”

Reech (1844b): “*Report in support of the Brandon Machinery Project*.” Chapter XVII has on page 119 the inscription: “Lorient, May 19, 1843... Navy Engineer F. Reech”. This highly interesting report on the paddle steamer “Brandon” of the French Navy was thus completed in 1843. On pages 27 to 29, the proposal is for the first time made to determine the resistance of a ship by experiments with geometrically similar models, and at the same time to demand that the appropriate speeds be met by the model law must be complied with the ratio of the square of the length scale. Furthermore, we find here the indication that Reech has taught these findings to his students at the *Ecole d’application du Génie Maritime* for 12 years, that is, since the beginning of his stay in Lorient in 1831.”

Reech (1852a): “*Mechanics course according to the generally flexible and elastic nature of bodies*.” “The description of the mechanics given here corresponds to the state of this science at that time. However, the final chapter V brings something completely new, which distinguishes

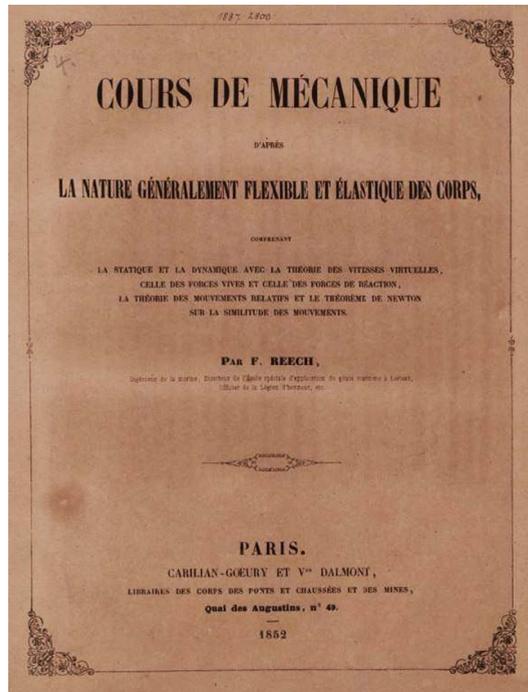


Figure 2. Cover of Reech’s Course on mechanics (1852), one of his foremost writings.

the mechanics of Reech with the other textbooks, namely “Newton’s theory of the similarity of movements, regarded as a universal rule” on all the problems of practical mechanics. From the comparison of the mass acceleration forces, which is based on Newton’s general similarity law, and the gravity forces as $\kappa = K : k = \rho F V^2 / \rho f v^2 = (\rho g) L^3 : (\rho g) l^3$ follows immediately Reech’s Law of Similitude of relative speeds as $V : v = (L/l)^{1/2}$.”

Reech (1854): “*General theory of the dynamic effects of heat.*” (Excerpt in the *Journal of Pure and Applied Mathematics*, Reech (1853a)). “Here, Reech takes a position on the fundamental ideas of [8] and [9] and on the ideas of various researchers such as Joule, Thompson, Rankine, Mayer, and Clausius. According to his explanations, they have attempted to improve the relations established by Carnot and Clapeyron. On page I of the foreword, he describes the objective of his theoretical studies on heat by stating: “I believe that in most of the studies on this subject, pure hypothesis has been given too much importance by logical structure of the considerations of Carnot. In my (Reech’s) view, this was not destroyed by Regnault’s throw-in, but only required to be completed from a new point of view. At least in what I have to say in this memorandum, it is completely clear and so clear that one can prove a general formula which will at the same time satisfy the experiences of Joule and those that Regnault recently published in the *Compte rendu de l’Académie des Sciences* 1853, volume XXXVI p. 680, first example).”

Reech (1855a): “*Air machine of a new system deduced from a reasoned comparison of the systems of Mr. Ericsson and Mr. Lemoine.* Here, Reech provides a heat-mechanical treatment of the heat-air machine and designs theoretical diagrams for the two types of hot air machines, those of Ericsson and Lemoine. He also proposes advantageous measures to improve the practical effect of these machines.”

Reech (1858a): “Demonstration of general properties of closed surfaces. In this paper, Reech proves for the purposes of ship stability a general “theorem on the number of normals”, about which [6] provides further information.”

Reech (1864a): “Constructions of metacentric develops to different loading states of a ship. *Memorial of the Maritime Genie* 1864, Issue 3, p. 168. In this fundamental work, Reech introduced the inclined ship in France to consider the wedges that were in and out of water. It is based on pure accounting methods of different load states of a ship. The computational interpolation procedure was set up in 1870 by the French naval engineer P. Risbec using calculation tables for practice. The Reech-Risbec method is described by [10] in Volume I, p. 161; compare there also with p. 116.”

Reech and Jordan (1864): “*Memoir on the rigorous and complete theory of the stability of floating bodies*. For this work, Reech was awarded from the French Academy of Sciences in 1864, at the same time as C. Jordan, for solving the task: “Give a rigorous and complete theory of the stability of floating bodies”. Further details are given by [6]; also compare with Pollard and Dudebout, volume I, p. XXII.”

“*Theory of rolling and pitching*, according to the records of his successor on the chair, Marine Engineer de Fréminville [11], made by Reech’s students. Schwartz [6] provides an appreciation of this theory. Pollard and Dudebout [10] express themselves in volume 1 on p. XXXIV as follows: In this epoch, we also put the extraordinarily complete and original theory, which Reech presented. It was at that time that this outstanding engineer brought them into their final version after several improvements.”

Weber at the end of this chapter adds: “In the volumes of the Catalogue of Scientific Papers, compiled and published by the Royal Society of London, a total of 19 scientific works by Reech with titles and years of publication are listed. I have not found any biography of Reech; to my knowledge that of Schwartz has not been published.” The last statement is wrong, because the first author has a copy of this publication.

Biography of Reech

Weber was the first who published Reech’s biography, mainly based on the account of [6]. He states: “In the following, I give the German translation of what Schwartz, Chief engineer of the French Navy, located at Lorient, published. The Report in French was given to me in 1941 through the exchange of the Master Board of Director Mendelssohn from the archives of the *Ecole du Genie Maritime* at Lorient. It will be referred in the following as ‘Report Schwartz’ [6].”

“When looking through the French fleet list of 1914, one realizes that the French Navy wanted to honor a certain number of scholars for several years by giving their names to submarines. In this list, we find the names of well-known, famous men, such as Newton, Laplace, Monge, Bernoulli, Joule etc. In addition to these, the Navy justifiably gave a place to those of its officers who did an excellent job in the service. They either promoted submarine navigation such as Gustave Zédé, Admiral Bourgois, Charles Brun, or contributed to the general field of the shipbuilding science and technology such as Sané, Dupuy de Lôme and engineer Joëssel.”

“After leaving the procedure of attaching the names of scholars to the sterns of our small boats, one is astonished not to find in that list the name Reech, who at the *Ecole d’application du Génie Maritime* was an excellent pioneer and professor with an originality of thoughts. The five engineers mentioned above were all his disciples, except for Sané, who was Inspector general when the Corps was organized in the Year VIII [Sic.: 1798 in our current time counting]. They entered the *Ecole du Génie Maritime* in the following years, when Reech already taught there: Dupuy de Lôme in 1837, Charles Brun in 1840, Gustave Zédé in 1845, and Joëssel [Sic.: Joseph-Emile Joëssel (1831–1898), French naval engineer] in 1856.”

