Available online at www.sciencedirect.com
SCIENCE DIRECT
ELSEVIER
C. R. Mecanique 331 (2003) 149–156
COMPTES RENDUS
MECANIQUE

On the stability of a Bingham fluid flow in an annular channel

Sur la stabilité de l'écoulement d'un fluide de Bingham dans une conduite annulaire

Nadjiba Kabouya, Chérif Nouar

LEMTA UMR 7563 (CNRS–INPL–UHP), INPL, ENSEM, 2, avenue de la Forêt de Haye, BP 160, 54504 Vandoeuvre-lès-Nancy, France

Received 12 June 2002; accepted after revision 3 December 2002

Presented by Évariste Sanchez-Palencia

Abstract

Linear stability of a fully developed Bingham fluid flow between two coaxial cylinders subject to infinitesimal axisymetric perturbations is investigated. The analysis leads to two uncoupled Orr–Sommerfeld equations with appropriate boundary conditions. The numerical solution is obtained using fourth order finite difference scheme. The computations were performed for various plug flow dimensions and radii ratios. Within the range of the parameters considered in this paper, the Poiseuille flow of Bingham fluid is found to be linearly stable. ***To cite this article: N. Kabouya, C. Nouar, C. R. Mecanique 331 (2003).***

Résumé

Une analyse de stabilité linéaire de l'écoulement de Poiseuille d'un fluide de Bingham dans une conduite annulaire vis à vis de perturbations infinitésimales axisymétriques est effectuée. L'analyse conduit à deux équations d'Orr–Sommerfeld découplées avec des conditions aux limites appropriées. La solution numérique est obtenue en utilisant un schéma aux différences finies d'ordre 4. Les calculs ont été effectués pour plusieurs dimensions de la zone « bouchon » et différents rapports de rayons. Dans la gamme de variation des paramètres considérés, nous n'avons pas trouvé d'instabilité. ***Pour citer cet article : N. Kabouya, C. Nouar, C. R. Mecanique 331 (2003).***

Keywords: Fluid mechanics; Linear stability; Bingham fluid

Mots-clés : Mécanique des fluides ; Stabilité linéaire ; Fluide de Bingham

E-mail address: cnouar@ensem.inpl-nancy.fr (C. Nouar).

1631-0721/03/$ – see front matter © 2003 Académie des sciences/Éditions scientifiques et médicales Elsevier SAS. Tous droits réservés.
doi:10.1016/S1631-0721(02)00015-3

Version française abrégée

Introduction

La présente étude est une contribution à l'analyse de la stabilité de l'écoulement de Poiseuille d'un fluide de Bingham. Dans un article récent, une analyse non linéaire a été effectuée par Nouar et Frigaard [1]. Les conditions critiques en dessous desquelles l'énergie cinétique d'une perturbation d'amplitude finie décroît en fonction du temps de façon monotone ont été déterminées. Afin de mieux appréhender les mécanismes de transition laminaire-turbulent il est nécessaire d'après Drazin [2], d'examiner le comportement de l'écoulement vis à vis de perturbations infinitésimales. C'est l'objectif de cet article. Il traite la stabilité de l'écoulement de Poiseuille d'un fluide de Bingham dans une conduite annulaire, vis à vis de perturbations infinitésimales. Bien que le théorème de Squire ne s'applique pas à cette étude, seul le cas axisymétrique est considéré. La littérature fournit des résultats dans deux situations limites. D'abord, dans le cas limite d'un fluide Newtonien en géométrie annulaire ; on retient globalement, que le nombre de Reynolds critique Re_c augmente lorsque le rapport des rayons $K = R_1/R_2$ diminue [3]. Ensuite, dans le cas limite de l'écoulement d'un fluide de Bingham entre deux plaques parallèles ; Frigaard et al. [4] ont déterminé les courbes de stabilité neutre et indiquent que le nombre de Reynolds critique augmente lorsque le nombre de Bingham B (rapport de la contrainte seuil à une contrainte visqueuse nominale) croît. Ce deuxième cas limite est réexaminé. On montre que les conditions aux limites utilisées dans [4] ne sont pas appropriées et qu'en fait l'écoulement de Poiseuille plan d'un fluide de Bingham est linéairement stable. Les résultats obtenus dans le cas d'une conduite annulaire, pour différents rapports de rayons, et pour différentes valeurs de B indiquent que l'écoulement de Poiseuille d'un fluide de Bingham reste stable vis à vis de perturbations infinitésimales axisymétriques.

