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Self-demodulation acoustic signatures for nonlinear propagation in glass beads

Signatures acoustiques de l'auto-démodulation basse fréquence lors de la propagation non linéaire dans des billes de verres

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Abstract

The present Note describes some experimental work related to the nonlinear propagation of acoustic waves in granular media such as unconsolidated glass beads. The studied nonlinear effect is a self-demodulation process performed with the operation of the so-called parametric transmitting antenna. The pump (or carrier) wave is generated by a high power ultrasonic broad-band transducer (100 kHz central frequency) which is LF (low frequency, i.e., a few kHz) amplitude modulated. As the attenuation of acoustic waves increases with frequency, only the LF demodulated wave can be transmitted. A parametric study is performed where the HF central frequency is monitored between 60 and 300 kHz. The LF demodulation profile versus the HF frequency is modified, its shape being temporally derived almost twice. A numerical analysis of the order of temporal derivation is done in the Fourier domain, its value varying from 1.25 to 2.7. Qualitative agreement with current theoretical models is described, and an advanced theoretical analysis by the same authors [Phys. Rev. E 66 (2002) 041303], taking into account absorption, nonlinearity, dispersion and scattering, is briefly discussed. ***To cite this article: V. Tournat et al., C. R. Mecanique 331 (2003).***

Résumé

La présente Note décrit un travail expérimental relatif à la propagation non linéaire d'ondes acoustiques dans des matériaux granulaires tels que des billes de verre non consolidées. Un transducteur ultrasonore large bande ayant une fréquence centrale de 100 kHz génère une onde de pompage (ou onde de transport) dont l'amplitude est modulée à basse fréquence (BF, autour de quelques kHz). L'atténuation des ondes acoustiques augmentant avec la fréquence, seule l'onde démodulée qui est basse fréquence peut être transmise sur l'épaisseur du milieu granulaire. Une étude paramétrique est ainsi menée où la fréquence de l'onde de pompage varie entre 60 et 300 kHz. Le profil du signal de démodulation BF en est modifié, son allure étant dérivée temporellement presque deux fois puis intégrée. Une analyse numérique de l'ordre de dérivation du signal est effectuée dans l'espace de Fourier, sa valeur variant de 1,25 à 2,7. Un accord qualitatif avec les modèles théoriques récents est décrit, et une

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analyse théorique approfondie par les mêmes auteurs [Phys. Rev. E 66 (2002) 041303], qui prend en compte l'absorption, la non linéarité, la dispersion et la diffusion, est brièvement discutée. ***Pour citer cet article : V. Tournat et al., C. R. Mecanique 331 (2003).***

Keywords: Acoustics; Granular materials; Demodulation process; Ballistic and diffusive regime; Temporal derivative of the LF wave shape

Mots-clés : Acoustique ; Matériaux granulaires ; Processus de démodulation ; Régime balistique et de diffusion ; Dérivation temporelle du profil de l'onde BF

Version française abrégée

L'acoustique non linéaire dans les matériaux granulaires non consolidés peut être décrite à partir de la théorie de Hertz du contact mécanique entre deux billes sphériques. L'équation (1) indique que la vitesse de phase des ondes acoustiques propagées dans une structure de billes est proportionnelle à la puissance $2/3$ de celle de la vitesse dans le verre plein, et à la puissance $1/6$ du chargement mécanique de cette structure. L'ordre de grandeur des vitesses propagées est de 200 à 300 m/s [3], ce qui indique que la fréquence de coupure, au delà de laquelle les modes acoustiques sont évanescents, se situe vers 300 kHz pour des billes de verre de diamètre 0,2 mm [3,2,4,5]. En raison de l'atténuation croissante avec la fréquence, seule l'onde BF associée à la modulation d'amplitude du paquet d'onde (sous forme d'un profil gaussien) pourra se propager dans le matériau granulaire, comme cela a pu être observé par divers auteurs [3,6–8], et expliqué théoriquement [1,8,9].