Below a short inset dealing with the role of Dupuy de Lôme, Weber continues: “The *Ecole du Genie Maritime* had been transferred to the Arsenal of Lorient by royal order of March 1830 and remained there until the imperial decision of April 11, 1854, by which the *Ecole* moved to Paris.

The instructor, who was able to raise the teaching quality at this school to a high scientific level and who saw how, in addition to the four above-mentioned engineers, numerous generations of students following each other before his catheter, was Friedrich [Sic.: Frédéric in German] Reech. His name deserved to be recalled and immortalized in the French Navy: He served her with rich devotion and in great simplicity during a career of forty-seven years. Reech was a representative of the hard-working population of Alsace, who gave France so many men who were distinguished in science, military and other fields of human activity.”

“Reech was born on September 9, 1805, at Lampertsloch (Department Bas-Rhin), a modest village of 400 souls, 18 km distant from Weissenburg [Sic.: Wissembourg in French]. The somewhat strange composition of the location with the final syllable “Loch [Sic.: English hole]”, clearly shows that the village had not been considered an important place for a long time. In 1823, at the age of 18, Reech was admitted in Strasburg to the *Ecole polytechnique*. At that time, the qualified pupils were arranged in alphabetical order for admission. At leaving it, he was the sixth among the ninety-five students. On November 12, 1825, he became a student of the Construction Officers Corps (*Génie Maritime*) and received access to the *Ecole spéciale* then located at Brest. The studies did not last very long there. After having passed the exams, he waited for a vacancy as a candidate for a 2nd grade engineer. He remained in port service in Brest until he was accepted to that degree three years later, on December 28, 1829. As a result of his appointment, he took a short stay in Cherbourg before taking up his service in Lorient on May 1, 1831. He remained in this port for twenty-three years without interruption, in order to exert decisive influence thanks to his high intelligence and his outstanding skills as an engineer and mathematician. He was appointed under the command of the Ministerial Conductor (*Directeur des Constructions Navales*) Chaumont and joined in Lorient the *Ecole d'application du Génie Maritime*, which had been moved there by order of March 28, 1830. One month after his arrival, on 1 June 1831, he was assigned to lecture at the *Ecole du Génie Maritime*, the fortunes of which he was to follow from that point on.”

After a description of various inventions of Reech during his first years at Lorient, [2] continues as: “Reech arrived in Lorient at a time of remodeling the workshops and their equipment, which offered the young engineer a wide field of study. The metal workshops had just been rebuilt from 1827 to 1829. The construction of the Clermont-Tonnerre Basin (1820 to 1833) was completed. The assistant-engineer Fauveau (student of the year 1813), who was ten years older than Reech, played a decisive role in this renewal of the arsenal. There was a nice cooperation in the future between these two men; the first owned shipyard practice, the second supplemented this practical side with all the vibrancy of his bold thoughts. The cooperation continued even when Fauveau moved to Brest in 1833. Now that Reech soon gave up his preoccupation with workshop questions, he applied all his activities to science and teaching at the *Ecole d'application*. In this area he had the happiest successes.”

Weber [2] details in the next paragraphs Reech’s works in steam engines and concludes: “However, Reech was not satisfied with the development of steam engines and propulsion equipment for ships only; he sought with equal zeal to define the hull of steamships with the best shape in order to achieve the least resistance. A chapter of the memoir on steam engines and their application in shipping, written in 1837 and published in book form in 1844, deals “with the shape of the steamships and their absolute dimensions”. After a mathematical study of hull shapes, Reech concludes that the current ships of the Royal Navy with a nominal power of 160 horsepower can only carry fuel for a maximum of 10 days; this is one of its greatest shortcomings for battle ships. They are also too weak to carry a battleship or frigate through the slightest bad weather, which is a second major error. After all, the machines and boilers are not protected against projectiles, and a single well-targeted cannon shot can jeopardize the floating ability of the entire ship. There is only one tool against all this, a simple one, namely to build much larger ships and equip them with much stronger steam engines.”

After having described the successes of one of Reech's students in the future design of ships, [2] comes back to him with the words: "Reech had been appointed chief engineer of second class on May 27, 1841, and chief engineer of the first class on January 25, 1846. He continued his lectures at the *Ecole d'application* and devoted himself to them with full dedication. He was mainly a scientist, open to mathematical studies and left the field of ingenious applications to others. He was particularly pleased to deal with the boldest theoretical questions of shipbuilding and knew no more difficulties when he arrived at the proverbial saying among his students: "Now we have nothing left over for us than to integrate." He gave his lectures with a natural handiness and had retained the Alsatian tone."

The next question treated by [2] deals with the theory of ships. He states: "In his lectures, of which unfortunately only a few parts are published, Reech informed his students of his valuable findings, which contributed to the great advances in the ship theory. It is sufficient to look through the scientific work of [10] to realize the outstanding place of Reech's work in shipbuilding."

Omitting here the description of the construction of the F-Curve, Weber states: "In 1864, Reech shared with Jordan [Sic.: Camille Jordan 1838–1922, French mathematician] the Grand Prize of Mathematics, which was awarded to them by the Academy of Sciences, Paris, for solving the task: "Give a strict and complete theory about the stability of floating bodies". Reech reopened the study of the stability of a ship under the influence of a pair of forces, which the engineer Baron Charles Dupin (1784–1873) used in his 1822 applications of geometry and mechanics for the purposes of the Navy. Reech was able to deplete more time on these questions than his predecessor, who devoted himself to mathematical sciences further distant, including economic and practical studies among others, not without success. Finally, Dupin had to give up scientific work as a result of the charges with high public offices, in contrast to Reech, who dedicated his whole life to them."

Omitting here further details on questions of ship stability, the role of his school during the development of ship design, and the description of rolling and pitching of ships, [2] concludes that Reech had been a pioneer on various fields of engineering sciences, the results of which were mainly presented in his lectures. He also gave further information on his life: "On April 12, 1854, Reech was promoted to the Director of the Naval Constructions at age 49. This shows how much his merits for the *Ecole d'application* were valued. Incidentally, the School had been transferred to Paris by decree of April 11, 1854. The Minister, in his report to the Emperor in support of that decree, expressly acknowledged the great devotion of the Director to his office."

"Reech, who moved to Paris in May 1854, retained the management of the *Ecole* there until he retired on September 9, 1872. He had been for 39 years professor and had taught some 200 students of the *Génie Maritime*, 54 of whom had risen to the degree of *officier général*. Considering the wide range of his works, Reech is the scientific old master of the Navy, as is Borda [Sic.: Jean-Charles de Borda 1733–1799, French mathematician and hydraulician] for the ship officers and Beautemps-Beaupré [Sic.: Charles-François Beautemps-Beaupré 1766–1854, French explorer and engineer] for the hydrographs. After his departure and the events of 1870 [Sic.: the French-Prussian War], Reech retired to Lorient, which had become his second hometown. Since he had published little (after the statement of Schwartz), he was not admitted to the Academy of Sciences... He maintained his exemplary qualities through kindness and simplicity until old age. He was married to Marie Buret in 1837; they remained childless. He had with her a faithful niece, who cared for him until the end of his life after the wife's death. He died on May 6, 1884, at the age of 79. According to his wishes, his earthly remains rest at Lampertsloch [Sic.: According to [2], who corrected a mistake of Schwartz]."

