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Singular nature of nonlinear macroscale effects in high-rate flow through porous media

Nature singulière des effets macroscopiques non linéaires dans un écoulement à hautes vitesses en milieux poreux

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Abstract

The quadratic law of laminar flow through porous media at high Reynolds numbers, which is well confirmed by the multiple experimental data, is shown to give rise to three fundamental paradoxes. All them can be resolved by assuming the singular structure of flow. The singularity is produced by the formation of jet brunches which invade the stagnant zones and sharply loss their kinetic energy. The numerical simulation confirms this effect. ***To cite this article: M. Panfilov et al., C. R. Mecanique 331 (2003).***

Résumé

On a montré que la loi quadratique d'un écoulement laminaire en milieu poreux aux grands nombres de Reynolds, qui est bien justifiée dans de nombreuses expériences, engendre trois paradoxes fondamentaux. Tous les trois peuvent être résolus en supposant une structure irrégulière de l'écoulement. La singularité est produite par la formation de certaines branches des jets qui entrent dans la zone stagnante et perdent brusquement leur énergie cinétique. Cet effet est confirmé par les simulations numériques. ***Pour citer cet article : M. Panfilov et al., C. R. Mecanique 331 (2003).***

Keywords: Porous media; Navier–Stokes; Nonlinear correction; Darcy's law; Forchheimer; Flow separation; Streamlines

Mots-clés : Milieux poreux ; Navier–Stokes ; Correction non linéaire ; Loi de Darcy ; Forchheimer ; Séparation de l'écoulement ; Lignes de courant

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Les données expérimentales sur les écoulements à grandes vitesses en milieux poreux justifient une correction du type quadratique de la loi de Darcy (Éq. (2)). Néanmoins, cette forme appelée loi de Darcy–Forchheimer engendre l'apparition de trois paradoxes fondamentaux.

Premièrement, il est facile de montrer que, dans un milieu isotrope, la correction en puissance paire de la vitesse dans une équation vectorielle est interdite. En effet, un vecteur ne peut être obtenu à partir d'une puissance paire de la vitesse que par son produit contracté avec un tenseur d'ordre 3 (voir Éq. (4)), or toutes les composantes d'un tel tenseur sont nulles pour un milieu isotrope.

Le deuxième paradoxe est lié à l'irréversibilité de la loi de Darcy/Forchheimer. En milieu isotrope, cette loi macroscopique doit être invariante par rapport à la direction de l'écoulement. Ainsi, l'inversion du sens de déplacement (changement du signe de la vitesse et du gradient de pression) ne doit pas changer cette loi. Pour une équation vectorielle contenant un terme quadratique, Éq. (4), l'invariance par rapport à la direction de l'écoulement ne peut être obtenue que si le tenseur d'ordre 3 – $A_{ijk}^{(3)}$ dans Éq. (4) – dépend de la vitesse. Cette contrainte ne se justifie pas pour un écoulement régulier pour lequel la loi de l'écoulement représente un développement tronqué régulier en puissance de la vitesse (Éq. (3)).

Le troisième paradoxe traduit que pour un milieu périodique le terme inertiel intégral est strictement égal à zéro, si l'écoulement (stationnaire) est supposé régulier. C'est une modification du paradoxe de D'Alembert formulé pour un écoulement idéal autour d'un obstacle. Cependant, aussi bien les données expérimentales que les résultats de calcul montrent que le terme inertiel intégral n'est pas nul.

L'explication de ces trois paradoxes se trouve dans l'hypothèse concernant la régularité de l'écoulement. Ainsi, le terme quadratique macroscopique est engendré par l'existence de points singuliers dans le champ des vitesses. On montre que, formellement, deux types de singularité sont responsables de la présence du terme quadratique : soit un champ de vitesse fini mais discontinu, soit une vitesse infinie et continue. Le premier type peut être réalisé par une rupture des lignes de courant provoquée par une instabilité. Le second type de singularité se manifeste sous la forme d'un faisceau de jets entrant ou sortant de façon « lisse » dans un point de contact (Fig. 1). Ces deux effets jouent un rôle essentiel dans la formation du terme quadratique.