Analyse linéaire de stabilité

On considère l'écoulement d'un fluide de Bingham dans une conduite annulaire Fig. 1. Les rayons adimensionnels des cylindres intérieur et extérieur sont notés respectivement R_1 et R_2. En régime laminaire établi, l'écoulement est caractérisé par la présence d'une zone dite « zone bouchon » où le matériau se déplace en bloc comme un solide rigide (modèle de Bingham). Cette zone est délimitée par les interfaces R^- du côté du cylindre intérieur et R^+ du côté du cylindre extérieur. La dimension relative de la zone bouchon est définie par $ap = (R^+ - R^-)/(R_2 - R_1)$. L'évolution de ap en fonction de B est décrite par a Fig. 2. A l'écoulement de base, on superpose une perturbation infinitésimale axisymétrique, Éq. (6). La linéarisation des équations du mouvement et la décomposition en modes normaux de la perturbation Éq. (13) conduisent à deux problèmes aux valeurs propres découplés : un problème aux valeurs propres dans la zone interne délimitée par R_1 et R^-, et un autre dans la zone externe délimitée par R^+ et R_2. L'écoulement est dit stable lorsque $C_i = \mathrm{Im}(C) < 0$, où C est la vitesse complexe de l'onde.

Résolution numérique, résultats et discussion

L'équation d'Orr–Sommerfeld (15) munie des conditions aux limites (16), est traitée par un schéma aux différences finies d'ordre 4. Le système aux valeurs propres généralisé obtenu est résolu par la méthode QZ implémentée dans MATLAB V. On s'intéresse particulièrement au mode le moins stable, $C_{i,\max}$. Le code de calcul est d'abord validé dans le cas particulier d'un fluide Newtonien, $ap = 0$. Ensuite, le cas de l'écoulement de Poiseuille plan d'un fluide de Bingham est réexaminé. Il est montré que les fonctions propres u doivent nécessairement satisfaire les conditions $u = Du = 0$ à l'interface liquide-solide, comme indiqué dans la Fig. 3. La représentation Fig. 4 des contours de $C_{i,\max}$ indique que l'écoulement est linéairement stable. Des résultats similaires sont obtenus dans le cas annulaire pour différents rapports de rayons, et pour les zones interne et externe. A titre d'exemple, la Fig. 5 montre pour $K = 0.5$ et $ap = 0.6$, la répartition des contours $C_{i,\max}$ obtenus dans la zone externe. L'augmentation rapide de l'écart entre les différentes courbes lorsque $C_{i,\max}$ croît est caractéristique de la stabilité de l'écoulement. En fait, dans la gamme considérée de variation du nombre d'onde axial, $0 < \hat{\alpha} \leqslant 5$, du nombre de Reynolds, $\widehat{Re} \leqslant 10^5$ et pour $0.3 \leqslant K \leqslant 0.9$ et $0.1 \leqslant ap \leqslant 0.9$, nous n'avons pas trouvé d'instabilité. L'effet de *ap* sur la stabilité de l'écoulement peut être apprécié à travers la Fig. 6. Des contours de $C_{i,\max}$ sont représentés dans le plan (ap, Re) pour $\alpha = 1$. L'allure des courbes montre clairement que l'écoulement est d'autant plus stable que *ap* est important.