As to the scientific works of Reech and his family, [2] contributes the following: "The discovery of the hitherto completely unknown creator of the main law for ship modeling in Germany was stimulated by the following events. In my lectures on similarity mechanics and model science [1],

I clearly dispersed the achievements of Isaac Newton and William Froude. Because the American [12], citing my name, expressed a somewhat different view, I have since 1930 again raised the problem of ship resistance from the point of view of similarity in careful consideration of a historical point of view. The result of this new research was summarized in a paper intended for the workshop of the Shipbuilding Technical Society. During the several years of preparations to investigate the extensive problem, an international meeting of the heads of the towing experimental institutes took place in Berlin from May 26 to 28, 1937. There, I had the opportunity to talk to Professor Barrillon [Sic.: Emile Barrillon 1879–1967, French naval scientist], engineer General of *Génie Maritime* and Director of the ship test basin in Paris, about the historical development of ship model experiments. He named the Alsatian Friedrich Reech a pioneer in this field and a forerunner of Froude. As former director of the *Ecole d'application* in Lorient—and later in Paris—he was highly regarded in naval circles of France for his great service to ship building, but was unknown in Germany.”

“Professor Barrillon has thus given the first suggestion for my research on the scientific work and life of Reech. Shortly after that conversation, I received a photograph of Reech, which depicts him in advanced age (Figure 3); it probably originates from the premises of the former site of Reech in Lorient or Paris. Next was attached a photographic print, reproduced here in Figure 2, page 273 of the textbook *Cours de Mécanique*, published in 1852. Its final chapter deals with the theory of mechanical similarity and, for the first time in literature, the main law of ship model experimentation, i.e. the Reech gravity model law of corresponding speeds.” Weber ends this chapter with many additional indications on the writings of Reech, and particularly on his family in Alsace. Weber mentions also the graveyard at Lampertsloch, where he found the graves of Reech and his father Georg Heinrich Reech (1781–1851) (Figure 4). In addition, he cites the text written on a heavy marble memorial stone with the inscription:

Frederic Reech
 Directeur des constructions navales
 Lampertsloch 1805
 Lorient 1884
 LUC. VI, 31 JEAN IX, 4.

“Underneath, a ship’s anchor is carved into the stone, apparently the badge of the French naval construction officials, which is framed by a finely crafted laurel wreath; on the laurel hangs the cross of the Legion of Honor, all coming out of the heavy marble slab. On behalf of the Shipbuilding Society, I had decorated the tomb of the great master of shipbuilding and ship building machinery with flowers during my visit.”

“So on September 23, 1941, I was at the destination of my trip and found that Friedrich Reech was born in Lampertsloch in the district of Weissenburg in Alsace and was buried there. He didn’t have descendants. In his youth, he was called in his hometown “the Reech Fritz”, and today he is called by his relatives “the uncle from Lorient”. Several gravestones of the relatives in the cemetery in Lampertsloch also bear the name Reech, who still live in Lampertsloch, who descend from the brothers of his father or grandfather; they all farm as their ancestors once did. According to older documents, the surname Reech was already used in Lampertsloch before the Thirty Years’ War, and the family name *Bauer* of Reech’s mother is one of the oldest of Lampertsloch and can be traced back until the 16th century. The inhabitants of Lampertsloch are of pure German origin and belong—like the inhabitants of all Alsace since the migration of peoples—to the tribe of the Alemans, that large Germanic tribe, whose name was already early used by the Romanesque neighbors as a name of all Germans that has become *Allemands*. In the families of Lampertsloch, only the German dialect, the elusive Alsatian Platt is spoken. The name Reech does not sound stretched, but briefly “Rech” or “Resch”.”

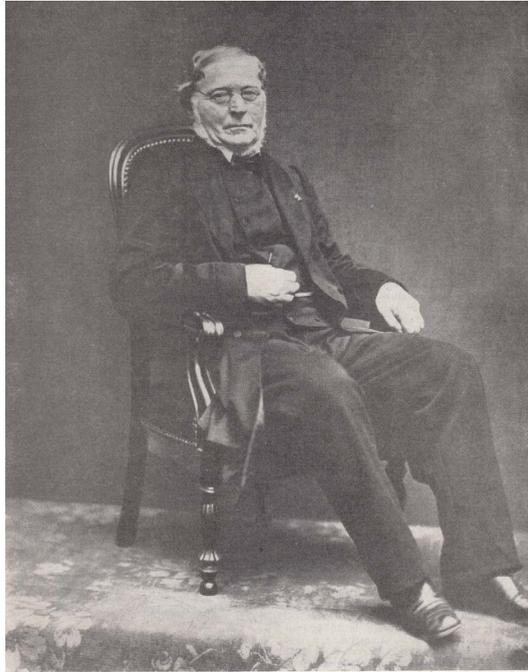


Figure 3. Aged Frederic Reech [2].

In conclusion, it can be stated that Weber, who initiated the systematic treatment of the theory of mechanical similitude, has had the interest and the capacity of revising his original proposal for the number describing the gravity effect. By understanding the professional career and the life of Frédéric Reech, he therefore made the step from the Froude to the Reech number. As evidenced by Hager and Castro-Orgaz [7], this is not an adequate selection, given that both have not at all worked in fields where this number is relevant. It was therefore proposed to leave the expression Froude number, or even Reech–Froude number, which should be effectively named Saint-Venant number. The latter is the true personality who significantly worked in this field of hydraulics, culminating in the presentation of the shallow water equations [3].

Correspondence between dSV and JB on Reech

Among the many topics discussed in this Correspondence, several letters deal with an approach of Reech on steady waves in deep water. These letters were written between 1869 and 1870. dSV, from 1868 president of the Mechanics Section of the Academy, had to deal with submissions for the main journals of the Academy. One of these submissions originated from Bertin [13].

In a Letter dated May 29, 1869, JB asks for the first time dSV whether he is able to pass him the address of Mr. Reech. At this time, JB was professor at the High School of Gap, in the French Savoy. The passage reads: “No one receives the journal *Comptes rendus* at Gap. I therefore eagerly take advantage of the information you give me, to ask Mr. Reech for a copy, if any, of his Note on periodic liquid waves. At the same time, I send him a copy of my memoir on the influence of friction in the regular movements of fluids.”