L'utilisation d'un code numérique (CFD-ACE), basé sur la discrétisation spatiale de type volumes finis des équations de Navier–Stokes (Éq. (1)), a permis d'examiner, pour différents nombre de Reynolds, l'évolution des lignes de courant dans un canal poreux de section irrégulière (Fig. 2). Pour des grands nombres de Reynolds, les simulations montrent que certains jets, provenant de la zone de transport, pénètrent dans la zone stagnante (Fig. 2). Comme le bilan de masse est conservatif, les lignes de courant sortant de la zone stagnante croiseront obligatoirement les lignes de courant y pénétrant. Ainsi, les lignes de courant prennent donc la configuration d'un double faisceau de jets (Figs. 1 et 3). On propose par conséquent une méthode approximative d'estimation du terme inertiel intégral basée sur le schéma intégral. Physiquement, l'effet de l'apparition du terme quadratique dans l'équation de l'écoulement en milieu poreux est provoqué par une perte irréversible d'une partie de l'énergie cinétique des jets par dissipation visqueuse dans les zones stagnantes.

1. Introduction

One of the basic theoretical problems related to the macroscale behavior of high-rate flow in porous media which consists in establishing a universal form for the nonlinear inertial correction to Darcy's law, remains open. The first theoretical results [1–6] have shown this correction to be cubic with respect to flow velocity in the isotropic case at rather low Reynolds numbers. At high Reynolds numbers no constructive physical results have been obtained apart from some mathematical theorems showing the existence of a macroscale behavior [7]. These results do not, however, confirm the classical Forchheimer equation suggesting a quadratic correction that is well justified by multiple experimental data. Up to now the theory has not been able to explain the quadratic law. Moreover within the frameworks of traditional representations, the quadratic law may be shown to be contradicting the fundamental symmetry properties of porous media. The structure of the present paper is the following: firts of all, based on the analysis performed in [1,3,8–10] we will show three fundamental paradoxes caused by the quadratic law (Section 2); secondly, we will show the new basic physical effect (Section 3) which is responsible for the quadratic integral inertia term and provides solution to all the paradoxes. This effect has a singular mathematical nature.

2. Problem formulation: paradoxes of quadratic flow

Let us examine the single-phase, incompressible flow in a single period of the porous medium $Y = Y_p \cup Y_s$ ($Y_p \cap Y_s = \phi$), where Y_p be the porous space, while Y_s be the solid body, ∂Y_{ps} is the interface between the solid and the porous space. The flow velocity $\boldsymbol{v}$ and the pressure p verify the Navier–Stokes equations in Y_p with no-slip conditions at ∂Y_{ps} and periodicity conditions for $\boldsymbol{v}$ and $p + x_1$:

$$\rho v_i \frac{\partial \boldsymbol{v}}{\partial x_i} + \rho \frac{\partial \boldsymbol{v}}{\partial t} = -\operatorname{grad} p + \mu \Delta \boldsymbol{v}, \quad \operatorname{div} \boldsymbol{v} = 0, \quad x \in Y_p; \qquad \boldsymbol{v} = 0, \quad x \in \partial Y_{ps} \tag{1}$$

where p and $\boldsymbol{v}$ are the pressure and the flow velocity, ρ and μ are fluid density and viscosity. The initial condition is a steady-state distribution of $\boldsymbol{v}$ and p. The macroscale flow is directed along x_1. Let us assume the medium is periodic with a period ε, which is small with respect to the macroscopic size of the domain. Let the medium be isotropic at the macroscale.

All experimental data show that the macroscale equation of flow has the form of:

$$-\operatorname{grad} P = \frac{\mu}{K} \boldsymbol{V} + \boldsymbol{F}(\boldsymbol{V}) \tag{2}$$

in domain $J = \{Re_{\min}, Re_{\max}\}$ when $Re_{\min} \to 0$, whereas $Re_{\max} \sim 150$–200 for granular media, where the characteristic length in Re is the grain diameter. Parameters P and $\boldsymbol{V}$ are the macroscopic pressure and velocity; the nonlinear correction to Darcy's law, vector-function $\boldsymbol{F}$, is a quadratic form of flow velocity. The objective of this paper is to suggest an explanation of the quadratic term, taking account of the fact that the quadratic form is very hard to explain theoretically. Moreover, if we accept that the quadratic model is true, this entails three serious paradoxes with respect to classical physics.