1. Introduction

The present study is a contribution to the stability analysis of the Poiseuille flow of Bingham fluid. Recently, nonlinear analysis was performed for plane channel and Hagen–Poiseuille flows, by Nouar and Frigaard [1], using energy method. The critical Reynolds number below which the kinetic energy of any finite amplitude disturbance decays monotonically in time, was determined. For understanding the mechanisms of laminar-turbulent transition, according to Drazin [2], it is necessary to analyze the flow stability on the basis of infinitesimal perturbations.

The present paper deals with the linear stability of the Poiseuille flow of Bingham fluid in an annular duct subject to infinitesimal perturbations. The plane channel geometry is recovered when the radius ratio $K = R_1/R_2 \to 1$. The Hagen–Poiseuille flow is not relevant, since the Newtonian flow is linearly stable. Although Squire's transformation cannot be used for our problem, it is nevertheless easier, to start with axisymetric situation. The literature provides us stability results for two limit situations. The first one concerns the Newtonian fluid flow in an annular channel. Mott and Joseph [3] showed that the critical Reynolds number, Re_c is a monotonic function of the radius ratio K, increasing without bound as $K \to 0$ (Hagen–Poiseuille flow). The second limit deals with the linear stability of Plane Poiseuille flow of Bingham fluid. Due to inappropriate boundary conditions, Frigaard et al. [4] find that the flow is linearly unstable above a certain Re_c. Neutral stability curves are obtained for various values of Bingham numbers, B (yield stress to nominal viscous shear stress ratio). This second case is revised here by using the correct boundary conditions. Then after, computations were performed for the annular geometry and several values of K and B. The results indicate that the Poiseuille flow of Bingham fluid in plane channel or in annular duct remains linearly stable to axisymmetric infinitesimal perturbations even when the Bingham number tends to zero.

2. Bingham–Poiseuille flow

We consider the flow of an incompressible Bingham fluid with a yield stress τ_s and a plastic viscosity μ_0 in an annular channel. The governing equations in dimensionless form are:

$$\nabla \cdot \mathbf{V} = 0 \tag{1}$$

$$\frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V} \cdot \nabla)\mathbf{V} = -\nabla p + \nabla \cdot \boldsymbol{\tau}(\mathbf{V}) \tag{2}$$

Here $\mathbf{V}$ is the velocity, p is the pressure and $\boldsymbol{\tau}$ is the deviatoric extra-stress tensor. The velocity vector is of the form $\mathbf{V} = u\mathbf{e}_r + v\mathbf{e}_\theta + w\mathbf{e}_z$, where u, v, w are the velocity components, and $\mathbf{e}_r$, $\mathbf{e}_\theta$, $\mathbf{e}_z$ are unit vectors in the radial r, circumferential θ, and axial z directions, respectively. The above equations are rendered dimensionless using half width of the annular space $(R_2^* - R_1^*)/2$ as the length scale, the maximum velocity W_0^* of the basic flow as velocity scale and ρW_0^{*2} for stress and pressure scale.

The constitutive equation (Oldroyd [5]), can be written after scaling as:

$$\boldsymbol{\tau} = \frac{1}{Re}\left[1 + \frac{B}{D_{\mathrm{II}}}\right]\mathbf{D} \Leftrightarrow \tau_{\mathrm{II}} > \frac{B}{Re} \tag{3}$$

$$D_{\mathrm{II}} = 0 \Leftrightarrow \tau_{\mathrm{II}} \leqslant \frac{B}{Re} \tag{4}$$

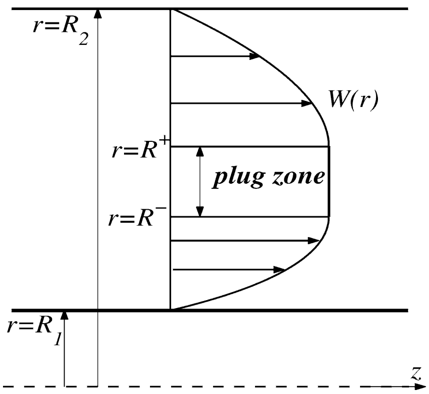

Fig. 1. Axial flow of Bingham fluid between two coaxial cylinders.

