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On the relation between the global modes and the spectra of drag and lift in periodic wake flows

Sur la relation entre les modes globaux et les spectres des forces hydrodynamiques dans un sillage périodique

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Abstract

In this Note we are interested in the relation between the symmetry properties of the global mode envelopes in wake flows and the spectra of the drag and lift forces. We consider the "impulse" formula for the hydrodynamic force and show that the drag force consists of contributions from the even harmonics, and the lift force of contributions from the odd harmonics, only. Our argument explains this well-known empirical fact and is also supported by the computational evidence we provide. Finally, we identify the unsteady wake flows, both controlled and uncontrolled, as belonging to a broader family of "streaming flows". ***To cite this article: B. Protas, J.E. Wesfreid, C. R. Mecanique 331 (2003).***

Résumé

Dans cette Note, nous nous intéressons à une relation entre les propriétés de symétrie qui caractérisent les enveloppes des modes globaux des sillages et les spectres des forces de traînée et de portance. Nous analysons ceux-ci avec l'aide de la « formule de l'impulsion » et montrons ainsi que les contributions à la traînée proviennent seulement des harmoniques paires, et les contributions à la portance, des harmoniques impaires. Notre argumentation explique ce fait empirique bien connu. De plus, celle-ci se trouve soutenue par les résultats des simulations numériques qui sont ici présentées. Enfin, nous caractérisons les écoulements stationnaires induits dans un sillage, contrôlé et non-contrôlé, comme appartenant à une famille assez riche d'écoulements rectifiés. ***Pour citer cet article : B. Protas, J.E. Wesfreid, C. R. Mecanique 331 (2003).***

Keywords: Fluid mechanics; Periodic wakes; Hydrodynamic forces; Streaming flows

Mots-clés : Mécanique des fluides ; Sillages périodiques ; Forces hydrodynamiques ; Écoulements rectifiés

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Nous analysons ici une propriété des forces hydrodynamiques dans le sillage d'un obstacle cylindrique. Nous présentons une explication d'un fait empirique bien connu, que le spectre de la force de traînée sur l'obstacle, se compose seulement des harmoniques paires et celui de la portance, des harmoniques impaires. Nous le démontrons en analysant les propriétés de symétrie des modes globaux dans le sillage. Lorsque le nombre de Reynolds est plus grand que la valeur critique $Re_{\mathrm{cr}} \cong 46$, l'écoulement derrière le cylindre est caractérisé par la naissance spontanée d'une allée de tourbillons de Bénard–von Karman. Les tourbillons advectés avec circulation négative ont un déplacement positif par rapport à l'axe de symétrie, et les tourbillons à circulation positive, présentent un déplacement négatif (Fig. 1).