In the Letter, also dated May 29, 1869, dSV presents an improved version of his theory on periodic liquid waves. It reads: “Having recently been involved in solving the problem of a regular system of periodic liquid waves in a horizontal channel of rectangular section and of indefinite length, I have come to results that seem to me to have an immediate relation with these of

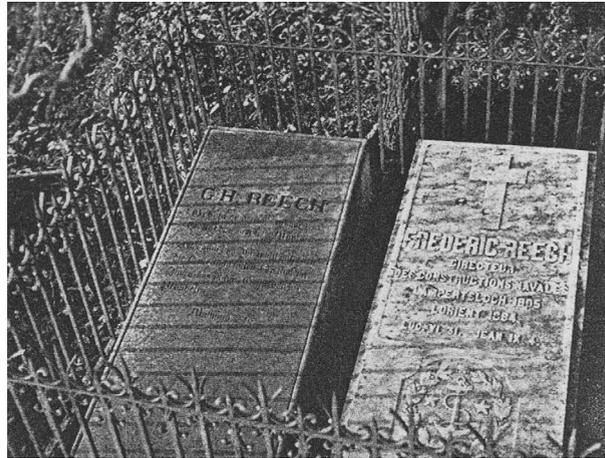


Figure 4. Tombstones of father Reech and Frédéric [2].

Mr. de Caligny's note in the *Compte rendu* of April 26, 1869, p. 980. According to my analysis, the condition of continuity of the liquid mass requires that the curves described by the liquid particles be circular and not elliptic. It follows to be so in the entire extent of the mass. At the bottom of the canal, the radii of the circles must be zero."

"Leaving aside my general equations, and reflecting directly on what is possible to conceive in a horizontal plane at the bottom of the canal, I have not been able to understand that, in this plane, there may be alternative movements. This includes a system of ellipses whose vertical axis reduced to 0, while the horizontal axis would retain a finite length, as Mr. de Caligny would like to admit. I agree with Mr. Boussinesq's more general theory as presented in a Note in the *Compte rendu* of April 19, 1869, p. 905."

"Assuming an indefinite depth of water, I managed to represent all the circumstances of the state of motion by the equations [Sic.: To keep the original numbering of equations, subscripts were added to the equations originating from the various letters]

$$\xi = x - he^{\pi y/l} \sin[(\pi x/l) - t(\pi g/l)^{1/2}], \quad (1_1)$$

$$\eta = y - he^{\pi y/l} \cos[(\pi x/l) - t(\pi g/l)^{1/2}], \quad (2_1)$$

$$(p - \Pi)/\omega = -y - (\pi h^2/2l)[I - he^{2\pi y/l}]. \quad (3_1)$$

Here, ξ, η are the variable coordinates of a liquid particle along the circle circumference described by this particle around point (x, y) as center. The radius of the circumference is $r = he^{2\pi y/l}$. The x coordinate is counted horizontally depending on the channel length. The ordinate y is counted vertically from bottom to top. [Sic.: t is time and g is the gravity acceleration]. By letting $y = 0$, we get, by means of (1₁), all the points ξ, η at the free surface. The line of these points is an epicycloid. The value $2l$ is the common length of the surface throughout the system; whereas $2h$ is its height at the free surface. It is necessary that one has $(h/l) \leq 1/\pi$, so that the line of the point ξ, η at the free surface is at most a cycloid, that is, a line with turning points, a limit beyond which the condition of continuity of the liquid mass would no longer be satisfied."

"At any depth z , i.e. at height $y = -z$, the wave height is $2r$, i.e., $2h/(e^{\pi z/L})$. With [Sic.: $L = 2l$] A as wave propagation speed and T half-time of the course in a circular circumference, one has

$$A = (gl/\pi)^{1/2} \quad \text{and} \quad T = (\pi l/g)^{1/2}. \quad (4_1)$$

Equation (3₁) provides the pressure p , which, at a certain time t , has no effect on (x, y) , but on the mobile point (ξ, η) , describing a circular circumference about point (x, y) as the center. The value

of Π is the atmospheric pressure at the free surface, and ω the liquid density. After having solved this simple case, I tried to deal with the general case of a finite water depth, but the equations became so complicated, that I had to give up, developing them explicitly and discussing them.”

Note that at about the same time, JB announced the publication of his large work on periodic waves [14], which was published three years later [15]. A comment of dSV is: “The article of Mr. Marquis de Caligny (now Correspondent of the *Institut*, living in Versailles, rue de l’Orangerie 18) was presented in the *Comptes rendus*, April 26, 1869, Vol. LXVIII, p. 980). JB finds that the movement in the higher portions of the top wave molecules have infinitely flat orbits; that in the upper wave portion the movement in the sense toward the channel bottom from the observation point and between extreme to the lower part of this article. This is what I communicated with various details to the *Philomathic Society* about 27 years ago. As to the wave shape, I did not find it circular near the surface; the axis y is vertical, but the case is not quite the same, especially since my expressions are mainly for fairly large waves.”

In the next letter, dated May 30, 1869, dSV states the following to JB: “Dear Sir, as it is unlikely that Mr. Reech will send you his article, I had let it copied by my son, and here it is, with the note of Mr. de Caligny. His experiments at the rest did not seem delicate enough that they can be taken high enough as a basis.”

“Mr. Reech is a strong man, but a little dreamy, and someone who does not finish anything. His reasons may seem to be weak. However, you will no doubt find that they deserve to be discussed in the new draft of your paper that I sent back to you that you will probably make before the end of the year (since Mr. Liouville until then will have no place in his journal).” [Sic.: Joseph Liouville was the editor of the *Journal de Mathématiques pures et appliquées*, in which JB originally intended to publish his research on this topic.]

dSV continues: “Therefore, I hope that I will be a little more free and will be put on the committee as reviewer, especially if you attach to your pages, for my use, the sketches of developed calculations, which exempt me from doing them, plus explanations that make my work easy. However, I wish to say with some heresy, that I have not read and studied Cauchy, Poisson and Corancez. On waves in general, it would be necessary, according to my estimate, to be able to take care in the report, that you explain the difference between the problem it deals with and that of you, if they are essentially different. Yours dSV.”

The response of JB to dSV, written on June 16, 1869, reads: “Sir and dear Master, I beg you to forgive me for the somewhat long delay I have put in writing to you. I exchanged several letters with Mr. Reech, I even sent him my memoir, and I was much interested in receiving his input at this point, which would hopefully end our little discussion, before telling you the details. As soon as it comes, I’ll do it.”

“When you told me that he had written an article for the *Comptes rendus*, I promptly asked him for a copy as you had advised me, and at the same time, I sent him my memoir on fluid friction. He answered me on the spot (May 27), attaching to his letter a handwritten copy of his article, and told me that he had not intended to reply to my note of April 19 but to that of Mr. de Caligny. Moreover, he added, that the research I had done is quite similar to yours.”

“Indeed, I recognized, from reading his article that he had dealt only with periodic plane waves, spreading in a channel of infinite depth. I do not fully understand this article, even though in the particular case he dealt with, his solution is similar to mine. I did not see how the condition of continuity, to the orbits the form suited, even at the bottom of the canal and whatever he says in the article that he could not conceive it otherwise. Rather, I would like to understand the opposite, as I explained in the last letter, which I sent to him. Here are the terms of this letter. It seems to me that he designed a flow having at the bottom a finite horizontal speed without an appreciable vertical component. Indeed, when a wave hits the liquid layer in which it will propagate, all the molecules of this layer must participate in its action, in the horizontal sense, in finite movements.