La Fig. 1 décrit le système expérimental mis en œuvre, et la Fig. 2 présente les résultats expérimentaux essentiels. Il s'agit du profil de l'onde démodulée (accélération mesurée) après propagation dans le milieu granulaire – profil qui dépend de la fréquence de l'onde de pompage (présenté pour 4 valeurs à 60, 160, 220, et 300 kHz respectivement). Le calcul de l'ordre de dérivation temporelle du profil est effectué dans le domaine de Fourier, les résultats étant présentés pour 13 valeurs de fréquence dans le Tableau 1. L'ordre de dérivation p varie de 1,25 à 2,7. Ceci atteste que l'enveloppe de modulation gaussienne est dérivée un peu plus d'une fois, 2, puis presque 3 fois successivement en fonction de la fréquence de l'onde de pompage. Des éléments théoriques empruntés à [1], concernant ce processus de démodulation non-linéaire, sont présentés ensuite. L'équation (2) représente la relation contraintes-déformations dans le cadre de la théorie de Hertz. La relation (3) donne l'équation d'onde non linéaire associée au champ de déplacement u, ce dernier étant divisé en deux parties : l'une u^{Ω} est associée aux basses fréquences (BF) et l'autre u^{ω} relative aux hautes fréquences (HF). Enfin, l'équation (4) issue de [1], donne la transformée de Fourier du champ de déplacement BF de l'onde démodulée lorsque l'onde de pompage HF se propage balistiquement.

Les données expérimentales présentées sur la Fig. 2 sont alors interprétées qualitativement sur la base de l'ensemble de ces résultats théoriques associés à d'autres considérations sur la diffraction [10], et la diffusion [1]. A basse fréquence, la dispersion est responsable de l'intégration du profil temporel gaussien de l'enveloppe de l'onde de pompage (comme prédite par l'équation (4)). Le détecteur sensible à l'accélération va ainsi capter, du fait de cette intégration, la dérivée première de l'enveloppe du champ de déplacement émis par le transducteur ultrasonore de puissance (cf. Fig. 2(a) à 60 kHz). Par la suite, à plus haute fréquence (160 kHz sur la Fig. 2(b)), l'atténuation de l'onde HF de pompage devenant plus importante, cet effet de dispersion s'évanouit, ce qui résulte en une dérivation temporelle supplémentaire. A fréquence plus haute encore (ici 220 kHz, cf. Fig. 2(c)), un effet de diffraction [10] apparaît, ce qui résulte en une 3ème dérivation. Celle-ci n'est pas complètement observée dans le travail présenté, puisque l'ordre de dérivation ne vaut que 2,7. Enfin, une intégration du profil intervient vers 300 kHz, et est interprétée comme une manifestation de la transition balistique-diffusion dans le transport des ondes de pompage HF.

Le travail conclu sur diverses améliorations devant être apportées pour prendre en compte les ondes de cisaillement [11,12], dont les signatures non linéaires sont a priori différentes (non-linéarité impaire au lieu de paire pour les ondes longitudinales). Il indique aussi les similarités qui existent pour ces problèmes de démodulation non linéaire avec des configurations de matériaux présentant des fissures, comme en atteste des publications récentes sur le sujet.

1. Position of the problem

Unconsolidated granular media are very common (natural sediments, sandy materials). The propagation of elastic waves in these media is a real challenge in many cases because fundamental processes are often inherently mixed and cross-linked. In the linear case, i.e., when the amplitude of the elastic waves (or alternatively the induced strain) is sufficiently small, very basic features are the transition from ballistic propagation to diffusion-like energy transfer when increasing frequency, velocity dispersion and frequency dependent absorption. Scattering (and then diffusive transport of acoustic waves) in granular materials appears due to deviations from the regular lattice, i.e., inhomogeneities of grains positions and of contacts between them, under the so-called cut-off frequency [1,2]. In the present work, 0.2 mm diameter glass beads have been used. The wave speed of short acoustical bursts in the grains, which can be calculated from the Hertzian contact theory, is given in Eq. (1):