Les forces hydrodynamiques peuvent être déterminées à l'aide de la « formule de l'impulsion » (Saffman [1]). Ainsi à 2D, l'on obtient (1), qui peut être approximé par l'expression (2). Par ailleurs, il a été proposé [2], que chaque quantité hydrodynamique dans un sillage périodique, puisse être décrite, pour $t \to \infty$, par la somme (3) comprenant un champ stationnaire (l'écoulement de base) et la superposition des harmoniques. Ainsi, pour le cas de la fonction de courant dont on peut deduire toutes les quantités hydrodynamiques en y appliquant les opérateurs différentiels correspondants, l'amplitude des oscillations de fréquence $n\gamma$ (c'est-à-dire, n-iéme harmonique) au point (x_0, y_0), est donnée par $2|c_n(x_0, y_0)|$, qui est appellée « l'enveloppe du mode global » (Zielińska et al. [4], Dušek [2]). La fréquence fondamentale de l'émission du détachement tourbillonnaire est γ et, après normalisation, correspond au nombre de Strouhal $St = \gamma D/|\vec{V}_\infty|$ (où D est le diamètre d'obstacle et $\vec{V}_\infty$ est la vitesse de l'écoulement). L'écoulement de base est une solution instable des équations de Navier–Stokes stationnaires obtenue pour une valeur supercritique du nombre de Reynolds et symétrique par rapport à l'axe de l'écoulement. Il a été déjà remarqué par Dušek [2], que les enveloppes spatiales des harmoniques $c_n(x, y)$ présentent quelques propriétés de symétrie très intéressantes. Par exemple, les enveloppes des harmoniques paires de vorticité sont symétriques, et celles des harmoniques impaires sont anti-symétriques (Éq. (4)). En utilisant la propriété (4) dans la décomposition (2), nous obtenons (5). L'on observe ainsi que, grâce aux propriétés de symétrie mentionnées au-dessus, les produits $\int_\Omega x g_m(x, y)\,\mathrm{d}\Omega$ avec m pair et $\int_\Omega y g_n(x, y)\,\mathrm{d}\Omega$ avec n impair, sont égaux à zéro. En conséquence, le spectre de la traînée ne contient que des harmoniques paires et le spectre de la portance, que des harmoniques impaires. Ces conclusions sont illustrées par nos simulations numériques d'un sillage bidimensionnel naturel et forcé à $Re = 150$. Nous avons utilisé pour les simulations numériques la méthode du vortex, qui est présentée dans la thèse [5] et dans [6]. Le cas contrôlé est effectué en faisant tourner le cylindre avec une vitesse qui varie sinusoïdalement. L'amplitude de la vitesse sur l'obstacle est égale à $2|\vec{V}_\infty|$. Nous montrons dans la Fig. 2 les spectres de la traînée et de la portance pour le cas sans contrôle (a), avec contrôle à une fréquence sous-harmonique (b) et avec contrôle à la fréquence résonante (c). Dans tous ces cas, nous voyons que, comme prévu, seules les harmoniques paires contribuent à la traînée et les harmoniques impaires à la portance. Ce que nous avons montré explique de même la raison pour laquelle la valeur moyenne de la traînée n'est pas égale à zéro alors que celle de la portance l'est. Cet effet est lié à la présence d'un champ stationnaire (« mean flow correction ») qui résulte de l'interaction de chaque harmonique avec sa conjugée complexe. Ces écoulements moyens induits appartiennent donc à une classe connue d'écoulements rèctifiés (« streaming flows »), dans laquelle une contribution stationnaire est produite par la présence des oscillations avec amplitude inhomogène dans l'espace et par la nonlinéarité. Il se vérifie que ces deux conditions sont satisfaites dans le problème étudié. Les propriétés des écoulements rectifiés ont été initialement discutées par Rayleigh [8] et Schlichting [9] et plus récemment revues par Petit et Gondrin [10], et Riley [11]. Une discussion plus complète des différents aspects de ce phénomène dans les sillages contrôlés est présentée dans [5] et [6].

In this Note we are interested in explaining a certain aspect of the wake flow phenomenology, namely the well-known empirical fact that the spectrum of the drag force acting on the obstacle consists solely of even harmonics, whereas the spectrum of the lift force has odd harmonics only. We will address here this fact using the symmetry properties of the global modes which are defined and characterized below. Then we show evidence

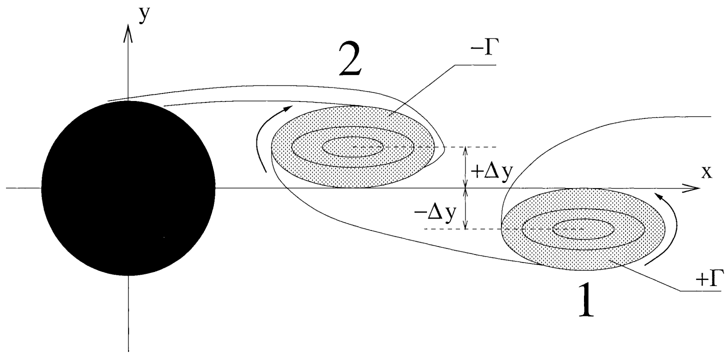

Fig. 1. Sketch illustrating the distinctive features of the vorticity distribution in the Bénard–von Kármán vortex street behind the obstacle. The symbols $\pm\Gamma$ denote the circulations of the vortices and $\pm\Delta y$ their separation from the centerline.

Fig. 1. Schéma illustrant les propriétés caractéristiques de la distribution de vorticité dans une allée de Bénard–von Kármán derrière un obstacle. Les symboles $\pm\Gamma$ représentent circulation des tourbillons et $\pm\Delta y$ leur déplacement par rapport à l'axe de l'écoulement.

based on numerical simulations which supports our arguments. In this study we consider plane wake flows behind cylindrical obstacles in the periodic regime. When the Reynolds number Re is higher than the critical value Re_{cr} (equal to about 46 in the case of the circular cylinder), the flow is characterized by the spontaneous appearance of a staggered array of counter-rotating vortices, the so-called Bénard–von Kármán vortex street. A crucial feature of this phenomenon is that these vortices are advected downstream at non-zero separations Δy from the centerline (the clockwise rotating vortex moves above the centerline and vice versa). Fig. 1 illustrates the distinctive features of the Bénard–von Kármán vortex street.