Being thus tightened, they extend by keeping their volume. Those that are at a very small height and above the bottom only have to climb by an amount of the order of ε , while those on the surface have to make room for all the others and will rise by the same order as their horizontal movements. The analogue occurs but with depressions instead of elevations, when the wave has passed and the horizontal velocity is declining. It appears to me that all other researchers who dealt with liquid waves, especially in shallow water, had admitted at the bottom horizontal movements much greater than in the vertical direction. Not knowing what has led Mr. Reech to such conclusions, I replied by giving him my solution for flat waves in a channel of finite depth, with enough explanations to show him that it verifies all the probable conditions. I also offered to send him my manuscript. He answered on the spot (May 31), thanking me for sending my solution, and on June 1, after a study, he sent me another letter in which he acknowledged to have read my memoir; at the same time, he told me that we disagreed on the condition of continuity.

After a long discussion on the issues of fluid incompressibility, JB concluded: "This proves that, if one does not assume very small motions, or that one wants to take into account the terms of second degree in u, v , it is possible to have for u, v expressions of the form $A \cos(f\eta t) + B \sin(f\eta t)$. However, this is what occurs for all vibrational motions. Expressions of this form are only possible during infinitely small motions, yielding linear equations, and they are only approached during rather small final displacements. That is pretty much what I said to Mr. Reech in my reply of June 8, to which he has not yet responded. On June 8, before he had received my letter, he returned my memoir, without, I believe, doing anything other than just looking at it. He told me, at the same time announcing this dismissal, that he finds my exact analysis only correct for infinitely small displacements. At the same time, he told me that when having studied the case of circular waves, he found that the equation of continuity cannot be satisfied by displacements of the form $A \cos(f\eta t) + B \sin(f\eta t)$. Nor can it be satisfied for plane waves, except for infinite depth. I believe that such difficulties, together with my reasons, will lead him to change his mind." JB thus was actively involved into this problem, and at the time certainly one of the best who could answer the questions of dSV.

The next Letter, dealing with Mr. Reech, was written on August 6, 1869, by JB to dSV. It reads: "I'm sending you a new copy of my essay 'On the theory of periodic liquid waves', with a few notes at the bottom of the pages, about Mr. Reech's theory, and more on the developments of all calculations on interspersed flying sheets. I recall to having seen in the library the first volume of the *Memoirs of the Foreign Savants*, but I did not find it. Therefore, I left in white a large part of page 2, hoping, during my visit to Montpellier, on August 11 and 12, to consult the Memoirs of Poisson and Cauchy, and to add appropriate material, to fill this page, that will contain what I would have to say and that I will beg you to tie up with a pin. At the same time, I'm sending you five copies of my theory of Savart's experiments."

"Since last time I wrote to you, I exchanged a few letters with Mr. Reech. He recognizes my exact theory in case the two movements are infinitely small, indeed the unique case that I studied there. However, in the last letters, he insisted to believe that his own approach alone applies to movements of finite amplitude, despite the denial that gives him, precisely for this case, the experiments of Mr. de Caligny. It seems to me that he has given up the idea, that one cannot conceive at first glance horizontal movements on the bottom of the channel, an idea that had struck me the most in his article of the *Compte-rendu* [Sic.: Reech 1869b]. His penultimate letter showed me at last that we had not agreed perfectly until then, because he calls x, y, z the coordinates of the center of the trajectory of each molecule, whatever that is, while I so designated the coordinates of the balance position. Therefore, my demonstration of the absolute impossibility of satisfying the equations, taking into account the terms of second order of smallness, with movements of the form $u = M_1 \cos(k_1 t) + M_2 \sin(k_2 t)$, v, w similar, started wrongly. This occurs albeit accurate in itself, since, from Mr. Reech's point of view, u, v, w do not

designate the displacements relative to the balance position, as I had thought, but you had to add to this x, y, z coordinates of the center, whatever this may be, the trajectory, to have the true coordinates. However, except for the case of flat waves in a channel of infinite depth, the only case he dealt with, the same impossibility arises from other considerations, at least for all the questions that my memoir deals with, and he did not respond to the letter, in which I exposed all this. In addition, I will detail at length, to the note of § I, to the first part of § III, and to that of § IV, with the most general equations, to clarify, so that you can, I hope, inform yourself.” As for the solution of many other problems, dSV actively engaged JB in the process of problem definition and solution, because dSV knew that JB was highly interested into these scientific challenges. JB, who would have hardly worked on the problem of periodic waves, accepted his role in finding the solution of a problem posed by his ‘Master’. The collaboration thus was outstanding, because both profited enormously from each other, and in addition became good friends.

On October 6, 1869, JB received a letter from Mr. Reech. His comments to dSV’s letter are as follows: “I received a letter this morning from Mr. Reech, although I have not sent him anything for several months. He tells me that *all efforts* to obtain from his point of view a solution to the problem of periodic liquid waves in the case of a finite depth channel, have failed. At the same time, he sent me a copy of his recent book, entitled ‘Theory of motor machines and the mechanical effects of heat’ [Sic.: Reech (1869a)], telling me that perhaps I would be interested. The correspondence I had with him will therefore not have misplaced him towards me.” Despite no letters exchanged between Reech and JB are contained in the Correspondence, JB traces a rather evident image of Reech. The latter, also much elder than JB, did not accept his criticisms. The highly versatile mathematician JB has certainly impressed Reech, who was not really able to cope with him. At least, as mentioned by JB, the entire contact remained friendly. Note also the strong comment of dSV on Reech as expressed in his letter to JB on May 30, 1869. The topic remained untouched for nearly a year, when dSV contacted JB again in this matter.

On September 1, 1870, dSV writes to JB the following: “I am studying Mr. Bertin’s memoir, on which I hope I can report. I find good things in it, but I do not think there will be a high approval because the drafting is confusing and defective. Besides, he has no priority, as he admits himself, since he is merely giving a demonstration, almost elementary but very misty, of the two equations, which Mr. Reech posed without demonstrating them. I understand that you did not want to be too noisy, for some simplifications of calculation that you indicated to Mr. Bertin when you had the great courtesy to study that work. But you should be quoted as having found this before Mr. Bertin and also presented to the Academy before Mr. Reech the equations of it that return everything that has found the mistake of Mr. Bertin, because you find:

(1) Let H be the water depth, z the distance of an arbitrary point to the surface, $2r$ the horizontal amplitude of the oscillation of this point about the balance position, and $2l = 2\omega_o T$ along the oscillation $2r = (C/2)(e^{\pi(H-z)/l} + e^{-\pi(H-z)/l})$. This is reduced for extremely large values of H to $2r = (C/2)(e^{\pi H/l} e^{-\pi z/l})$, or to $2he^{-\pi z/l}$ by letting the constant be $2h = (C/2)e^{\pi H/l}$, representing the surface amplitude, or for $z = 0$.