Fig. 1. Écoulement axial d'un Fluide de Bingham entre deux cylindres coaxiaux.

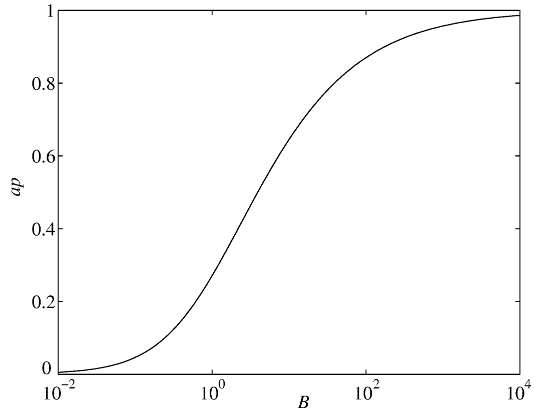

Fig. 2. Dimensionless plug zone width *ap* as a function of Bingham number: $K = 0.5$.

Fig. 2. Évolution de *ap* en fonction du nombre de Bingham pour $K = 0{,}5$.

where D_{II} and τ_{II} are respectively the second invariant of the strain rate $\mathbf{D}$ and of the deviatoric stress tensor $\boldsymbol{\tau}$. The quantity $(1 + B/D_{\mathrm{II}})$ is a dimensionless effective viscosity. The dimensionless parameters B and Re are respectively the Bingham and the Reynolds number. They are defined by:

$$B = \frac{\tau_s(R_2^* - R_1^*)}{2\mu_0 W_0^*}, \quad Re = \frac{\rho W_0^*(R_2^* - R_1^*)}{2\mu_0} \tag{5}$$

For one-dimensional shear flow, only the axial component of velocity W is non-zero, depending on the radial coordinate. The analytical expression of the axial velocity profile $W(r)$ as well as the details of the calculations can be found in the paper of Bird et al. [6] and Fordham et al. [7]. An axial velocity profile is illustrated in Fig. 1. The dimensionless radius of the inner and outer cylinders are respectively denoted by R_1 and R_2. At the interfaces R^- and R^+, between the liquid-like zone and the solid-like zone: $|\tau_{rz}| = \tau_s$. One has to note that the interface is not a material surface. Fig. 2, shows the evolution of the relative dimension of the plug core defined by $ap = (R^+ - R^-)/(R_2 - R_1)$ as a function of B for $K = 0.5$. Calculations performed for K ranging between 0.1 and 0.9 indicate a very slight effect of K on *ap*.

3. Linear stability analysis

Following the usual linear stability analysis, an infinitesimal perturbation $(\varepsilon\mathbf{v}', \varepsilon \mathrm{p}')$, having an amplitude $(\varepsilon \ll 1)$, is imposed on the basic flow $(\mathbf{V}, P)$. The perturbed flow is then given by:

$$(\mathbf{V} + \varepsilon\mathbf{v}', p) = (\varepsilon u', \varepsilon v', W + \varepsilon w', P + \varepsilon p') \tag{6}$$

Hereafter, it is assumed that the disturbance is axisymmetric. Wherever the yield stress is exceeded, the effective viscosity of the perturbed flow is expanded about the basic flow:

$$\mu(\mathbf{V} + \varepsilon\mathbf{v}') = 1 + \frac{B}{|DW|} - \varepsilon\left(\frac{\partial w'}{\partial r} + \frac{\partial u'}{\partial z}\right)\frac{B}{DW|DW|} \tag{7}$$

where, $D \equiv \mathrm{d}/\mathrm{d}r$. It is clear that $|\tau_{ij}(\mathbf{V} + \varepsilon\mathbf{v}') - \tau_{ij}(\mathbf{V})| = \mathrm{O}(\varepsilon)$, which means that the perturbed flow can only linearly perturb the yield surface position from their initial position:

$$r^{\pm}(\mathbf{V} + \varepsilon\mathbf{v}') = R^{\pm}(\mathbf{V}) + \varepsilon h^{\pm}(z, t) \tag{8}$$

