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Self-switching of displacement waves in elastic nonlinearly deformed materials

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Abstract The problem of self-switching plane waves in elastic nonlinearly deformed materials is formulated. Reduced and evolution equations, which describe the interaction of two waves the power pumping wave and the faint signal wave are obtained. For the case of wave numbers matching the pumping and signal waves, a procedure of finding the exact solution of evolution equations is described. The solution is expressed by elliptic Jacobi functions. The existence of the power wave self-switching is shown and commented. ***To cite this article: J. Rushchitsky, C. R. Mecanique 330 (2002) 175–180.*** © 2002 Académie des sciences/Éditions scientifiques et médicales Elsevier SAS

acoustics / waves / two waves interaction / self-switching of waves

Self-commutation d'ondes de déplacement dans les milieux élastiques non-linéaires

Résumé Nous formulons le problème de self-commutation d'ondes dans des matériaux élastiques déformés non linéairement. Nous déduisons les équations d'évolution complètes et réduites décrivant l'interaction entre deux ondes, l'onde de pompage et une onde faible transportant le signal. Dans le cas de nombres d'onde concordants entre l'onde de pompage et l'onde-signal, on décrit la procédure de construction de la solution exacte aux équations d'évolution. Cette solution s'exprime en termes de fonctions elliptiques de Jacobi. On montre l'existence de la self-commutation de l'onde de pompage et on commente le résultat. ***Pour citer cet article : J. Rushchitsky, C. R. Mecanique 330 (2002) 175–180.*** © 2002 Académie des sciences/Éditions scientifiques et médicales Elsevier SAS

acoustique / ondes / interaction de deux ondes / self-commutation des ondes

1. Two waves interaction (statement of the problem, reduced and evolution equations)

We consider the interaction of two waves as a special case of the analysis carried out in [1–3]. These references deal with the interaction of many waves, propagating in elastic nonlinear deformed materials. The present analysis has much in common with studies in nonlinear optics, concerning wave interactions [4–6]. New wave phenomena were uncovered [7], which are to surprisingly close to traditional nonlinear wave phenomena [3–6]. This was designated as self-switching of electromagnetic waves.

For light waves, this phenomenon has the following features: A. It is observed in fiber optic wave guides; B. The system involves two coupled waves; at the entrance to the medium or wave guide one wave is

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powerful (pumping wave with a large amplitude), whereas the other wave is weak (control signal with a very small amplitude); C. Parameters of waves are fixed at a finite distance from the entrance (at the outlet from the medium); it is stated that under certain conditions the very small change of a control signal causes the fast change of wave intensities at the outlet. The pumping wave can switch completely from its own frequency to the frequency of the control signal and then return to its initial value.

This study concerns displacement waves (acoustic waves) in materials; the goal is analysis of the self-switching phenomenon within the framework of the classical nonlinear approach.

Let us choose the longitudinal plane wave propagating in the direction of the Ox_1 axis. The material is assumed to be elastic and features a quadratic nonlinearity with respect to deformation. This corresponds to the Murnaghan elastic potential. Then the initial equation will be [1–3]

$$\rho\frac{\partial^2 u_1}{\partial t^2} - (\lambda + 2\mu)\frac{\partial^2 u_1}{\partial x_1^2} = N_1 \frac{\partial^2 u_1}{\partial x_1^2}\frac{\partial u_1}{\partial x_1} \tag{1.1}$$

and solution of its linear variant will have the form of the harmonic wave $u_{1\,\mathrm{lin}}(x_1, t) = A_{1\,\mathrm{lin}}\,\mathrm{e}^{\mathrm{i}(k_{1\,\mathrm{lin}}x_1 - wt)}$ with all parameters constant (amplitude, frequency, wave number).

We consider only the case, where at the entrance to a medium two longitudinal waves with distinct frequencies are assigned. As usual in multiple wave interaction problems, we will use the van der Pol scheme – the scheme of slowly varying amplitudes. One assumes that the entrance signal wave and pumping wave are harmonic with amplitudes depending on the spatial coordinate $A_{\mathrm{pum}}(x)$, $A_{\mathrm{sign}}(x)$, fixed frequencies w_{pum}, w_{sign}, and fixed wave numbers k_{pum}, k_{sign}:

$$\begin{aligned} u_{\mathrm{pum(sign)}}(x,t) &= \mathrm{Re}\left\{A_{\mathrm{pum(sign)}}(x)\,\mathrm{e}^{\mathrm{i}(k_{\mathrm{pum(sign)}}x - w_{\mathrm{pum(sign)}}t)}\right\} \\ &= a_{\mathrm{pum(sign)}}(x)\cos\left[k_{\mathrm{pum(sign)}}x - w_{\mathrm{pum(sign)}}t + \varphi_{\mathrm{pum(sign)}}(x)\right] \end{aligned}$$