(2) To obtain the propagation speed ω_o or U , the equation yields [Sic.: Note that here both ω_o or U describe the wave celerity, whereas [15] only uses ω_o , and that tanghyp is currently the tanh function]

$$\frac{\text{tanghyp} \cdot \frac{H}{2\pi 2l}}{\frac{H}{2\pi 2l}} = \frac{V^2}{gH}, \quad \text{or} \quad \frac{l}{\pi H} \frac{\exp \frac{\pi H}{l} - \exp \frac{-\pi H}{l}}{\exp \frac{\pi H}{l} + \exp \frac{-\pi H}{l}} = \frac{V^2}{gH},$$

or, if H is large,

$$\frac{V^2}{gH} = \frac{l}{\pi H}, \quad \text{from where } V = \sqrt{\frac{gl}{\pi}}.$$

These are the two principle results of Mr. Reech and Mr. Bertin. I am unable, it is true, in the notes, which I conserved on your memoir, to understand the results following the formulae of Mr. Reech. One has to take with x_o, z_o as the coordinates of the rotational center of point (x, y) , $2T = 2l/U$ in time, at position $x - x_o = he^{-\pi z_o/l} \cdot \sin[(\pi/T)(t - x_o/U)]$, $z - z_o = -he^{-\pi z_o/l} \cdot \cos[(\pi/T)(t - x_o/U)]$. I do not realize anymore in your approach for the displacements u, w , which are with $x - x_o, z - z_o$,

$$u = \frac{d\varphi}{dx} = C \left[\exp\left(\pi \frac{H - z_o}{l}\right) + \exp\left(-\pi \frac{H - z_o}{l}\right) \right] \frac{2\pi \cos\theta'}{2TU} \sin\left(\frac{\pi}{T}\right) \left[t - \frac{x \cos\theta' + y \sin\theta'}{U} - \psi \right],$$

$$w = \frac{d\varphi}{dz} = C \left[\exp\left(\pi \frac{H - z_o}{l}\right) - \exp\left(-\pi \frac{H - z_o}{l}\right) \right] \frac{2\pi \cos\theta'}{2TU} \cos\left(\frac{\pi}{T}\right) \left[t - \frac{x \cos\theta' + y \sin\theta'}{U} - \psi \right].$$

Here, $\cos\theta' = 1$, $\sin\theta' = 0$, because the wave propagation and the wave speed do not vary with x . Hence, for an infinite value of H , $C(e^{\pi(H-z_o)/l} \pm e^{-\pi(H-z_o)/l}) = 4he^{-\pi z_o/l}$, whereas the other yields

$$u = x - x_o = +\frac{4\pi}{l} h \exp\left(-\frac{\pi z_o}{l}\right) \sin\left[\frac{\pi}{T}\left(t - \frac{x}{U} - \psi\right)\right],$$

$$w = z - z_o = -\frac{4\pi}{l} h \exp\left(-\frac{\pi z_o}{l}\right) \cos\left[\frac{\pi}{T}\left(t - \frac{x}{U} - \psi\right)\right].$$

These expressions have in addition to those of Mr. Reech the coefficient $4\pi/l$. Where from does it originate? Does C represent another thing in the last equations than in the first? And for ψ , has it to be equal to zero? I do no more remember."

After additional comments, dSV concludes: "I understand that you find it ingenious to reduce to hydrostatic conditions, considering surfaces perpendicular to the resulting inertial forces ρg to forces of inertia $-\rho d^2x/dt^2, -\rho d^2y/dt^2$, a hydrodynamic problem such as that of periodic waves, and therefore you approve the statements of Mr. Reech and Mr. Bertin. However, it was good, as you did, to solve the problem also by the differential equations of hydrodynamics. Farewell, dear Sir, to the best times. My tributes to Madame Boussinesq, your friend Claude dSV." What a friendly statement of dSV, and here the first (and the only) time that he uses the initial of his first name, Claude. He indeed admired his young scholar, who was again able to solve a complicated problem by his mathematical genius.

The responses to the above questions of dSV remain answered in the letter of JB, dated October 5, 1870, reading: "Here's how to fit all my results relating to the swell with those of Mr. Reech. I find, in my memoir, by using your notations and calling x, z the coordinates of the center of the orbit of a molecule

$$\varphi = C \left[\exp\left(\frac{\pi(H-z)}{L}\right) + \exp\left(\frac{-\pi(H-z)}{L}\right) \right] \cos\left(\frac{\pi}{T}\right) \left(t - \frac{x}{U} - \text{const. } \psi \right),$$

from where

$$x - x_o = u = \partial\varphi/\partial x = C \frac{\pi}{L} \left[\exp\left(\frac{\pi(H-z)}{L}\right) + \exp\left(\frac{-\pi(H-z)}{L}\right) \right] \sin\left(\frac{\pi}{T}\right) \left(t - \frac{x}{U} - \psi \right).$$

As the sine function varies between -1 and $+1$, the horizontal diameter of the orbit is equal to the double of the expression $C(\pi/L)[e^{\pi(H-z)/L} + e^{-\pi(H-z)/L}]$. By letting r be half of this diameter,

$$r = C(\pi/L)[e^{\pi(H-z)/L} + e^{-\pi(H-z)/L}] = (\text{for } H \rightarrow \infty) C \frac{\pi}{L} \left(\exp \frac{\pi H}{L} \exp \frac{-\pi z}{L} \right).$$

One has also, for the identical case of $H \rightarrow \infty$,

$$w = \frac{d\varphi}{dz} = -C \frac{\pi}{L} \left(\exp \frac{\pi H}{L} \exp \frac{-\pi z}{L} \right) \cos \frac{\pi}{T} \left(t - \frac{x}{U} - \psi \right).$$

By replacing $C(\pi/L)(e^{\pi H/L})$ with h , z by z_o , u and w by $x - x_o$ and $z - z_o$, respectively, and by letting $\psi = 0$, which is equivalent to admit, as Mr. Reech did that the z -axis passes at the wave maximum for $t = 0$, where w has its maximum, the formulae of Mr. Reech are reproduced as

$$\begin{aligned}x - x_o &= r \cdot \sin(\pi/T)(t - x_o/U), \\z - z_o &= -r \cdot \cos(\pi/T)(t - x_o/U), \\r &= h \cdot e^{-\pi z_o/L}.\end{aligned}$$

By admitting for the free surface profile the expression

$$-z = \frac{2\pi C}{\tau \omega_o} \left[\exp\left(\frac{2\pi h}{\tau \omega_o}\right) - \exp\left(\frac{-2\pi h}{\tau \omega_o}\right) \right] \cos\left(\frac{2\pi}{\tau}\right) \left(\tau - \frac{x}{\omega_o} - \psi \right).$$

I neglect at this location the effect of second order quantities of displacements. In any case, I will improve my description as follows: «The values of w for $z = 0$ yield the vertical ordinates of the free surface directed downward, corresponding to the horizontal coordinates $x + u$, $y + v$ of liquid molecules located on this surface. The equation of this surface then is $-z = \text{etc.}$. . . as indicated above. As found by you [Sic.: dSV], that the movements of a molecule in its orbit are not uniform except for a circular orbit».