2.1. Symmetry paradox: interdiction on even power-value of velocity

According to the vector origin of the Darcy law, any regular nonlinear correction to pressure gradient can be represented in the form of a generic series:

$$-\operatorname{grad} P = A^{(2)}_{ij} V_i \boldsymbol{e}_j + A^{(3)}_{ijk} V_i V_j \boldsymbol{e}_k + A^{(4)}_{ijkm} V_i V_j V_k \boldsymbol{e}_m + A^{(5)}_{ijkmn} V_i V_j V_k V_m \boldsymbol{e}_n + A^{(6)}_{ijkmnp} + \cdots \tag{3}$$

where V_i are the components of the macroscale flow velocity vector; $\boldsymbol{e}_i$ are the unit vectors along the axes; $A^{(n)}$ are the tensors of rank n, which constitute the intrinsic system parameters, i.e., are independent of the flow velocity components V_i. Let $A^{(n)}$ be responsible for the medium properties only. For an isotropic medium, an intrinsic tensor is non-zero if its rank is even. All the odd-rank tensors are responsible for some asymmetry or anisotropy. Hence, in an isotropic medium, Eq. (3) should take the form:

$$-\operatorname{grad} P = A^{(2)}_{ij} V_i \boldsymbol{e}_j + A^{(4)}_{ijkm} V_i V_j V_k \boldsymbol{e}_m + A^{(6)}_{ijkmnp} V_i V_j V_k V_m V_n \boldsymbol{e}_p + \cdots$$

with a consecutive simplification of the tensor structure (in isotropic media, $A^{(2)}_{ij} \to A^{(0)}$, etc.). Thus the even power-values of velocity do not appear.

However, the numerous experimental data show that the nonlinear correction is strictly quadratic (w.r.t. velocity) within a large range of Reynolds numbers whatever the degree of medium isotropy. The most general form of this experimental fact for an isotropic medium is in 3D case:

$$-\operatorname{grad} P = A^{(0)}\boldsymbol{V} + A^{(3)}_{ijk} V_i V_j \boldsymbol{e}_k \tag{4}$$

In a 1D case at the macroscale, a third-rank tensor transforms to a first-rank tensor (vector), therefore Eq. (4) takes the form:

$$-\operatorname{grad} P = A^{(0)}\boldsymbol{V} + \boldsymbol{A}^{(1)} V^2 \tag{5}$$

This is the well-known empirical Forchheimer equation, where, however, parameter $\boldsymbol{A}^{(1)}$ is a vector.

To remove this contradiction between theory and experimental data, known as the symmetry paradox [9,10], we must assume that: (i) either tensor $A^{(3)}_{ijk}$ describes not only the porous medium but also the flow pattern which may be anysotropic; or (ii) the hypothesis of flow regularity is not true.

2.2. Irreversibility paradox: dependence of inertia tensor $A^{(3)}_{ijk}$ on flow velocity

Based on experimental data, let us assume as above that the flow pattern is regular and the quadratic w.r.t. velocity law is valid. Then relation (3) or its truncated form (4) is true. Let us assume that tensor $A^{(3)}_{ijk}$ describes not only the porous medium but also the flow pattern. One particular property of an isotropic medium consists in the fact that the flow law must be invariable with respect to flow direction. This means that a simultaneous change of the sign before the pressure gradient and the velocity vector does not change the flow law. Hence we obtain immediately for Eq. (3) or (4) that tensor $A^{(3)}_{ijk}$ must depend on velocity and must satisfy the property: $A^{(3)}_{ijk}(\boldsymbol{V}) = -A^{(3)}_{ijk}(-\boldsymbol{V})$. In a 1D case the same property should be rewritten for vector $\boldsymbol{A}^{(1)}$.

However in a regular series (3), tensor $A^{(3)}_{ijk}$ and all other tensors $A^{(n)}$ are independent of velocity, by definition of a regular function. This is a second contradiction between theory and experiment.

2.3. D'Alembert paradox: full annihilation of integral inertia terms for regular flow

Let the surface ∂Y_p of sub-domain Y_p consist of two non-intersecting parts: $\partial Y_p = \partial Y_{ps} \cup \partial Y^*$, where ∂Y_{ps} is the interface between the solid and the porous space, and ∂Y^* is that part of the external boundary of the whole period Y which crosses sub-domain Y_p. Let the flow velocity $\boldsymbol{v}$ defined in (1) be regular everywhere in $Y_p \cup \partial Y_p$. It then becomes easy to show that $\langle v_i \partial \boldsymbol{v}/\partial x_i \rangle_{Y_p} = 0$, where symbol $\langle \cdot \rangle_{Y_p}$ stands for the value averaged over domain Y_p. Indeed:

$$\begin{aligned}\left\langle v_i \frac{\partial \boldsymbol{v}}{\partial x_i}\right\rangle_{Y_p} &= \left\langle \frac{\partial (v_i \boldsymbol{v})}{\partial x_i} - \boldsymbol{v}\frac{\partial v_i}{\partial x_i}\right\rangle_{Y_p} = \left\langle \frac{\partial (v_i \boldsymbol{v})}{\partial x_i}\right\rangle_{Y_p} = \int_{Y_p} \frac{\partial (v_i \boldsymbol{v})}{\partial x_i}\,\mathrm{d}x = \int_{\partial Y_p} v_n \boldsymbol{v}\,\mathrm{d}x \\ &= \int_{\partial Y_{ps}} v_n \boldsymbol{v}\,\mathrm{d}x + \int_{\partial Y^*} v_n \boldsymbol{v}\,\mathrm{d}x = 0\end{aligned} \tag{6}$$

The integral along ∂Y_{ps} is nil due to the no-slip conditions, the integral along ∂Y^* is zero due to the periodicity conditions.

The value inside the angle brackets in the left-hand side of (6) is the inertia term in the Navier–Stokes equations (1).

This theorem says that the integral inertia effect in porous medium is zero, whatever the structure of flow inside the porous space. In other words, various manifestations of the inertia effects such as flow acceleration/deceleration, inertial shift and streamline deformation, formation of vortexes, formation of stagnant zones – do not explain the nonlinear term in Eq. (2) (as the averaged magnitude of all these effects is nil) while the flow remains regular. Note that condition (6) remains valid for any structure of porous medium, isotropic or not.

This theorem is a modification of the well-known D'Alembert paradox for ideal flow around a spherical particle in an infinite domain, which says that the total resistance force is nil (if the flow remains regular).

Thus, the experiment shows the existence of a nonlinear term in the integral equation, whilst the theory says that this term should be zero.

Physically, relation (6) means that the total kinetic energy of regular periodic flow is conserved whatever the local kinetic energy variations. In other words, all the variations of the kinetic energy are totally reversible.

3. Explanation of the paradoxes. Singulary mechanisms to form the macroscale inertia terms

3.1. Formal procedure to obtain the quadratic correction

All the three paradoxes are based on the common hypothesis about the regular structure of flow velocity field and the periodic boundary conditions. In [7] it has been proved that a periodic solution to the stationary Navier–Stokes equations in a periodic porous medium exists in a very large range of *Re*. Our numerical experiments entirely affirm the periodic structure of flow in a periodic medium at any Reynolds numbers at least while the flow remains laminar. Thus, the unique way to resolve the paradoxes is to exclude the hypothesis about flow regularity. Then the singular expansion (4) or (5) may take place at the macroscale, which removes the irreversibility and symmetry pradoxes. It remains to explain the D'Alembert paradox.

It can be seen from (6) that the integral inertia term is nonzero if one of two possibilities occurs: (i) if only a point exists where the velocity is discontinuous, or (ii) if only a point exists where the velocity is infinite but the product $v_i\,\partial \boldsymbol{v}/\partial x_i$ is integrable. In the first case, the derivative of velocity is a delta-function, and integration (6) yields a nonzero value. Any velocity jump is admissible only when viscosity becomes negligible. Otherwise the Laplace operator in the Navier–Stokes equations equals the derivative of the delta-function at the point of discontinuity. Hence the integral viscosity term is diverging. However discontinuity is probable at very high Reynolds numbers. A typical example is the shock of a highly accelerated jet against the pore wall. Let M be the discontinuity surface. Let the flow velocity sharply decrease from the finite value $\boldsymbol{v}_M$ just before surface M up to zero just behind it. Eq. (8) then yields the following value for the integral inertia term: $\langle v_i\,\partial \boldsymbol{v}/\partial x_i\rangle = \int_M v_{nM}\boldsymbol{v}_M\,\mathrm{d}x$, where v_{nM} is the normal flow velocity just before the jump. If we assume that $\boldsymbol{v}_M = \lambda \boldsymbol{V}$ (λ does not depend of $\boldsymbol{v}$), then $\boldsymbol{F}(\boldsymbol{V}) = V^2\boldsymbol{A}^{(1)}$, where $\boldsymbol{A}^{(1)} = \int_M \lambda^2(x)\boldsymbol{n}_M\,\mathrm{d}x$, $\boldsymbol{n}_M$ is a unit normal vector to surface M. Thus, the shock of a jet determines the quadratic integral form of function $\boldsymbol{F}(\boldsymbol{V})$ in (2).