$$v_s = \left(\frac{3}{2}\right)^{1/2}\left[\frac{\tilde{n}(1-\phi)}{3\pi(1-\nu)^2}\right]^{1/3} v_{\text{glass}}^{2/3}\left(\frac{P_0}{\rho}\right)^{1/6} \tag{1}$$

where $\tilde{n}$ is the average number of contacts per grain, ν is the Poisson ratio for glass, v_{glass} is the velocity of the longitudinal sound waves in bulk glass, P_0 is the static pressure applied on the granular assembly, ϕ the porosity of the medium, and ρ the density of the medium. Numerical values of the phase wave are commonly predicted around 300 m/s (for $\tilde{n} = 6$), while experimental results are in the 200–300 m/s range depending on the mechanical load [3]. Consequently, the estimated cut-off frequency equal to $f_c = \pi v_s/(2a)$ where a is the bead radius, lies around 300 kHz, as confirmed by experimental observation [3,2,4,5]. Accordingly, the propagation of acoustic waves with frequency above that limit will become evanescent. Attenuation becomes drastic over one single wavelength (i.e., 1 mm at 300 kHz), and the HF components will be completely damped out. Here, we consider pump wave frequencies under this cut-off frequency limit, i.e., propagative acoustic modes.

If an amplitude modulation is done at sufficiently low (e.g., a few kHz) frequency, then this component can be transmitted easily in the glass beads medium, indicating in turn that a strong nonlinear collective process is taking place due to the inter-beads Hertzian contacts. Such nonlinear demodulation has been readily observed by several authors [3,6–8] and explained theoretically [1,8,9]. It is the aim of this paper to present and discuss some significant experimental results related to the change of the temporal profile of the LF transmitted signal versus the high frequency of the carrier wave which is used to feed the ultrasonic pump transducer.

2. Experimental observations and numerical treatment

The usual set-up is shown on Fig. 1. It is a standard configuration where the carrier wave (or pump wave) is launched from one side of the glass beads assembly. After propagation through it, the demodulated LF wave is detected on the other side of the granular medium by an ultrasonic receiver imbedded in the glass beads. There is no change in the directivity pattern of the latter transducer for low frequencies (from 0 Hz to 40 kHz), and velocity amplitude at its surface decreases as the frequency. Received electrical signals are thus proportional to the acceleration field of the acoustic wave in this low frequency range.

The carrier wave is initially modulated by a Gaussian envelope. The choice of this modulation profile is due to its easiness of temporal integration (or alternatively derivation) as well as its Fourier transformation. Comparison of this initial signal shape to the demodulation profile after transmission is shown on Fig. 2, at various frequencies (60, 160, 220 and 300 kHz). Briefly stated, a derivative close to the first is observed at 60 kHz (on Fig. 2(a)), then a second and almost third derivative (on Figs. 2(b) and 2(c) respectively at 160 and 220 kHz).

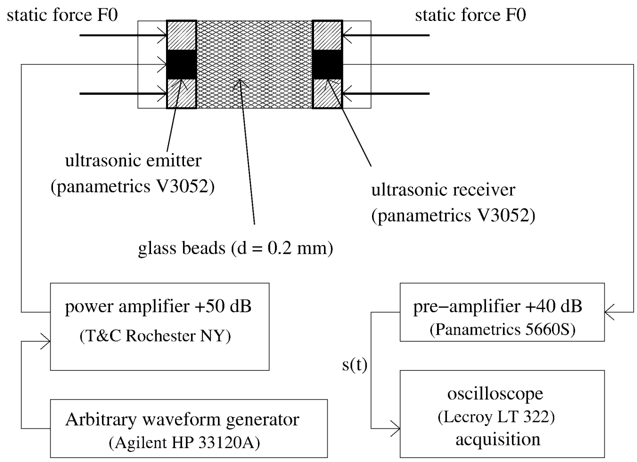

Fig. 1. Schematic for the experimental set-up.