As you say, it is obvious that a swell of which each wave is symmetrical, can only be propagated by the forward velocity of the liquid layers, raised, by a speed communicating to the following liquid layers and in turn lifts them by tightening, and by the backward speed of the depressed liquid layers. This speed fills the void in front of them, and thus carries the depression further. If, at some point, the liquid free surface and the velocities of the molecules were symmetrical to certain vertical planes, these would always remain exactly as if they were becoming solid, non-frictional walls. This is due to reasons of symmetry, even taking into account the places of the highest or lowest points of the waves, which could only go up or down on the spot. This results if friction is neglected, and where movements are supposed to be very small and without initial speed, simple integrals of the equations of the problem, equations that are then

$$\begin{aligned}\rho \frac{\partial^2 u}{\partial t^2} &= -\frac{\partial(P - \rho g w)}{\partial x}, \quad \rho \frac{\partial^2 w}{\partial t^2} = -\frac{\partial(P - \rho g w)}{\partial z}, \quad \frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0 \\ \text{for } z = 0, P = 0; \quad \text{for } z = h, w = 0, \quad \text{for } t = 0, \frac{\partial u}{\partial t} &= \frac{\partial w}{\partial t} = 0.\end{aligned}\tag{12}$$

The resulting simple integrals, analogous to those of Laplace, are

$$\begin{aligned}u &= A \left[\frac{\exp k(h - z_o) + \exp -k(h - z_o)}{\exp(kh) - \exp(-kh)} \right] \sin k(x_o - \xi) \cos k' t, \\w &= A \left[\frac{\exp k(h - z_o) - \exp -k(h - z_o)}{\exp(kh) - \exp(-kh)} \right] \cos k(x_o - \xi) \cos k' t, \\p &= \rho g w - \rho \left(\frac{k'}{k} \right) A \left[\frac{\exp k(h - z_o) + \exp -k(h - z_o)}{\exp(kh) - \exp(-kh)} \right] \cos k(x_o - \xi) \cos k' t,\end{aligned}\tag{22}$$

where A , k and ξ are arbitrary constants satisfying the system of equations (1₁), with

$$k' = \left[kg \frac{\exp(kh) - \exp(-kh)}{\exp(kh) + \exp(-kh)} \right]^{1/2} = [kg \tanh(kh)]^{1/2}.\tag{32}$$

The temporal variations of u , w are proportional to $\cos kx$, and the free surface $z = A \cos \kappa(x - x_\kappa) \cos \kappa' t$ is a constant at each sinusoidal time, thereby conserving its highest and lowest points always at the same vertical plane. The formula of Newton $T = \pi(L/g)^{1/2}$ is evidently wrong given that the direct analysis and the various experiments of Mr. Pâris cited in the brochure of Mr. Bertin (p. 23) yield $T = (\pi L/g)^{1/2}$.

All of Mr. Reech's results were correct in my submission to the Academy, but they were only established for very small movements, while Mr. Reech gave them as the first in case of a large

amplitude. I then demonstrated as the first, in my new draft, that for a finite amplitude, they cannot be studied, as Mr. Reech appears to indicate in his note, for the movement of periodic waves in basins of arbitrary depth, because oscillations then cannot be merely periodic.

I believe that this problem of periodic liquid waves, taken in its generality, including, for example, the diffraction phenomena that I have reported, could only be dealt with as I did. However, Mr. Bertin's more basic method (and I do not know if I should add Mr. Reech) might well suit me to the simple problem of the swell in high seas. It seems to me that it is necessary, in a report on the work of Mr. Bertin, to describe this merit, or you risk having little to say in particular in favor of the memoir analyzed. It is by renewing also on the part of my wife, the proposal contained in my letter of yesterday, that I have the honor to say to myself, Mr. Count and dear Master, with the most vivid and respectful gratitude, Your devoted disciple, JB."

The answer of dSV to JB is contained in the letter written on October 9, 1870. It reads: "Dear Sir, the only objection I made in my letter of October 1 against your wave theory, compared to that of Mr. Bertin and Mr. Reech, in that (from the notes I have) the upper surface of the waves having by equation

$$-z = \frac{2\pi C}{\tau\omega_o} \left[\exp\left(\frac{2\pi H}{\tau\omega_o}\right) - \exp\left(\frac{-2\pi H}{\tau\omega_o}\right) \right] \cos\left(\frac{2\pi}{\tau}\right) \left[t - \frac{x}{\omega_o} - \psi \right]$$

appears to me to be a sinusoid, while it should be a generalized cycloid. This objection was unfounded, for time t was in the equation I want to write down; it must be distinguished between this equation and a similar equation containing x , to have in the equation both x and z of the next curve giving the function of the waves. However, the result of this last issue will be to give a cycloid, as do the equations of Mr. Reech and Mr. Bertin. Yet, I will ask you something else.

(1) The equations of these two gentlemen can also be written, with x , z as the coordinates of a point describing a circle whose center has the coordinates x_o , z_o , with $2h$ as the wave height, $2T$ as the wave period and $2L$ as the wave length (such that the propagation speed is $V = L/T$), as

$$x = x_o + h \exp\left(-\frac{\pi z_o}{L}\right) \sin \pi \left(\frac{t}{T} - \frac{x_o}{L}\right), \quad z = x_o - h \exp\left(-\frac{\pi z_o}{L}\right) \cos \pi \left(\frac{t}{T} - \frac{z_o}{L}\right). \quad (1_3)$$

These values of x and z perfectly satisfy the continuity equation to be verified by assuring that $d\sigma/(dx_o dz_o) = (dz/dz_o)(dx/dx_o) - (dx/dz_o)(dz/dx_o)$ of Mr. Bertin does not vary with time [Sic.: with σ as auxiliary variable]. Indeed, $dz/dz_o = 1 + (h\pi/L) \exp(-) \cos(-)$; $dx/dx_o = 1 - (h\pi/L) \exp(-) \cos(-)$; $dx/dz_o = -(h\pi/L) \exp(-) \sin(-)$; $dz/dx_o = -(h\pi/L) \exp(-) \sin(-)$, from where follows well independent of time the relation $d\sigma/(dx_o dz_o) = 1 - (h^2\pi^2/L^2) \exp(-2\pi z_o/L)$.

(2) In assuring that with the velocity components $u = dx/dt$, $w = dz/dt$, the ordinary continuity condition $(du/dx) + (dw/dz) = 0$ is satisfied, because $x = x_o + \dots$, $z = z_o + \dots$ [Sic.: see equations above] gives $dx/dt = u = (h\pi/T) \exp(-) \cos(-)$, $w = dz/dt = (h\pi/T) \exp(-) \sin(-)$."

The following analysis is a complicated approach of dSV, not to be discussed here in detail. The letter continues: "Now, I would like to put in my report on [Sic.: the publication of] Mr. Bertin that, before Mr. Reech's Note was published in the *Compte-rendu* of May 10, 1869, preceding Mr. Bertin's first publication. In fact, you had found earlier not only their formulas, yet even more general expressions in this regard, applying to a finite water depth. In this case one has, instead of looking for trajectory ellipses of the same eccentricity of the frames at the bottom, from which this movement comes from the background that Mr. de Caligny observed and on which Mr. Reech doubts."

