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Quasi-static versus dynamic failure instabilities in fluid-saturated porous media

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Abstract Using a linear perturbation approach, we show that under quasi-static conditions, unbounded growth of perturbations coincides with localization under drained or undrained conditions. Under dynamic loadings, unbounded growth is related either to the emergence of stationary discontinuities (and these are set by drained conditions) or to the appearance of the flutter phenomenon (acceleration waves). For associative behaviour the inception of unbounded growth is always set (under both static and dynamic conditions) by the singularity of the drained acoustic tensor. It is only for non-associative flow that unbounded growth may correspond to undrained localization in quasi-static conditions and to flutter under dynamic conditions. ***To cite this article: A. Benallal, C. Comi, C. R. Mecanique 330 (2002) 339–345.*** © 2002 Académie des sciences/Éditions scientifiques et médicales Elsevier SAS

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Modes de rupture sous conditions quasi-statiques et dynamiques dans les milieux poreux saturés

Résumé En utilisant une méthode de perturbation linéaire, on montre qu'en conditions quasi-statiques la croissance illimitée de perturbations correspond exactement aux conditions de localisation en conditions drainées ou non drainées. Lorsque l'on prend en compte les effets d'inertie, elle correspond soit à l'émergence de discontinuités stationnaires ou à l'apparition du phénomène de balancement (flutter) pour les ondes d'acceleration. Pour un écoulement associé, sa première apparition correspond toujours à la singularité du tenseur acoustique drainé. Un écoulement non-associé est nécessaire pour qu'elle apparaisse en premier à la singularité du tenseur acoustique non-drainé (quasi-statique) ou à l'apparition du balancement (dynamique). ***Pour citer cet article : A. Benallal, C. Comi, C. R. Mecanique 330 (2002) 339–345.*** © 2002 Académie des sciences/Éditions scientifiques et médicales Elsevier SAS

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La méthode de perturbation linéaire est utilisée ici pour l'analyse des phénomènes d'instabilité dans les milieux poreux saturés. Seule la croissance illimitée des perturbations est considérée ici d'une part parce que l'utilisation de la méthode linéaire est justifiée dans ce cas et d'autre part parce qu'elle peut servir à la définition de critère de rupture. Le comportement des matériaux considérés est représenté par la description de Biot [1] et Coussy [2] et résumé par les relations (7)–(9). Les équations de champ sont données par la conservation de la masse (5), les équations du mouvement (3)–(4) et la compatibilité (2).

La linéarisation des relations de comportement et des équations de champs autour de la solution de référence et la recherche de perturbations de la forme (16) conduisent à la condition d'instabilité (17) contenant le taux de croissance η, la direction $\mathbf{n}$ et le nombre d'onde ξ. La croissance illimitée de perturbations correspond à $\mathrm{Re}(\eta) \to \infty$.

En condition dynamiques, elle a lieu lorsque (24) est satisfaite. Cette condition n'est autre en fait que l'équation caractéristique pour les célérités c des ondes d'accélération si on pose $\eta = -\mathrm{i}\xi c$. De là on tire que cette croissance illimitée ne peut se produire que si les carrés c^2 de ces célérités sont négatifs ou complexes. Elle arrivera donc pour la première fois au cours d'un chargement à l'apparition des ondes d'accélération stationnaires ($c = 0$) ou à l'apparition du phénomène de balancement (flutter, c^2 complexe) pour les ondes d'accélération. D'autre part elle correspond toujours à $\xi \to \infty$ et cela suggère que le mode de rupture est localisé.