The van der Pol procedure is based on slowness in amplitudes changes and the absence of energy influx. It is necessary to specify the types of interactions, which should be taken into account. In our case when interaction of power and weak waves is studied, the assumptions needed are specific and consist of two groups. The first group of assumptions is taken into account mainly in the reduced equation:

1. The weak wave has a frequency which is half that of the pump wave $2w_{\mathrm{sign}} = w_{\mathrm{pum}}$;
2. One considers the influence of a self-generation of the weak wave on the pump wave, since this will be just the wave with a frequency of the pump wave. One does not consider self-generation of the pump wave, since that will be another wave effect, not linked with the self-switching effect;
3. The usual nonlinear interaction of two waves should be taken into account, as well.

Now, the reduced equation can be obtained in the form

$$\begin{aligned} &(\lambda + 2\mu)\left[k_{\mathrm{pum}}\,\mathrm{e}^{\mathrm{i}(k_{\mathrm{pum}}x - w_{\mathrm{pum}}t)}\frac{\mathrm{d}A_{\mathrm{pum}}}{\mathrm{d}x} + k_{\mathrm{sign}}\,\mathrm{e}^{\mathrm{i}(k_{\mathrm{sign}}x - w_{\mathrm{sign}}t)}\frac{\mathrm{d}A_{\mathrm{sign}}}{\mathrm{d}x}\right] \\ &\quad = -k_{\mathrm{pum}}k_{\mathrm{sign}}(k_{\mathrm{pum}} + k_{\mathrm{sign}})N_1 A_{\mathrm{pum}}A_{\mathrm{sign}}\,\mathrm{e}^{\mathrm{i}[(k_{\mathrm{pum}}+k_{\mathrm{sign}})x - (w_{\mathrm{pum}}+w_{\mathrm{sign}}t)]} \\ &\qquad - (k_{\mathrm{sign}})^3 N_1 (A_{\mathrm{sign}})^2\,\mathrm{e}^{2\mathrm{i}(k_{\mathrm{sign}}x - w_{\mathrm{sign}})t} \end{aligned} \tag{1.2}$$

Comment 1.1. – The reduced Eq. (1.2) differs from the analogous equation for electromagnetic waves in the case of quadratically nonlinear medium [5]. This is due to the different representations of nonlinearity in optics and acoustics – the square of the polarization in optics and a product of the first derivative by the second derivative in acoustics. However, the procedure used in [5] is quite general and is adopted in what follows.

In the next step, the evolution equations should be obtained from the reduced equation. Here, the second group of assumptions must be formulated: (1) The influence of the weak wave on the pump wave is negligible. (2) The direct influence of the pump wave on the weak wave is considered to be essential.

In these circumstances, the evolution equations take the form of two uncoupled equations:

$$\frac{\mathrm{d}A_{\mathrm{pum}}}{\mathrm{d}x} = \frac{N_1}{\lambda + 2\mu}\frac{(k_{\mathrm{sign}})^2}{k_{\mathrm{pum}}}(A_{\mathrm{sign}})^2\,\mathrm{e}^{\mathrm{i}(2k_{\mathrm{sign}}-k_{\mathrm{pum}})x}$$
$$\frac{\mathrm{d}\overline{A}_{\mathrm{sign}}}{\mathrm{d}x} = -\frac{N_1}{\lambda + 2\mu}k_{\mathrm{pum}}(k_{\mathrm{pum}}+k_{\mathrm{sign}})\overline{A}_{\mathrm{pum}}A_{\mathrm{sign}}\,\mathrm{e}^{\mathrm{i}(2k_{\mathrm{sign}}-k_{\mathrm{pum}})x} \qquad (1.3)$$