The linear perturbation equations in the two yielded regions are derived by substitution of (7) into the momentum and continuity equations by retaining only terms of order ε. The boundary conditions, for the perturbation velocity, are obtained from the nonslip and nonpenetration conditions at the walls.

$$u' = v' = w' = 0 \quad \text{at } r = R_1 \text{ and } r = R_2 \tag{9}$$

The continuity of stress at the yield surface requires that $D_{\mathrm{II}}(\mathbf{V} + \varepsilon \mathbf{v}') = 0$. Therefore,

$$D_{ij}(\mathbf{V} + \varepsilon \mathbf{v}') = 0 \quad \text{at } r = r^{\pm},\ \forall i, j \tag{10}$$

Expanding and linearising about $R^{\pm}$ give:

$$u'_{,r}(R^{\pm}, z, t) = w'_{,z}(R^{\pm}, z, t) = u'(R^{\pm}, z, t) = 0 \tag{11}$$

$$w'_{,r}(R^{\pm}, z, t) + u'_{,z}(R^{\pm}, z, t) = h^{\pm}\frac{\mathrm{d}^2 W}{\mathrm{d}r^2}(r = R^{\pm}) \tag{12}$$

In the normal modes theory, the solutions are of the following form:

$$(u', v', w', p', h^{\pm}) = \big(u(r), v(r), w(r), p(r), h^{\pm}\big) \times \mathrm{e}^{\mathrm{i}\alpha(z - Ct)} \tag{13}$$

Where, α, is the axial wave number, $C = C_r + \mathrm{i}C_i$ is the complex wave speed and αC_i is the growth rate of the perturbation. Substituting Eq. (13) into the linearized perturbation equations and eliminating the pressure terms, yields the Orr–Sommerfeld equation for Bingham fluid in each yielded region. With increasing the Bingham number, the width $(1 - ap)$ of the sheared fluid zones decreases. To take into account this geometrical effect, the following reduced parameters are introduced:

$$\hat{\alpha} = \alpha \times (1 - ap), \quad \widehat{Re} = Re \times (1 - ap), \quad \widehat{B} = B \times (1 - ap), \quad r = \eta \times (1 - ap) \tag{14}$$

This last transformation appears more naturally in the case of plane channel, when the yielded zone is mapped in $[0, 1]$. Finally, the Orr–Sommerfeld equation is obtained as follows:

$$[W - C]\,Lu + \left(\frac{\widehat{D}W}{\eta} - \widehat{D}^2 W\right)u = -\frac{\mathrm{i}}{\hat{\alpha}\widehat{Re}}L^2 u + 2\frac{\mathrm{i}\hat{\alpha}\widehat{B}}{\widehat{Re}}\left[2\frac{\mathrm{d}}{\mathrm{d}\eta}\left(\frac{\widehat{D}(\eta u)}{\eta|\widehat{D}W|}\right) - \frac{u}{\eta}\frac{\mathrm{d}}{\mathrm{d}\eta}\left(\frac{1}{|\widehat{D}W|}\right)\right] \tag{15}$$

where, $\widehat{D} \equiv \mathrm{d}/\mathrm{d}\eta$ and $L \equiv (1/\eta)\widehat{D}(\eta\widehat{D}) - \alpha^2 - 1/\eta^2$.