From the physical point of view, the velocity discontinuity means that the kinetic energy of fluid is sharply and irreversibly lost (or increased) at this point. Such an irreversible loss of the fluid energy determines the appearance of the nonlinear term in the macroscopic flow equation.

From this point of view, not any singularity of the flow field can explain the annihilation paradox. In particular, the well-known phenomenon of jet separation from the solid body does not cause the macroscopic inertia effects, as the velocity in the separation point is zero, although the velocity derivative in infinitely high.

The second case is that of an infinite velocity with an integrable product $V_i\,\partial \boldsymbol{V}/\partial x_i$. The typical example of such a structure is a "bundle of jets" which is formed by a set of jets focused into a single jet or leaving a jet in such a way that all the streamlines remain smooth in the contact point. The jet bundle is shown in Fig. 1. According to the mass conservation principle, the flow velocity along the united jet must be infinite. Contact point N is singular as the flow velocity becomes infinite there (in a continuous way). If s is the length of a streamline of the bundle, then the inertia term can be written along the streamline as the product $V_s\,\partial \boldsymbol{V}/\partial x_s$. Therefore the inegrability requirements for this product determine the admissible types of streamline singularity at the contact point.

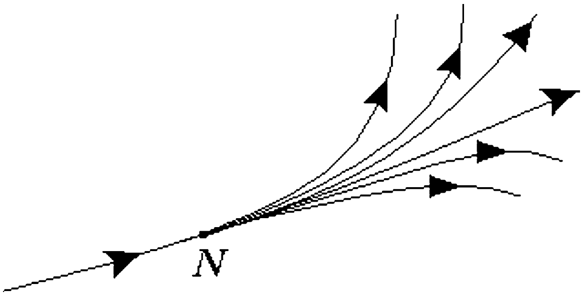

Fig. 1. A bundle of jets.

Fig. 1. Un faisceau de jets.

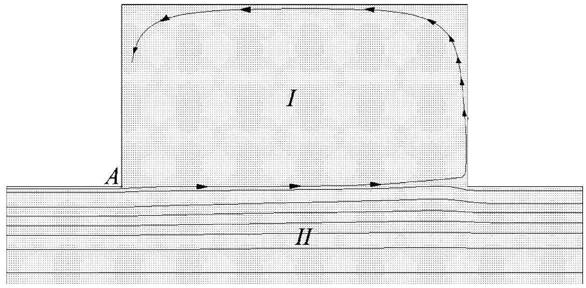

Fig. 2. Invasion of a transport jet into the stagnant zone (numerical simulation).

Fig. 2. Invasion d'un jet de transport dans la zone stagnante (simulation numérique).

3.2. Effect of jet invasion into the stagnant zone

Using a commercial package CFD-ACE based on the finite volume method, we have analyzed the structure of Navier–Stokes flow at various Reynolds numbers in a periodic pore channel of non-uniform cross section as in Fig. 2, where a single medium period is presented (the flow is directed from the left to the right). It is known that starting from a critical value of *Re*, a liquid jet is separating from the pore wall near point A. The separation streamline divides the flow into two domains: the transport zone II and the stagnant zone I. Although the liquid in the stagnant zone is engaged in a vortex rotation, this movement does not make any contribution to transport through the channel. The traditional flow pattern assumes that no mass exchange between these zones exists, in such a way that a non-renewable mass of liquid is rotating in the stagnant zone.

Our numerical experiments show another result. Starting from a critical Reynolds number (which is sufficiently higher than 1), at least one jet coming from the transport zone enters into the stagnant zone, as illustrated in Fig. 2.

The effect of a laminar jet invasion into the stagnant zone has been observed experimentally [11]. It is easy to show that this effect gives rise to the formation of velocity discontinuities.

3.3. Conservative scheme of jet invasion

According to the principle of mass conservation, if a mass of liquid enters the stagnant zone, the same mass must leave it. Geometrically this is possible only if two jets cross one other. The general approximate scheme ensuring such a conservative jet invasion is shown in Fig. 3.

Generally speaking, two points of jet branching may exist within this scheme, M and N. The jet entrance at point M and the jet exit from point N is smooth, as said above. If any other jet enters the stagnant zone, it is forced to passe across point M and N. Otherwise the intersection of two jets, b and c, will be non-smooth, which is impossible. Thus, in general case, the infinite number of jets may enter the stagnant zone, by forming two jet bundles related by a single streamline MN. A bundle of jet exits from point N, and a second jet bundle enters in point M. Points M and N, being those of velocity discontinuity, contribute to the non-zero integral inertia term in (2).