En conditions quasi-statiques, la condition d'instabilité devient (29) et le taux de croissance η est donné par (31). De là on conclut que la croissance illimitée des perturbations correspond soit à la singularité du tenseur acoustique non drainé et le nombre d'onde ξ est arbitraire ou alors à $\det(\mathbf{n}\cdot\mathbf{H}^d\cdot\mathbf{n})\det(\mathbf{n}\cdot\mathbf{H}^u\cdot\mathbf{n}) < 0$ et dans ce cas le nombre d'onde ξ est infini. Dans ce cas le mode de rupture est soit diffus, soit localisé. Au cours d'un chargement, la croissance illimitée des perturbations aura lieu pour la première fois au passage des conditions de localisation en conditions drainées ($\det(\mathbf{n}\cdot\mathbf{H}^d\cdot\mathbf{n}) = 0$) ou non drainées ($\det(\mathbf{n}\cdot\mathbf{H}^u\cdot\mathbf{n}) = 0$).

Pour les modèles associés, le balancement n'existe pas [4] et la singularité du tenseur acoustique drainé précède toujours celle du tenseur acoustique non drainé [6]. Sous toutes conditions, la croissance illimitée se produit donc pour la première fois au passage de la condition de localisation en conditions drainées. Les autres cas évoqués plus haut ne peuvent survenir donc qu'en présence d'un écoulement non-associé pour le squelette.

1. Governing equations

Consider the dynamic evolution of a saturated porous body of volume V and boundary S consisting of a porous solid skeleton s and of the filled fluid f. In the framework of the Biot's formulation [1], extended to take into account plastic deformations [2], the basic static variables are the total Cauchy stress $\boldsymbol{\sigma}$ in the combined solid and fluid mix and the pore fluid pressure p. The kinematic variables are the strain $\boldsymbol{\epsilon}$ in the skeleton and the variation of fluid content ζ (i.e., the volume change of fluid per unit volume of mixture). In the poro-elastic-plastic context the kinematic variables are both partitioned into elastic and plastic parts so that $\boldsymbol{\epsilon} = \boldsymbol{\epsilon}^e + \boldsymbol{\epsilon}^p$ and $\zeta = \zeta^e + \zeta^p$. We denote by ϕ the porosity of the mixture (defined as the ratio of the fluid volume to the total volume) and by ρ_s, ρ_f the masses per unit volume of the solid phase and of the fluid phase respectively; $\rho = (1-\phi)\rho_s + \phi\rho_f$ is the mass per unit volume of the assembly. The displacements of the solid skeleton and the fluid are respectively $\mathbf{u}_s$ and $\mathbf{u}_f$. Let us also introduce

$$\mathbf{w} = \phi\,(\mathbf{u}_f - \mathbf{u}_s) \qquad (1)$$

such that $\dot{\mathbf{w}}$ is the fluid flux ($\dot{w}_i$ being the fluid volume crossing in the time unit the unit surface normal to the i-th axis).

Assuming small strains in the solid, the geometric compatibility conditions are:

$$\boldsymbol{\epsilon} = \frac{1}{2}\left(\mathbf{grad}\,\mathbf{u}_s + \mathbf{grad}^{\mathrm{T}}\,\mathbf{u}_s\right) \tag{2}$$

With $\rho\mathbf{b}$ the body force per unit volume of the mixture solid–fluid, the equation of motion for the total system is:

$$\operatorname{div}\boldsymbol{\sigma} + \rho\boldsymbol{b} - (1-\phi)\rho_s\ddot{\mathbf{u}}_s - \phi\rho_f\ddot{\mathbf{u}}_f = \mathbf{0} \tag{3}$$

The equation of motion of the fluid specifies the relative movement of the fluid with respect to the skeleton. Assuming the classical linear Darcy's law for the viscous drag, and denoting k the permeability (under isotropic conditions, assumed constant in this work) this equation can be written:

$$-\mathbf{grad}\,p + \rho_f\mathbf{b} = \rho_f\ddot{\mathbf{u}}_f + \frac{\dot{\mathbf{w}}}{k} \tag{4}$$

The final equation is supplied by the conservation of mass and takes the form:

$$\operatorname{div}\dot{\mathbf{w}} + \dot{\zeta} = 0 \tag{5}$$

The combination of (4) and (5) yields:

$$\dot{\zeta} - k\left[\nabla^2 p - \operatorname{div}\rho_f(\mathbf{b} - \ddot{\mathbf{u}}_f)\right] = 0 \tag{6}$$