The boundary conditions at the walls, $\eta = \eta_1$ and $\eta = \eta_2$, and at the interfaces, $\eta = \eta^-$ and $\eta = \eta^+$, are:

$$u = \widehat{D}u = 0 \tag{16}$$

The substitution of Eq. (13) into Eq. (11) indicates also that $w = 0$ at the interfaces. Thus the motion of the unyielded region of fluid is not perturbed at the leading order. Therefore, the stability problems (Eqs. (15) and (16)) in the internal fluid region, $[\eta_1, \eta^-]$, and in the external fluid region, $[\eta^+, 1]$, are completely uncoupled. Thus, they can be analyzed separately.

Remark. Similar Orr–Sommerfeld equation can be derived for plane Poiseuille flow [4]:

$$[W - C]\mathcal{L}u - \big(D^2 W\big)u = -\frac{\mathrm{i}}{\alpha Re}\mathcal{L}^2 u + \frac{4\mathrm{i}\alpha B}{Re}D\left[\frac{Du}{|DW|}\right] \tag{17}$$

Here, $D \equiv \mathrm{d}/\mathrm{d}y$ and $\mathcal{L} \equiv D^2 - \alpha^2$. The differential equation (17) is defined in the flow domain $\mathcal{D}$, delimited by the interfaces at $y = \pm a$ and the channel walls at $y = \pm 1$, i.e., $\mathcal{D} = [-1, -a] \cup [a, 1]$. The boundary conditions are:

$$u(\pm a) = Du(\pm a) = u(\pm 1) = Du(\pm 1) = 0 \tag{18}$$

Eq. (17) is symmetric and the boundary conditions (18) are homogeneous, then the solution may be split up into an odd part u_i and an even part u_p: $u = u_p + u_i$. Hence, if $u(\pm a) = 0$, then: $u_p(\pm a) = 0$ and $u_i(\pm a) = 0$, separately.

Similarly for the first derivative: i.e., $Du_p(\pm a) = 0$ and $Du_i(\pm a) = 0$. It is clear that the two fluid regions are uncoupled and equivalent. Hence, for the Bingham fluid, it is sufficient to consider only one region, $[a, 1]$ say. Following previous studies for Newtonian fluids, Frigaard et al. [4] want to focus only on the even modes. For this, they use the same procedure as for Newtonian fluids by considering $Du_p(a) = D^3u_p(a) = 0$, instead of $u_p(a) = Du_p(a) = 0$. To our best knowledge, the obtained results are not correct for two reasons: (i) the condition $u_p(a) = 0$ is not satisfied; (ii) the condition $D^3u_p(a) = 0$, cannot be used here, since the interface is not a symmetry axis. Actually, the distinction between the odd and even solutions for the Bingham fluid is not useful, since the two fluid regions are uncoupled.

4. Numerical method

Following Mott and Joseph's [3] method, the eigenvalue problem (15), (16) is solved using finite difference method with a fourth order centered scheme. In the present analysis, C was treated as the eigenvalue for given value of α and *Re*. The resulting generalized eigenvalue problem was solved using QZ-algorithm implemented in MATLAB V. For a good accuracy, after several tests, it was found that $N = 400$ is the optimal number of subdivisions between the two cylinders.

5. Results and discussion

In order to validate our MATLAB code, calculations are first performed for Newtonian fluid ($B = 0$). The critical Reynolds number is determined for different radii ratio $0.3 \leqslant K \leqslant 0.9$. The difference between our results and those obtained by Mott and Joseph [3] is within 0.5%. The case of Plane Poiseuille flow of Bingham fluid is then considered using appropriate boundary conditions. Fig. 3 represents the real and imaginary parts of the eigenfunction u for the least stable mode corresponding to $\hat{\alpha} = 1$, $ap = 0.6$ and $\widehat{Re} = 10^4$. The domain $[a, 1]$ is mapped into $[0, 1]$. For comparison, the corresponding results for Newtonian fluid, u_{new}, are plotted in the same figure. It is important to note that $u_{\text{new}}(\eta = 0) \neq 0$, however for Bingham fluid $u(\eta = 0) = 0$. In Fig. 4, results are displayed for $ap = 0.6$, in terms of contours of $C_{i,\max}$, (least stable mode). For fixed axial wave number, the rapid increase