This conservative scheme of bundle formation is however a steady-state approximation to the true pattern. In reality, the infinite velocity at surface MN is impossible, therefore the excessive liquid withdrawal from the stagnant zone is performed by means of a rupture of interface MN and an exit of isolated drops through these ruptures. Due to this the interface is oscillating in the time.

Note that the d'Alembert paradox in a problem of flow around a spherical particle was explained in [12] using the idea of a reverse jet encroachment on the stagnant zone formed just behind the particle. This jet was considered as a branch of the streamline separated from the solid body. However this scheme was not conservative, as no jet mechanism leaving the stagnant zone was suggested.

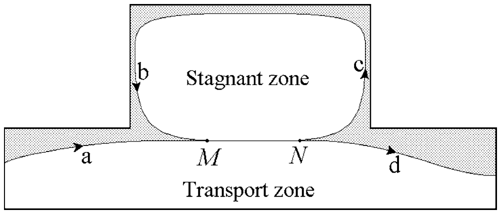

Fig. 3. Conservative scheme of jet invasion into the stagnant zone.

Fig. 3. Schéma conservatif d'invasion du jet dans la zone stagnante.

3.4. Approximate calculation scheme for the integral inertia term

The suggested conservative scheme of jet encroachment on the stagnant zone resolves all the paradoxes. A justification that this scheme really leads to a quadratic correction to Darcy's law can be obtained in the same way, as it is done traditionally for the D'Alembert paradox in the theory of ideal flow around solid body [11–13]. This traditional way is based on an integral approach.

Let us examine the domain between the upper and the lower streamlines forming the double jet bundle as shown in Fig. 3. Let v_a, v_b, v_c and v_d be the flow velocity in the appropriate jet of this bundle averaged over the jet cross section in the close vicinity of contact points M and N. Let v_M^+ and v_N^- be the velocity behind point M and before point N. Then we obtain for this domain:

$$\left\langle \frac{v_i \partial \boldsymbol{v}}{\partial x_i} \right\rangle = \frac{1}{2L}\left\lfloor \left(v_M^+\right)^2 - \left(v_N^-\right)^2 \right\rfloor \boldsymbol{e} \approx \frac{1}{4L}\left\lfloor \left(v_c^2 + v_d^2\right) - \left(v_a^2 + v_b^2\right) \right\rfloor \boldsymbol{e}$$
$$= \frac{1}{4L}\left\lfloor v_c^2 + v_d^2 - v_a^2 - v_b^2 \right\rfloor \boldsymbol{e}$$

where L is a characteristic length between points M and N, $\boldsymbol{e}$ is the unit vector directed along MN.

Due to periodicity, $v_a \approx v_d$. Point M is situated in a much slower flow domain, the kinetic energy there is completely lost due to the high viscous dissipation along the overall trace of jet $c-b$ in the stagnant zone. Therefore $v_b \approx 0$. It is clear that v_c is proportional to the averaged velocity: $v_c \approx \lambda V$. Finally, the integral inertia term is approximately equal to $\lambda V^2 \boldsymbol{e}/4L$ or to the kinetic energy of that branch of the transport jet which is lost in the stagnant zone. Thus, the macroscopic integral inertia effect is quadratic in velocity.

4. Conclusions

The general nonlinear law of flow through porous media can now be represented in the following way. At rather low Reynolds numbers the nonlinear correction is due to the coupled inertia-viscous effect which consists in the variation of the integral viscous dissipation caused by the inertia deformation of the streamline pattern. The nonlinear term is close to cubic and is rather low. The pure integral inertia effect is zero due to the regularity of flow. Starting from a critical Reynolds number, which is very different for various medium structures but is close to that of the flow separation, the streamline pattern becomes unstable and singular. The singularities are related with ruptures of the interface between the transport and the stagnant zones. The averaged (in the time) behavior of the system is steady-state. Starting from a second critical Reynolds number, this oscillating flow pattern becomes independent of *Re*. As the streamline pattern is stabilized, the coupled viscous-inertial effect remains invariable w.r.t. *Re*. The nonlinear correction has now another physical nature, being caused by the singular effect of transport jet encroachement on the stagnant zone which may be formally described as double jet bundle formation. Thus, this term is now quadratic. In this range of *Re*, the apparent permeability of flow is different from the real permeability appropriated to a purely viscous flow, due to various viscous dissipation characterizing these two limit cases of flow.

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