2. The solution of evolution equations

Let us now rewrite the evolution equations (1.3) in terms of real amplitudes. The two equations describing phases are combined into a single one for the phase difference. It is convenient to introduce the following definitions

$$S_{\mathrm{pum}} = \frac{N_1}{\lambda + 2\mu}\frac{(k_{\mathrm{sign}})^2}{k_{\mathrm{pum}}}, \qquad S_{\mathrm{sign}} = \frac{N_1}{\lambda + 2\mu}k_{\mathrm{pum}}(k_{\mathrm{pum}}+k_{\mathrm{sign}})$$

and to introduce the transferred wave number $\Delta k = 2k_{\mathrm{sign}} - k_{\mathrm{pum}}$. One obtains the following set of equations:

$$\big(\rho_{\mathrm{pum}}(x)\big)' = S_{\mathrm{pum}}\big(\rho_{\mathrm{sign}}(x)\big)^2\cos\varphi(x) \qquad (2.1)$$

$$\big(\rho_{\mathrm{sign}}(x)\big)' = -S_{\mathrm{sign}}\rho_{\mathrm{sign}}(x)\rho_{\mathrm{pum}}(x)\cos\varphi(x) \qquad (2.2)$$

$$\big(\varphi(x)\big)' = \Delta k - \left(2S_{\mathrm{sign}}\rho_{\mathrm{pum}}(x) - S_{\mathrm{pum}}\frac{(\rho_{\mathrm{sign}}(x))^2}{\rho_{\mathrm{pum}}(x)}\right)\sin\varphi(x) \qquad (2.3)$$

One may now introduce the quantities $I_{\mathrm{pum(sign)}}(x) = |A_{\mathrm{pum(sign)}}(x)|^2 = (\rho_{\mathrm{pum(sign)}}(x))^2$, the pump and signal wave intensities. With these intensities the so-called Manley–Rowe relationship can be derived

$$\frac{I_{\mathrm{pum}}(x)}{S_{\mathrm{pum}}} + \frac{I_{\mathrm{sign}}(x)}{S_{\mathrm{sign}}} = F \qquad (2.4)$$

This is in fact the first integral of system (2.1)–(2.3). The quantity F is the arbitrary constant and can be related to the energy transported by waves.

The third equation (2.3) may now be transformed.To this purpose, let us find first the coefficients S_{pum}, S_{sign} from Eqs. (2.1), (2.2)

$$S_{\mathrm{pum}} = \frac{(\rho_{\mathrm{pum}}(x))'}{(\rho_{\mathrm{sign}}(x))^2\cos\varphi(x)}, \qquad S_{\mathrm{sign}} = \frac{(\rho_{\mathrm{sign}}(x))'}{\rho_{\mathrm{sign}}(x)\rho_{\mathrm{pum}}(x)\cos\varphi(x)}$$

and substitute the quantity obtained into (2.3). After simple transformations, one finds the new form of (2.3)

$$\big(\varphi(x)\big)' = \Delta k - \frac{\sin\varphi(x)}{\cos\varphi(x)}\left[\ln\big(\big(\rho_{\mathrm{sign}}(x)\big)^2\rho_{\mathrm{pum}}(x)\big)\right]'$$

Use now the normalized intensities

$$\hat{I}_{\mathrm{pum(sign)}}(x) = \frac{I_{\mathrm{pum(sign)}}(x)}{S_{\mathrm{pum(sign)}}F} = \frac{(\rho_{\mathrm{pum(sign)}}(x))^2}{S_{\mathrm{pum(sign)}}F}$$

and the new variable $\xi = \sqrt{S_{\mathrm{sign}}F}\,x$. All basic equations can be written in a simpler more convenient form. First of all, Manley–Rowe relationship becomes

$$\hat{I}_{\mathrm{pum}}(\xi) + \hat{I}_{\mathrm{sign}}(\xi) = 1 \qquad (2.5)$$

The evolution equations will be also simply written using the normalized intensities:

$$\big(\sqrt{\hat I_{\mathrm{pum}}(\xi)}\,\big)'_\xi = -\hat I_{\mathrm{sign}}(\xi)\cos\varphi(\xi) \tag{2.6}$$

$$\big(\sqrt{\hat I_{\mathrm{sign}}(\xi)}\,\big)'_\xi = \sqrt{\hat I_{\mathrm{pum}}\hat I_{\mathrm{sign}}(\xi)}\cos\varphi(\xi) \tag{2.7}$$

$$\big(\varphi(\xi)\big)'_\xi = \Delta\hat k - \tan\varphi(\xi)\Big[\ln\Big(\hat I_{\mathrm{sign}}(\xi)\sqrt{\hat I_{\mathrm{pum}}(\xi)}\,\Big)\Big]'_\xi \quad \big(\Delta k = \sqrt{S_{\mathrm{sign}}F}\,\Delta\hat k\big) \tag{2.8}$$