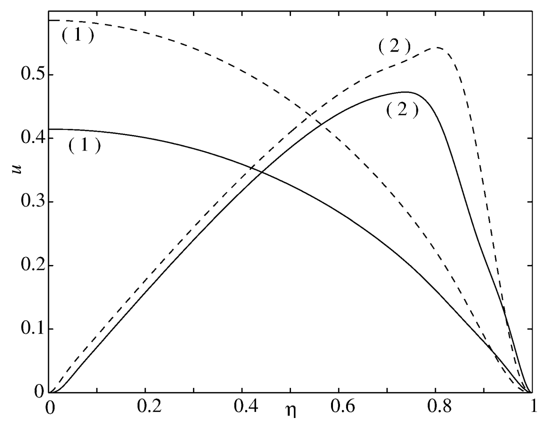

Fig. 3. Real (continuous line) and imaginary (dashed line) parts of the eigenfunction $u(r)$: (1) Newtonian fluid; (2) Bingham fluid.

Fig. 3. Partie réelle (ligne continue) et imaginaire (tirets) de la fonction propre $u(r)$: (1) Fluide Newtonien ; (2) Fluide de Bingham.

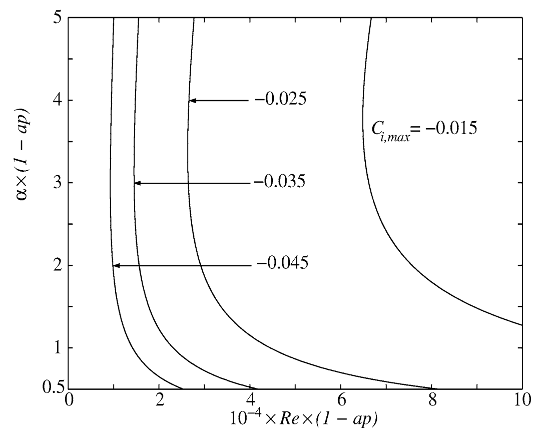

Fig. 4. Contours of $C_{i,\max}$ (least stable mode). Case of Plane Poiseuille flow of Bingham fluid with $ap = 0.6$.

Fig. 4. Contours de $C_{i,\max}$ (le mode le moins stable) : Cas de l'écoulement de Poiseuille plan d'un fluide de Bingham avec $ap = 0{,}6$.

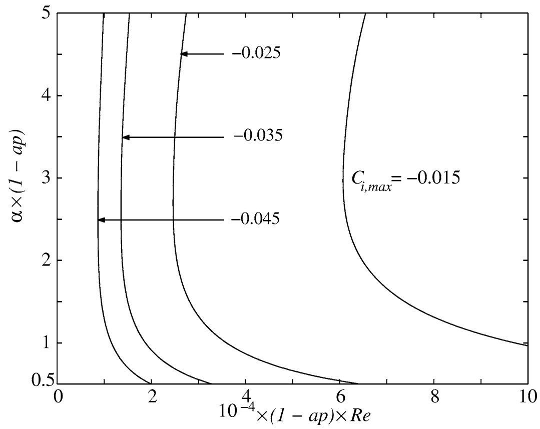

Fig. 5. $C_{i,\max}$ contours for $K = 0.5$ and $ap = 0.6$.

Fig. 5. Contours de $C_{i,\max}$ pour $K = 0{,}5$ et $ap = 0{,}6$.