Regarding the constitutive behaviour, the free energy potential considered here is defined by:

$$\rho\Psi\left(\boldsymbol{\epsilon} - \boldsymbol{\epsilon}^p, \zeta - \zeta^p, \boldsymbol{\alpha}\right) = \frac{1}{2}\boldsymbol{\epsilon}^e : \mathbf{E}^d : \boldsymbol{\epsilon}^e + \frac{1}{2}b^2M\left(\operatorname{Tr}\left(\boldsymbol{\epsilon}^e\right)\right)^2 + \frac{1}{2}M\zeta^{e2} - bM\zeta^e\operatorname{Tr}\left(\boldsymbol{\epsilon}^e\right) + \frac{1}{2}\boldsymbol{\alpha}\cdot\mathbf{h}\cdot\boldsymbol{\alpha} \tag{7}$$

$\mathbf{E}^d = (K^d - 2G/3)\mathbf{1}\otimes\mathbf{1} + 2G\mathbf{I}$ is the isotropic drained elastic tensor with G the shear modulus and K^d the drained bulk modulus. M is the Biot modulus, b is the Biot coefficient of effective stress and $\boldsymbol{\alpha}$ represents the vector of supplementary internal variables describing various dissipative phenomena. The quadratic and uncoupled dependence on these internal variables is chosen just for simplicity; more complex dependencies may be considered without any alterations of the results. $\mathbf{1}$ and $\mathbf{I}$ are respectively the second order and fourth order symmetric unit tensors while $\operatorname{Tr}(\cdot)$ is the trace operator. The stress $\boldsymbol{\sigma}$, the pore pressure p and the thermodynamical forces $\boldsymbol{\chi}$ associated to $\boldsymbol{\alpha}$ are given by:

$$\boldsymbol{\sigma} = \rho\frac{\partial\Psi}{\partial\boldsymbol{\epsilon}} = \mathbf{E}^d : \boldsymbol{\epsilon}^e + bM\left[b\operatorname{Tr}\left(\boldsymbol{\epsilon}^e\right) - \zeta^e\right]\mathbf{1}, \quad p = \rho\frac{\partial\Psi}{\partial\zeta} = M\left[\zeta^e - b\operatorname{Tr}\left(\boldsymbol{\epsilon}^e\right)\right], \quad \boldsymbol{\chi} = -\rho\frac{\partial\Psi}{\partial\boldsymbol{\alpha}} \tag{8}$$

Evolution of the internal variables is given by introducing the plastic potential $F(\boldsymbol{\sigma}, p, \boldsymbol{\chi})$ and normality

$$\dot{\epsilon^p} = \dot{\lambda}\frac{\partial F}{\partial\boldsymbol{\sigma}}, \quad \dot{\zeta^p} = \dot{\lambda}\frac{\partial F}{\partial p}, \quad \dot{\boldsymbol{\alpha}} = \dot{\lambda}\frac{\partial F}{\partial\boldsymbol{\chi}} \tag{9}$$

where $\dot{\lambda}$ is the plastic multiplier satisfying the classical Kuhn–Tucker relations $\dot{\lambda} \geqslant 0$, $f \leqslant 0$, $\dot{\lambda}f = 0$ with $f = f(\boldsymbol{\sigma}, p, \boldsymbol{\chi})$ the yield function. When the *material is in plastic loading* using relations (8) and (9) and the consistency condition $\dot{f} = 0$, the rate constitutive equations may be given the following two alternative forms:

$$\dot{\boldsymbol{\sigma}} = \mathbf{H}^d : \dot{\epsilon} - \mathbf{K}\dot{p}, \qquad \dot{\zeta} = \mathbf{L} : \dot{\epsilon} + N\dot{p} \tag{10}$$