Fig. 1. Système expérimental.

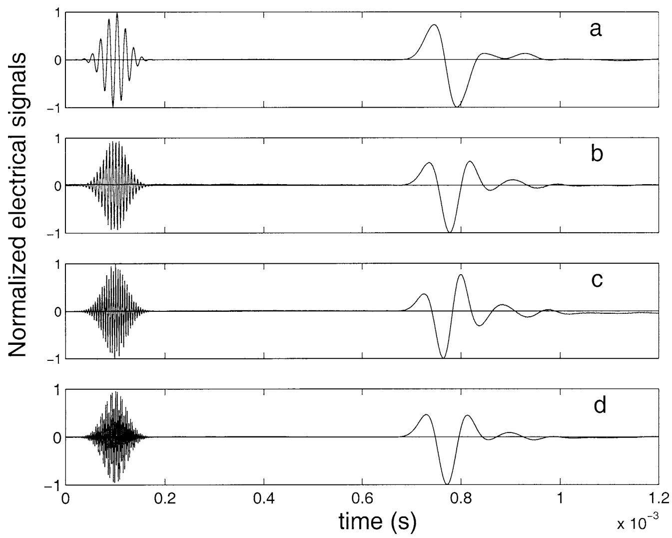

Fig. 2. Emitted HF signals and corresponding demodulated LF profiles, obtained experimentally, for pump wave frequencies of (a) 60 kHz, (b) 160 kHz, (c) 220 kHz, and (d) 300 kHz.

Fig. 2. L'onde BF émise et l'onde démodulée correspondante, expérimentales, pour les fréquences de pompage de (a) 60 kHz ; (b) 160 kHz ; (c) 220 kHz ; (d) 300 kHz.

Finally, at 300 kHz, a second derivative is observed (on Fig. 2(d)). To confirm this experimental trend, a very simple numerical treatment has been implemented, where the partial order of temporal derivation is computed in the Fourier domain. As a matter of fact, the Fourier transform of the modulation envelope can be written $S(f) = FT[s(t)] = (-j2\pi f)^p \, \mathrm{e}^{-f^2\tau_m^2/4} \, \mathrm{e}^{-2\pi jf\tau_0}$, quantity which is used in order to obtain the best fit at the same time on the fractional order p of temporal derivation, as well as on the time parameters τ_0, the time delay of the demodulated signal, and τ_m, the characteristic duration of the demodulated signal. The computations of the best fit parameters are collected on the Table 1 for 12 different frequencies ranging from 60 to 300 kHz.

Table 1
Calculated results

Tableau 1
Les résultats calculés

Pump wave frequency (kHz)	Order of derivation p	Modulation time τ_m (10^{-4} s)	Time delay τ_0 (10^{-3} s)
60	1.25	0.38	0.780
100	1.45	0.35	0.776
120	1.65	0.34	0.776
140	1.85	0.34	0.775
160	2.05	0.34	0.778
180	1.95	0.33	0.774
200	2.2	0.33	0.774
220	2.7	0.33	0.776
240	2.5	0.33	0.772
260	1.95	0.33	0.768
280	1.95	0.33	0.770
300	1.95	0.33	0.768

A slow decrease on the time parameters is observed. Moreover, the estimates of the fractional derivation order confirm the results qualitatively predicted by theoretical considerations, that is an initial first order derivative at low frequency (here at 60 kHz), followed by a further successive second, then eventually third additional derivations and finally an integration. Some of these trends of temporal derivation or integration of the modulation envelope were already anticipated in the field of nonlinear underwater acoustics [10]. It was further confirmed for propagation in unconsolidated granular materials (in sandy materials) by Zaitsev et al. [7,8]. In both cases the authors were not facing two or three consecutive temporal derivations of the wave profile, but instead merely one single derivation. In our configuration, we observe this multiple derivation process and then this integration because several different mechanisms are taking place. The next section provides a very brief description of a relevant theoretical model. Then in the last section, we propose some further qualitative arguments, in view to interpret the measurements which have been done.