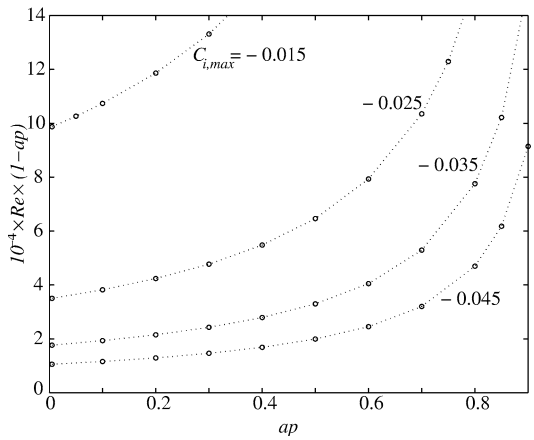

Fig. 6. Effect of the plug zone thickness *ap*, on the flow stability.

Fig. 6. Effet de l'épaisseur *ap*, de la phase « solide » sur la stabilité de l'écoulement.

in the spacing between the curves as $C_{i,\max}$ is increased, indicates that the flow is linearly stable. In fact, within the range of parameters considered here ($\hat{\alpha} \leqslant 5$, $\widehat{Re} \leqslant 10^5$), instabilities do not exist. Similar results are obtained for an annular duct with different radii ratios, in the internal and external fluid regions. As an example, Fig. 5 gives for the external fluid region, contours of $C_{i,\max}$, for a radius ratio of 0.5 and $ap = 0.6$. As for plane channel, the characteristic of the flow stability is clearly shown by the spacing between the contours.

The effect of the plug zone thickness on the flow stability for a given perturbation characterized by an axial wave number α can be appreciated through Fig. 6. Contours of $C_{i,\max}$ for $\alpha = 1$, are depicted in the plane $(ap, \widehat{Re})$. For each contour, $\widehat{Re}$ increases with increased *ap*, indicating that the larger *ap* is, the stronger the stability. Similar variations are observed if one considers others values of α. Let us consider now the case where $ap \to 0$, i.e., $B \to 0$. Fig. 6 shows for the lowest value of ap considered here, $ap = 0.005$, that the flow remains linearly stable, contrary to the Newtonian case. We are not surprised by this result, in fact, as long as $ap \neq 0$, there is a plug zone, and the condition $D_{\mathrm{II}} = 0$ has to be satisfied at the solid–liquid interface to ensure continuity of stress. This condition leads to $u = 0$ at the interface, while for Newtonian fluid u_{new} is maximum as shown in Fig. 3. Therefore, the eigenfunction (for the least stable mode) for Bingham fluid as $B \ll 1$ is different from that corresponding to Newtonian fluid ($B = 0$).

To draw the conclusion from the present study, it is found that the Poiseuille flow of Bingham fluid is linearly stable to infinitesimal axisymmetric perturbations, and ($B = 0$) is a singularity limit. For future work, linear stability to asymmetric perturbations will be considered.

Acknowledgements

The authors thank J. Dusek for fruitful discussions.

References

[1] C. Nouar, I. Frigaard, Nonlinear stability of Poiseuille flow of a Bingham fluid; theoretical results and comparison with phenemenological criteria, J. Non-Newtonian Fluid Mech. 100 (2001) 127–149.
[2] P.G. Drazin, W.H. Reid, Hydrodynamic Stability, Cambridge University Press, 1995.
[3] J.E. Mott, D.D. Joseph, Stability of parallel flow between concentric cylinders, Phys. Fluids A 11 (1968).

[4] I.A. Frigaard, S.D. Howison, I.J. Sobey, On the stability of Poiseuille flow of a Bingham fluid, J. Fluid Mech. 263 (1994) 133–150.
[5] J.G. Oldroyd, A rational formulation of the equations of plastic flow for a Bingham solid, Proc. Cambridge Philos. Soc. 43 (1947) 100–105.
[6] B. Bird, G.C. Dai, B.J. Yarusson, The rheology and flow of viscoplastic material, Rev. Chem. Engrg. 1 (1983) 1–101.
[7] E.J. Fordham, S.H. Bittleston, M.A. Tehrani, Viscoplastic flow in centered annuli, pipes and slots, Ind. Engrg. Chem. Res. 39 (1991) 517–524.