It is convenient to distinguish two cases of the self-switching phenomenon: (I) The transferred wave number Δk is small; (II) Wave numbers are matched ($\Delta k = 0$). Let us consider the second case and begin from the third equation (2.8). If $\Delta k = 0$ (mismatching is absent), this equation can be integrated very simply

$$\begin{gathered}\hat I_{\mathrm{sign}}(\xi)\sqrt{\hat I_{\mathrm{pum}}(\xi)}\,\sin\varphi(\xi) = G\\ G \equiv \hat I_{\mathrm{sign}}(0)\sqrt{\hat I_{\mathrm{pum}}(0)}\,\sin\varphi(0) = \hat I_{\mathrm{sign}}(0)\sqrt{\hat I_{\mathrm{pum}}(0)}\,\sin\big[2\varphi_{\mathrm{sign}}(0) - \varphi_{\mathrm{pum}}(0)\big]\end{gathered} \tag{2.9}$$

Expression (2.9), where G is an arbitrary constant, can be treated together with (2.5) as an integral of system (2.6)–(2.8). So, we have two integrals for the system of three unknown functions. Thus, it is necessary to find one of these functions only. For this, let us take into account the previous integrals in equations (2.6), (2.7). One needs in fact the second equation (2.7), only. Let us transform it as follows:

$$\big(\sqrt{\hat I_{\mathrm{sign}}(\xi)}\,\big)'_\xi = \sqrt{\hat I_{\mathrm{pum}}(\xi)\hat I_{\mathrm{sign}}(\xi)}\,\cos\varphi(\xi) \to \big(\hat I_{\mathrm{sign}}(\xi)\big)'_\xi = \pm 2\sqrt{\big(1-\hat I_{\mathrm{sign}}(\xi)\big)\hat I^2_{\mathrm{sign}}(\xi) - G^2} \tag{2.10}$$

The solution of (2.10) is

$$\xi = \pm\frac{1}{2}\int_{\hat I_{\mathrm{sign}}(0)}^{\hat I_{\mathrm{sign}}(\xi)} \frac{\mathrm{d}\hat I_{\mathrm{sign}}}{\sqrt{\hat I_{\mathrm{sign}}(1-\hat I_{\mathrm{sign}})^2 - G^2}} \tag{2.11}$$

Comment 2.1. – All three roots of the cubic equation $\hat I_{\mathrm{sign}}(1-\hat I_{\mathrm{sign}})^2 - G^2 = 0$ are normalized intensities, and hence they are proper fractions and can take values only from segment [0; 1].

Consider the simplest case: $G = 0$. There are three possibilities here: (1) The initial intensity of the power wave is zero $\hat I_{\mathrm{pum}}(0) = 0$. (2) The initial intensity of the signal wave is zero $\hat I_{\mathrm{sign}}(0) = 0$. (3) The initial difference of phases is zero $2\varphi_{\mathrm{sign}}(0) - \varphi_{\mathrm{pum}}(0) = 0$ (phases are mismatched). With respect to the roots of the cubic equation, this means that it is simplified $\hat I_{\mathrm{sign}}(1-\hat I_{\mathrm{sign}})^2 = 0$ and its roots are $\hat I_{\mathrm{sign}(3)} = \hat I_{\mathrm{sign}(2)} = 1$, $\hat I_{\mathrm{sign}(1)} = 0$.

In the case $G = 0$ the elliptic integral (2.11) can be calculated in terms of elementary functions

$$\begin{gathered}\xi = \pm\frac{1}{2}\int_{\hat I_{\mathrm{sign}(0)}}^{\hat I_{\mathrm{sign}}(\xi)} \frac{\mathrm{d}\hat I_{\mathrm{sign}}}{\sqrt{\hat I_{\mathrm{sign}}(1-\hat I_{\mathrm{sign}})^2}} = \int_{\hat I_{\mathrm{sign}}(0)}^{\hat I_{\mathrm{sign}}(\xi)} \frac{\mathrm{d}\sqrt{\hat I_{\mathrm{sign}}}}{1-(\sqrt{\hat I_{\mathrm{sign}}})} = \mathrm{Arth}\sqrt{\hat I_{\mathrm{sign}}(\xi)}\Big|_{\hat I_{\mathrm{sign}}(0)}^{\hat I_{\mathrm{sign}}(\xi)}\\ \to \xi - \xi_0 = \mathrm{Arth}\sqrt{\hat I_{\mathrm{sign}}(\xi)}\ \big(\xi_0 = \mathrm{Arth}\sqrt{\hat I_{\mathrm{sign}}(0)}\,\big) \to \sqrt{\hat I_{\mathrm{sign}}(\xi)} = \mathrm{th}(\xi+\xi_0)\end{gathered} \tag{2.12}$$