Out of several ways that allow one to calculate the hydrodynamic force in unbounded external flows, we choose to analyze the 2D "impulse formula" (e.g., Saffman [1]), as here the link with vortex dynamics in the wake is most evident

$$\vec{F} = -\frac{\mathrm{d}}{\mathrm{d}t}\int_{\Omega} (\vec{r}\times\vec{\omega})\,\mathrm{d}\Omega \tag{1}$$

where the flow domain Ω extends to infinity and the fluid density was set as $\rho = 1$. This formula associates the force with the time derivative of the vorticity impulse, i.e., the integrated cross-product of the position vector $\vec{r} = [x, y]$ and vorticity $\vec{\omega} = \omega\vec{k}$ ($\omega = \partial v/\partial x - \partial u/\partial y$, where u and v are the two velocity components, and $\vec{k}$ is the versor perpendicular to the plane of motion). We note that in 3D the analogous formula is slightly different, as there is the factor of $1/2$ in front of it. Using the following approximation for the horizontal (drag) component in formula (1) (cf. Fig. 1)

$$F_D \cong -\frac{\mathrm{d}}{\mathrm{d}t}\sum_i \Gamma_i\Delta y_i = -\frac{\mathrm{d}}{\mathrm{d}t}\big[(+\Gamma_1)(-\Delta y_1) + (-\Gamma_2)(+\Delta y_2) + \cdots\big] \tag{2}$$

we immediately see that the drag force is intrinsically related to the presence of the staggered array of counterrotating vortices behind the cylinder. During every vortex shedding cycle two new vortices are created in the recirculation zone behind the obstacle: one with negative circulation above the centerline (i.e., with $-\Gamma$ and Δy) and another one with positive circulation below the centerline (i.e., with Γ and $-\Delta y$). Consequently, they contribute negative vorticity impulse, which upon differentiation with respect to time and reversing the sign (cf. Eq. (2)), yields the observed drag.

It was proposed in [2] that every hydrodynamic quantity in the saturated periodic wake flow (i.e., for $t \to \infty$) be represented as a sum of a certain steady field, i.e., the basic flow, and a superposition of harmonics. This proposition presupposes that the velocity field in the whole flow domain oscillates with the same single global frequency and its harmonics, a property known as the *global mode* behavior (e.g., Wesfreid et al. in [3], Zielińska and Wesfreid in [4]). Here we give this representation in terms of the streamfunction Ψ_∞

$$\Psi_\infty(x, y, t) = \Psi_b(x, y) + \sum_{n=-\infty}^{\infty} c_n(x, y)\, \mathrm{e}^{\mathrm{i}n\gamma t} \tag{3}$$

where Ψ_b is the streamfunction corresponding to the basic flow and c_n are the spatial envelopes associated with the particular harmonics. Conjugate symmetry is assumed to hold among the coefficients c_n, i.e., $c_n = c_{-n}^*$ for all n (star denotes complex conjugation), so that relation (3) involves only real quantities. All relevant flow quantities can be recovered by applying suitable differential operators to Ψ_∞, and this procedure naturally carries over to the spatial envelopes $c_n(x, y)$ of the harmonics. The amplitude of oscillations with frequency $n\gamma$ (i.e., the n-th harmonic) at the point (x_0, y_0) is given by $2|c_n(x_0, y_0)|$. This quantity thus represents the spatial variation of the oscillation amplitude (with the frequency $n\gamma$) and therefore has often been referred to as the *global mode* amplitude (e.g., Zielińska et al. [4], Dušek [2]). All oscillations vanish at infinity, hence we have the property $\lim_{(x,y)\to\infty} |c_n| \to 0$. The symbol γ denotes the fundamental frequency of vortex shedding at saturation (i.e., for $t \to \infty$). This frequency is often normalized to give the Strouhal number of vortex shedding $St = \gamma D/|\vec{V}_\infty|$, where D is the obstacle diameter and $\vec{V}_\infty$ is the free stream at infinity. As a matter of fact, relation (3) is the limiting case (for $t \to \infty$) of a more general relation allowing for a slow evolution in time of the envelopes c_n, the so called "slow dynamics". Here we consider the saturated case when all transients have died out, hence the time-dependence of the envelopes c_n is suppressed. The basic flow Ψ_b is the unstable solution of the steady-state Navier–Stokes system obtained for a supercritical value of *Re*. Structurally, it is related to the solution obtained in the subcritical conditions and is characterized by symmetry with respect to the centerline.