3. Current theoretical models and interpretation

The full theoretical model can be found in [1]. Here a shortened version is briefly reviewed, including additional comments in order to explain the observed experimental results. One starts by writing the 1-D Hertz nonlinear stress-strain relationship which is valid for the inter-grain mechanical contacts:

$$\sigma = C\left(\varepsilon_0 + \frac{\partial u}{\partial x}\right)^{3/2} \tag{2}$$

where σ denotes the stress, C a constant proportional to the Young modulus of the beads, ε_0 the static strain and u the dynamic displacement along axis x. For a material which is mechanically pre-stressed, the modulus of the static strain ε_0 is much larger than the dynamic term $\varepsilon = \partial u/\partial x$. A second order series expansion of Eq. (2) easily gives rise to a quadratic nonlinearity. The stress is then substituted in the 1-D equation of motion, in order to obtain the propagation equation for u. By noting $u = u^{\Omega} + u^{\omega}$, where u^{Ω} and u^{ω} represent the low frequency (LF) and the high frequency (HF) components of the displacement fields, the wave equation is rewritten after averaging (as shown by the symbol $\langle\rangle$) over one period of the HF carrier wave:

$$\left[\frac{\partial^2}{\partial t^2} - c_0^2\frac{\partial^2}{\partial x^2}\right]u^{\Omega} = -c_0^2\Gamma\frac{\partial}{\partial x}\left\langle\left(\frac{\partial u^{\omega}}{\partial x}\right)^2\right\rangle \tag{3}$$

with c_0 as the phase velocity, and where Γ represents the coefficient of quadratic nonlinearity. The right-hand side of Eq. (3) can as well be expressed in terms of the energy density profile of the HF carrier wave $\langle W^\omega \rangle$, in the form $\frac{\Gamma}{\rho}\langle W^\omega \rangle_{,x}$. According to this result, the demodulated LF wave is excited by the gradient of the energy density profile of the carrier wave. The carrier wave itself is next described either by a propagation or by a diffusion equation [1]. When the carrier wave is mostly attenuated by absorption instead of scattering, the major contribution for the demodulated signal comes from the carrier wave ballistic propagation. In such a case, the LF displacement field outside the excitation region, can be derived in the Fourier domain in the following form [1]:

$$\tilde{u}^\Omega = K\tilde{\phi}(\Omega)\left[\frac{1}{-j\Omega(1+c_\omega/c_0)+1/\tau} + \frac{1}{-j\Omega(1-c_\omega/c_0)+1/\tau}\right]\mathrm{e}^{j(\Omega/c_0)x} \tag{4}$$

with τ the attenuation time of the carrier wave, $\tilde{\phi}(\Omega)$ the Fourier transform of the Gaussian modulation function, c_ω the group velocity of the carrier wave, and K a constant associated with the characteristics of the medium. Dispersion of the waves velocities inside the HF spectrum can be neglected when the spectrum width is narrow. In contrast, the difference between the velocities of HF waves and LF demodulated wave should be taken into account. Accordingly, velocity dispersion can be responsible for the integration of the profile of the demodulated wave. This integration of Eq. (4) occurs solely when both factors in the brackets are proportional to $1/(j\Omega)$, which is the case when $|-j\Omega(1-c_\omega/c_0)| \gg |1/\tau|$. Further details on these considerations as well as an in depth discussion can be found in [1].

The effect of group velocity dispersion on the HF wave packet is a broadening of the signal, i.e., an increase of the Gaussian modulation characteristic width τ_m. It can be observed on the demodulated wave in the numerical fits of Table 1 for the lowest pump waves frequencies.