The solution (2.12) may be used to calculate the intensity of the pumping wave from the Manley–Rowe relationships (2.5) $\sqrt{\hat I_{\mathrm{pum}}(\xi)} = \mathrm{csh}(\xi+\xi_0)$.

Comment 2.2. – If the initial intensity of the signal wave is equal to zero (the initial amplitude of the signal wave is equal to zero), then the arbitrary constant ξ_0 also equals zero. Thus, the initial intensity of the pumping wave decreases with time; and initial intensity of the signal wave increases with time, when all power of the pump wave will go into the signal wave.

Let us come back to the case $G \neq 0$ and order roots $\hat{I}_{\text{sign}(3)} \geqslant \hat{I}_{\text{sign}(2)} \geqslant \hat{I}_{\text{sign}(1)}$. Also, introduce the new function

$$z^2(\xi) = \frac{\hat{I}_{\text{sign}}(\xi) - \hat{I}_{\text{sign}(1)}}{\hat{I}_{\text{sign}(2)} - \hat{I}_{\text{sign}(1)}} \quad \left(z^2(\xi_1) = 0,\ z^2(\xi_2) = 1\right)$$

and denote

$$\theta = \frac{1}{z^2(\xi_3)} = \frac{\hat{I}_{\text{sign}(2)} - \hat{I}_{\text{sign}(1)}}{\hat{I}_{\text{sign}(3)} - \hat{I}_{\text{sign}(1)}} \leqslant 1$$

Then the solution of (2.10) takes the standard form of the elliptic integral of the third kind

$$\xi = \pm \frac{1}{\sqrt{\hat{I}_{\text{sign}(3)} - \hat{I}_{\text{sign}(2)}}} \int_{z(0)}^{z(\xi)} \frac{\mathrm{d}z}{\sqrt{(1-z^2)(1-\theta^2 z^2)}}$$

In this case the function introduced above $z(\xi)$ will have the form of Jacobi elliptic function

$$z(\xi) = \operatorname{sn}^2\Big] \sqrt{\hat{I}_{\text{sign}(3)} - \hat{I}_{\text{sign}(1)}}(\xi - \xi_0), \theta\Big] \quad \text{or}$$
$$\hat{I}_{\text{sign}}(\xi) = \hat{I}_{\text{sign}(1)} + \left(\hat{I}_{\text{sign}(2)} - \hat{I}_{\text{sign}(1)}\right) \operatorname{sn}^2\Big[\sqrt{\hat{I}_{\text{sign}(3)} - \hat{I}_{\text{sign}(1)}}(\xi - \xi_0), \theta\Big] \tag{2.13}$$

Then according to the Manley–Rowe relationship (2.5) the intensity of the pumping wave will be determined by the formula

$$\hat{I}_{\text{pum}}(\xi) = 1 - \hat{I}_{\text{sign}}(\xi) \tag{2.14}$$

3. Self-switching of the pumping wave

Let us return to relationship (1.4) and assume $w_{\text{sign}} = 2w$, $w_{\text{pum}} = w$ and $k_{\text{sign}} = 2v_{\text{ph}}w$, $k_{\text{pum}} = v_{\text{ph}}w$ ($v_{\text{ph}} = \sqrt{(\lambda + 2\mu)/\rho}$ is the phase velocity of plane longitudinal elastic waves). We are still interested in the values of intensities and the differences of phases at the outlet – at the distance l from the entrance.

Comment 3.1. – One may note that in the case of elastic medium wave numbers mismatching is equal to zero $\Delta k = 0$; and therefore the difference of phases could be evaluated as follows $\varphi(\xi) = 2\varphi_{\text{sign}}(\xi) - \varphi_{\text{pum}}(\xi)$.