Our argument here is derived from the interesting observations made by Dušek in [2], namely that the spatial envelopes $c_n(x, y)$ of the harmonics (cf. Eq. (3)) are characterized by some non-trivial symmetry properties. In that study evidence was presented showing that the spatial envelopes of the velocity harmonics follow the pattern: $u_n(x, y) = (-1)^n u_n(x, -y)$ and $v_n(x, y) = (-1)^{n+1} v_n(x, -y)$ for $n = 0, \ldots,$ where $u_n = \partial c_n/\partial y$ and $v_n = -\partial c_n/\partial x$ are the longitudinal and the transversal components, respectively. Consequently, the envelopes of the vorticity harmonics $g_n(x, y) = -\Delta c_n(x, y)$ satisfy the relation

$$g_n(x, y) = (-1)^{n+1} g_n(x, -y) \quad \text{for } n = 0, \ldots \tag{4}$$

which means that even vorticity harmonics are antisymmetric, and odd harmonics are symmetric, with respect to the centerline. Note that, due to the conjugate symmetry of the different fields involved, all of the properties mentioned above also extend to negative values of n. Exhaustive discussion of analogous symmetry properties of the velocity field in a periodic wake flow is given in [2]. Using the decomposition (3) in (1) we obtain

$$\vec{F} = \vec{F}_b - \sum_{n=-\infty}^{\infty} \left[\mathrm{i}n\gamma\, \mathrm{e}^{\mathrm{i}n\gamma t} \int_{\Omega} \left(\vec{r} \times (g_n \vec{k}) \right) \mathrm{d}\Omega \right] \tag{5}$$

where $\vec{F}_b$ represents the contribution of the basic flow Ψ_b. We observe that due to the fact that $g_n = g_{-n}^*$, relation (5) involves only real quantities. Now we see that, owing to the symmetry properties of the spatial envelopes $g_n(x, y)$ of the vorticity harmonics (cf. Eq. (4)), the products $\int_\Omega x g_m(x, y)\, \mathrm{d}\Omega$, m-even, and $\int_\Omega y g_n(x, y)\, \mathrm{d}\Omega$, n-odd, vanish. This implies that only even vorticity harmonics contribute to drag and only odd harmonics contribute to lift. Thus, the well-known facts about the spectral content of the lift and drag time series turn out to be a necessary consequence of the symmetry properties characterizing the spatial envelopes of the harmonics. Evidence confirming these observations is shown below.

In Fig. 2 we show the spectra of drag and lift obtained in the numerical simulations of the 2D wake behind a stationary and rotationally oscillating cylinder at $Re = 150$. For this value of *Re* the unforced flow is in the periodic regime. Complete discussion of this study along with a description and validation of the numerical method used