Transition from ballistics to diffusion in the HF waves transport manifests itself, by an integration of the demodulated profile, as can be seen in [1]. This integration occurs when pump waves are scattered before being absorbed, i.e., when the condition $\tau_a \gg \tau_m \gg \tau_s$ is fulfilled, where τ_a is a characteristic absorption time of the HF pump waves, τ_m the width of the gaussian function of modulation, and τ_s the characteristic scattering time of the HF. It means that at this transition, information about these characteristic times can possibly be extracted.

4. Further discussions and conclusions

As explained in Section 2, the demodulated wave profile is close to the first, second and third derivatives of the gaussian enveloppe modulation of the carrier wave recorded at three different frequencies (respectively at 60, 160 and 220 kHz). It has been outlined in the table that the order of derivation varies over the 60–220 kHz bandwidth of the carrier wave from 1.25 to 2.7. Almost two further derivatives when increasing the carrier wave frequency are thus observed. When increasing further the frequency (300 kHz), an integration is finally observed.

At 60 kHz, the carrier wave is slightly attenuated. The cumulative effect of linear dispersion is responsible for a temporal integration of the demodulation wave shape. Consequently, the acceleration (as detected by the ultrasonic receiver for low frequencies at one side of the granular assembly) is in fact proportional to the first derivative of the Gaussian enveloppe.

When the frequency is further increased, the attenuation of the carrier wave becomes stronger, and this in turns decreases the penetration distance of the carrier HF wave. The cumulative dispersion effect on the demodulation ends up vanishing, which implies a derivation of the demodulated LF profile Gaussian (as shown in Fig. 2(b), when the carrier wave frequency is 160 kHz). A second temporal derivative is thus observed for the detected LF acceleration.

The third derivation is observed at even higher frequencies of the carrier wave (at 220 kHz, as seen in Fig. 2(c)). This last derivation is due to the diffraction of the demodulated wave, as explained in [10]. The HF carrier wave in this last case is confined, due to the very strong attenuation, nearby the ultrasonic emitting transducer, which is the source for the demodulated wave. This is directly seen in Eq. (3) where the right term is the source of the wave equation. In fact this last derivation is not yet completed at 220 kHz, because the index of derivation is 2.7 instead of being truly 3. An explanation for such a feature is the effect of frequency dependent absorption on the LF demodulated wave propagation. High frequencies of the demodulated LF wave spectrum are more attenuated than the low ones, tending to decrease the partial derivative order of the experimentally obtained profile. Obviously, this effect acts mainly when LF propagate over a long distance, i.e., virtual sources are localized nearby the ultrasonic transducer. Thus, it concerns only the high frequencies of the pump wave.

Finally, an integration is observed for higher frequencies of the pump wave (on Fig. 2(d) for 300 kHz). At this frequency, the wavelength approaches the typical size of inhomogeneities in the granular medium. The pump wave is then strongly scattered and may transport energy diffusively. As predicted in [1], diffusion of the pump wave when $\tau_a \gg \tau_m \gg \tau_s$ implies integration of the demodulated signal. Consequently, observation of this transition provides information on the relative weight between scattering and absorption.

Qualitative agreements with the theoretical model developed in [1] are promising for a quantitative analysis in order to extract parameters of the acoustic wave propagation in the granular media. However, it is necessary (particularly for the diffraction effects) to take into account the three-dimensional geometry of the problem, i.e., changes in the directivity pattern of the ultrasonic emitter with the pump wave frequency. This numerical work is at this time in progress.

It is evident that the present analysis is not entirely sufficient. Each single inter-grain Hertzian contact acts as a mode converter between longitudinal and shear waves. This problem is under very active investigations both in linear and in nonlinear propagation [11,12]. The behavior of each individual (longitudinal or shear) wave is certainly different in the demodulation process. For example, in a cubic lattice of beads, their major contributions in nonlinearity are not identical, being mainly even for longitudinal polarisation and odd for shear.

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