Since we have already the solution (2.13), (2.14), then intensity values can be calculated

$$\hat{I}_{\text{sign}}(\xi_l) = \hat{I}_{\text{sign}(1)} + \left(\hat{I}_{\text{sign}(2)} - \hat{I}_{\text{sign}(1)}\right) \operatorname{sn}^2\Big[\sqrt{\hat{I}_{\text{sign}(3)} - \hat{I}_{\text{sign}(1)}}(\xi_l - \xi_0), \theta\Big]$$
$$\hat{I}_{\text{pum}}(\xi_l) = 1 - \hat{I}_{\text{sign}}(\xi_l), \qquad \xi_l = l\sqrt{S_{\text{sign}}F}, \qquad \varphi(\xi_l) = \arcsin \frac{G}{\hat{I}_{\text{sign}}(\xi_l)\sqrt{\hat{I}_{\text{pum}}(\xi_l)}}$$

It can be shown that self-switching phenomena occurs when two conditions are fulfilled

$$\theta = \frac{\hat{I}_{\text{sign}(2)} - \hat{I}_{\text{sign}(1)}}{\hat{I}_{\text{sign}(3)} - \hat{I}_{\text{sign}(1)}} \approx 1, \qquad \mathrm{e}^{\sqrt{\hat{I}_{\text{sign}(3)} - \hat{I}_{\text{sign}(1)}}(\xi_l - \xi_0)} \gg 1 \tag{3.1}$$

One may write the assumption more explicitly, when the pumping wave has a large amplitude and the signal wave is weak: $\hat{I}_{\text{pum}}(0) \approx 1$, $\hat{I}_{\text{sign}}(0) \ll 1$ and $G^2 \approx (\hat{I}_{\text{sign}}(0)) \sin^2\varphi(0)$.

Consider the cubic equation $\hat{I}_{\text{sign}}(1 - \hat{I}_{\text{sign}})^2 - G^2 = 0$, when the phase mismatch is small. Its roots can be written approximately $\hat{I}_{\text{sign}(1)} \approx (\hat{I}_{\text{sign}}(0)) \sin^2\varphi(0)$, $\hat{I}_{\text{sign}(2,3)} \approx 1 \mp \sqrt{\hat{I}_{\text{sign}}(0)} \sin\varphi(0)$. Then the two conditions (3.1) can be checked. The first condition is fulfilled and the second condition should be traded off by the inequality $e^{\xi_l} \gg 1$ (it is valid for $\xi_l \geqslant 4$, for example).

In optics [4,7], a useful approximate formula is found for the signal wave intensity at the waveguide outlet. For acoustic waves this can be transformed as follows

$$\hat{I}_{\text{sign}}(\xi_l) \approx \left(\frac{1-U}{1+U}\right)^2, \quad U \approx 4\hat{I}_{\text{sign}}(0)\sin^2\varphi(0)\,e^{2\xi_l} \tag{3.2}$$

Two cases are interesting: (1) Initial intensity of the signal wave is zero $\hat{I}_{\text{sign}}(0) = 0$. Then from (3.2) it follows that $\hat{I}_{\text{sign}}(\xi_l) = 1 = \hat{I}_{\text{pum}}(0)$. Then the power of the pumping wave is injected in the signal wave. We observe the situation, when given at the entrance first harmonics is transformed at the outlet into the second harmonics. Wave is switched from the first harmonics into the second one. (2) The initial intensity of the signal wave is small, but non zero. Let us calculate the signal wave normalized intensity at the outlet on the medium, supposing the very small transition of the power wave intensity into the signal wave intensity

$$\hat{I}_{\text{sign}}(\xi_l) \approx \left(\frac{1-U}{1+U}\right)^2 \ll 1,$$

$$U \approx 4\hat{I}_{\text{sign}}(0)\sin^2\varphi(0)\,e^{2\xi_l} \approx 1 \to \hat{I}_{\text{sign}}(\xi_l) \approx \frac{64\hat{I}_{\text{sign}}(0)}{\sin^2\varphi(0)}\,e^{-2\xi_l} \ll 1 \quad (\sin\varphi(0) \approx 1)$$

One may then observe a different situation. When the signal wave of a very small intensity is excited, then the power wave passes through the medium almost without change; and we have at the outlet the pumping wave with the frequency ω and the weak signal wave. When the weak signal wave intensity is made to vanish, then we have at the outlet the signal wave with the frequency 2ω. *So, we observe the phenomenon of self-switching of the displacement wave.*

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