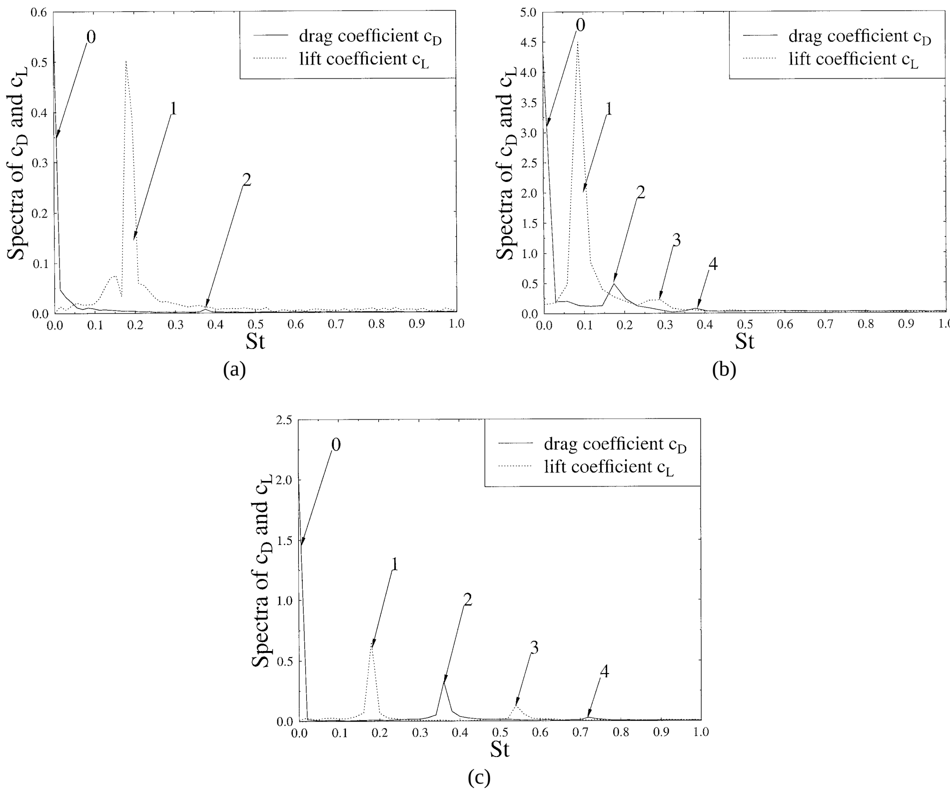

Fig. 2. Spectra (in arbitrary units) of the drag and lift coefficients. The figures represent the flows at $Re = 150$ with the following parameters: (a) no control, (b) $St_f = 0.09$ (0.5), $W = 2.0$, and (c) $St_f = 0.18$ (1.0), $W = 2.0$. Numbers in parentheses are the relative focing frequencies (i.e., St_f/St_{nat}). In the figures we identify the harmonics associated with the particular peaks.

Fig. 2. Spectres de la traînée et de la portance. Les figures représentent les écoulements pour $Re = 150$ forcés aux fréquences indiquées : (a) sans contrôle, (b) $St_f = 0.09$ (0.5), $W = 2.0$ et (c) $St_f = 0.18$ (1.0), $W = 2.0$. Les nombres entre parenthèses sont les fréquences relatives du forçage (c'est à dire St/St_{nat}). Dans ces figures nous avons indiqué les harmoniques auxquelles les pics correspondent.

is given in Protas [5] and in Protas and Wesfreid [6]. Here the data is shown for the uncontrolled flow and for the flows with the normalized forcing frequency $St_{\mathrm{f}} = 0.09$ ($St_{\mathrm{f}}/St_{\mathrm{nat}} = 0.5$ – subharmonic forcing) and $St_{\mathrm{f}} = 0.18$ ($St_{\mathrm{f}}/St_{\mathrm{nat}} = 1.0$ – resonant forcing). In all the cases the normalized rotation amplitude is $W = 2.0$, i.e., the peak circumferential velocity is twice bigger than the free stream at infinity. In Fig. 2(a), corresponding to the natural vortex shedding, we see that drag peaks at the zeroth and the second harmonic, whereas lift peaks at the first harmonic. These effects are magnified when the flow is forced at the natural frequency of $St_{\mathrm{nat}} = 0.18$ (Fig. 2(c)) resulting in a resonant state. Here we see that drag c_D also peaks at the fourth harmonic, whereas lift has an additional peak at the third harmonic. In the subharmonic case shown in Fig. 2(b), we observe the "lock-on" state in which the flow oscillates with the forcing frequency equal to half of the natural frequency. Therefore, the first harmonic is now at the normalized frequency of $St = 0.09$ (i.e., $\frac{1}{2}St_{\mathrm{nat}}$) and corresponds to forcing. The second harmonic in this regime corresponds to the first harmonic in the unforced case (cf. Fig. 2(a)). In the subharmonic case as well drag peaks at the zeroth, second and fourth harmonic, while lift peaks at the first and the third harmonic. Thus, all these findings support the arguments formulated above. In [6] we showed evidence that the presence of an oscillating velocity field related to vortex shedding augments the mean drag already due to the basic flow, but does not affect the mean lift. As a result, the mean drag coefficient can be expressed as $c_D = c_D^b + c_D^0$, where c_D^b and c_D^0 are the mean contributions of the basic flow and the unsteady vortex shedding, respectively. In the controlled flows analyzed here the "mean flow correction" increases the mean drag above the uncontrolled level, however as we showed in [6], for a certain range of the forcing frequencies the mean drag can actually be reduced.

The "mean flow correction" arises in oscillatory flows when a given harmonic interacts with its complex conjugate yielding a field of Reynolds stresses that produce a steady correction as a result [7]. Consequently, this phenomenon belongs to a broader class of "streaming flows" already discussed in the context of wall-bounded viscous flows by Rayleigh [8] and Schlichting [9]. The distinguishing features of the "streaming flows" are quadratic interactions and spatial inhomogeneity that are necessary to produce a steady field. Both of these ingredients are present in our problem: quadratic terms are obtained upon plugging representation (3) into the nonlinear term in the Navier–Stokes equation, whereas inhomogeneity is hidden in spatial variation of the envelopes $c_n(x, y)$ (which vanish at infinity, hence have to be spatially varying so as not to be zero everywhere). Detailed discussion of the mean flow characteristics of controlled wakes was given by the authors in [6]. Generic properties of "streaming flows" are reviewed by Petit and Gondrin in [10] and by Riley in [11].

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