"However, trying with your formulae involving ψ as an unknown function of x_o , z_o , and with \sinh , \cosh denoting the sine-hyperbolic, cosine-hyperbolic functions, A an unknown constant, and

H the total water depth, then

$$x = x_o - A \operatorname{cosh} \left[\frac{\pi(H - z_o)}{L} \right] \sin \pi \left(\frac{t}{T} - \frac{x_o}{L} - \psi \right), \quad (2_3)$$

$$z = z_o - A \operatorname{sinh} \left[\frac{\pi(H - z_o)}{L} \right] \cos \pi \left(\frac{t}{T} - \frac{x_o}{L} - \psi \right). \quad (3_3)$$

This is because you denote with u, w the displacements $x - x_o, z - z_o$. I say, in trying whether these formula satisfy the continuity condition, I find, neither by using the condition $(dz/dz_o)(dx/dx_o) - (dx/dz_o)(dz/dx_o)$ be a constant based on Mr. Bertin, nor by applying the ordinary condition $(du/dx) + (dw/dz) = 0$, that these two formulas not at all satisfy the above conditions except for $\psi = 0$. Further, in conserving ψ , it is impossible to attribute a value in x_o, z_o such that either of the two conditions were satisfied. Could you then, dear Sir, tell me what according to your analysis are the formulas by which the coordinates x and z , at time t , of a molecule oscillating about a point whose fixed coordinates are x_o, z_o , when the periodic movement is given." As a consequence, it appears that JB was the first having provided the general solution to the permanent water wave problem in finite water depth. His 1872 publication [15] discusses also the approach of Reech. On p. 576ff, it is written: "Mr. Reech, in his note of 1869, *Comptes rendus*, volume 68, p. 1099, studies a periodic wave system propagated in a channel. However, without assuming that the trajectory centers coincide with the equilibrium positions of the molecules, but placing themselves at the point of view of the current Note. The idea of this Note even came to me only by studying his article and trying to generalize his equations, some of which he kindly communicated to me in a correspondence that he gave the honor of maintaining with me."

"Mr. Reech proposes, in this work, to study the waves propagated along a rectangular channel of indefinite length, assuming the movements parallel to the sides, and the same on the whole extent of any straight perpendicular to these sides. If the x axis is taken in the longitudinal direction, we will have everywhere $v = 0$, and u, w will not depend on y ." After some computations, Mr. Reech deduces: "The trajectories will be circular. This takes place... but applies only in the case of infinite depth h ."

Boussinesq's [15] work has certainly made some rumor against him by Mr. Reech. As to Louis Emile Bertin (1840–1924), he developed to one of the foremost naval engineers in France. In the 1870s, he left the naval theory in favor of a great naval designer, thereby winning great international recognition as a leading naval architect.

Conclusions

The professional career and the biography of Frédéric Reech are described based on two sources: (1) A hardly known paper of Moritz Weber, namesake of the Froude number first, and after this research proposing the Reech number for gravity-inertial processes in hydrodynamics. In addition, numerous details on the career of the naval engineer Reech are presented. These include his role at the Maritime School in Lorient and later in Paris, his various publications dealing with steam machines for large vessels, stability aspects of ships, dynamic effects of heat, the measurement the power of machines, next to his works in the determination of the resistance of ships. Weber identifies clearly Reech as the founder having proposed the law that is currently assigned to the English naval engineer William Froude. The biography of Reech is also detailed mainly based on a paper written by Schwartz.

(2) Weber did not at all describe a work of Reech directed to the theory of periodic waves. Given that dSV in his role as president of the Mechanics Section, the Academy of Sciences, Paris, had to review a submission to this topic, he engaged his colleague Boussinesq with the computational details. The latter exchanged a number of letters contained in the Correspondence with his mentor dSV, thereby strongly developing this theory. It appeared finally in the top French journal

Mémoires présentés par divers Savants, evidencing the genius of Boussinesq's mathematical brain. The interaction between the two, and the contact of Boussinesq with Reech, thus close the circle shedding light onto an essential portion of developments in hydraulics and hydrodynamics.

Given the scientific work of Reech (see biography below), the approach of Reech in this entirely theoretical field is somewhat astonishing. None of his other writings deals with similar problems of physics. It also appears surprising that once Boussinesq started to interact with Reech, the latter hardly answered the questions to the fundamental issues of periodic steady waves. This attitude hardly detracts from Reech as a pioneer in the field of ship resistance, but dSV is definitely the father of the number which is currently attached to both Reech and Froude.

Acknowledgments

The Authors would like to thank the Library of *Académie des sciences*, Paris, in particular to its Directress, Mrs. Françoise Bérard, and its perpetual Secretary, Mrs. Hélène Carrère d'Encausse, for having given us the possibility to access the Correspondence, as well as their interest in publishing this work.

Appendix

Extract of [14] on his 'Essay on the theory of liquid periodic waves'. Commissioners: Delauney, Bonnet, Jamin.

"I consider a homogeneous liquid at rest, limited below by a horizontal bottom supporting at its top a constant pressure, and indefinite laterally. I assume that the molecules of this liquid contained in a fairly small space generate periodic movements. These movements will spread all around, and after a certain period of time, during which the effects due to the initial conditions will disappear, the entire environment will be subjected to vibrations of the same period. I first seek the laws of these vibrations, limiting myself to the points located at a sufficient distance of vibration for the movements to be very small and very continuous."

"I start by giving the general equations of the continuous movements of any matter, and those that govern the small periodic agitations of liquids. I then address the proposed issue that leads to the following laws:

(1) Each molecule describes, around its equilibrium position, an ellipse whose shape is determined by the depth at which the molecule is located. This ellipse, located in the vertical plane that passes through the center of the vibrations, also has its large horizontal axis, and a constant focal distance of all molecules located on the same vertical. Infinitely flattened for those at the bottom, it is noticeably circular on the surface, provided the vibrations are not too slow; I then conclude that the molecule, by travelling the upper half of its trajectory, moves away from the center of the vibrations, while it approaches it by traversing the other half.

(2) The surfaces of the waves, i.e. the geometric locations of the points where the molecules are at the same stage of their vibration, are about circular cylinders, which have as common axis the vertical of the center of vibrations, and of which the radius increases over time. The speed of this increase, i.e. the rate of wave propagation, is significantly constant as soon as the radius contains a large number of times the wavelength. The speed of propagation of an isolated wave is also very different as long as the vibrations are fast.

(3) Finally, for all molecules located on the same horizontal plane, the amplitude of motion varies arbitrarily from a radius emanating from the axis to the neighboring radii, while, on the same radius, it is the opposite of the square root of the distance to the axis."

"I then study the case where the vibrations, instead of being produced in a small container, are on an entire cylinder of arbitrary shape, with vertical generators, and I show that the general

movement is then equivalent to the superposition of an infinity of circular waves, which would have as axes the generators of a cylinder very similar to these proposed.”

“I apply this theory to periodic waves emanating from a point where a wide enough obstacle intercepts a significant part of it. These waves, as they propagate beyond, barely enter the area protected by the obstacle. However, they produce, somewhat outside of this region, very elongated hyperbolic wrinkles, analogous to the fringes that gives a screen by intercepting some of the rays emanating from a bright spot. The main laws of these phenomena are the result of an analytical study of the defined integrals of diffraction, in which I borrow nothing from the various approximation methods used for their numerical calculation.”

A Commission headed by dSV [16] accepted the above proposal in a Report to the Academy of Sciences, resulting finally in the large paper of JB [15].

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