$$\dot{\boldsymbol{\sigma}} = \mathbf{H}^u : \dot{\epsilon} - \frac{\mathbf{K}}{N}\dot{\zeta}, \qquad \dot{p} = \frac{\mathbf{L}}{N} : \dot{\epsilon} + \frac{1}{N}\dot{\zeta} \tag{11}$$

having introduced the drained tangent modulus $\mathbf{H}^d$ relating the strain rate to the stress rate under drained conditions $\dot{p} = 0$ and the undrained tangent modulus $\mathbf{H}^u$ relating the strain rate to the stress rate under undrained conditions $\dot{\zeta} = 0$. These are given by

$$\mathbf{H}^d = \mathbf{E}^d - \frac{\mathbf{E}^d : \partial F/\partial\boldsymbol{\sigma} \otimes \partial f/\partial\boldsymbol{\sigma} : \mathbf{E}^d}{H^d},$$

$$\mathbf{H}^u = \mathbf{E}^u - \frac{\mathbf{E}^u : (\partial F/\partial\boldsymbol{\sigma} - bM/(3K^u)\partial F/\partial p\mathbf{1}) \otimes (\partial f/\partial\boldsymbol{\sigma} - bM/(3K^u)\partial f/\partial p\mathbf{1}) : \mathbf{E}^u}{H^u} \tag{12}$$

with the notations

$$H^d = h^d + \frac{\partial f}{\partial\boldsymbol{\sigma}} : \mathbf{E}^d : \frac{\partial F}{\partial\boldsymbol{\sigma}}, \qquad H^u = h^u + \left(\frac{\partial F}{\partial\boldsymbol{\sigma}} - \frac{bM}{3K^u}\frac{\partial F}{\partial p}\mathbf{1}\right) : \mathbf{E}^u : \left(\frac{\partial f}{\partial\boldsymbol{\sigma}} - \frac{bM}{3K^u}\frac{\partial f}{\partial p}\mathbf{1}\right) \tag{13}$$

$\mathbf{E}^u = \mathbf{E}^d + b^2M\mathbf{1} \otimes \mathbf{1}$ is the isotropic *undrained* elastic tensor, $K^u = K^d + b^2M$ the undrained bulk modulus. Finally, h^d and h^u are the drained and undrained plastic hardening moduli, respectively, related by

$$h^d = -\frac{\partial f}{\partial\boldsymbol{\chi}}.\mathbf{h}.\frac{\partial F}{\partial\boldsymbol{\chi}}, \qquad h^u = h^d + M\frac{K^d}{K^u}\frac{\partial f}{\partial p}\frac{\partial F}{\partial p} \tag{14}$$

In (10) and (11) we have also set

$$\mathbf{K} = b\mathbf{1} + \left(\frac{\partial f/\partial p - b\partial f/\partial\boldsymbol{\sigma} : \mathbf{1}}{H^d}\right)\mathbf{E}^d : \frac{\partial F}{\partial\boldsymbol{\sigma}}, \qquad \mathbf{L} = b\mathbf{1} + \left(\frac{\partial F/\partial p - b\partial F/\partial\boldsymbol{\sigma} : \mathbf{1}}{H^d}\right)\frac{\partial f}{\partial\boldsymbol{\sigma}} : \mathbf{E}^d$$

$$N = \frac{1}{M} + \frac{(\partial F/\partial p - b\partial F/\partial\boldsymbol{\sigma} : \mathbf{1})(\partial F/\partial p - b\partial F/\partial\boldsymbol{\sigma} : \mathbf{1})}{H^d} \tag{15}$$

2. Perturbation analysis

We consider an infinite poro-elastic-plastic medium and assume uniform physical properties within it. This body is remotely and uniformly loaded in such a way that a homogeneous solution in terms of stresses and strains prevails throughout it. To detect instabilities a perturbation approach is used where an infinitesimal perturbation is superposed to the solution at a generic instant of the evolution and the behaviour of the perturbation is analyzed. Stability is assured if small perturbations produce only limited changes in the solution. We denote by a superscript 0 all the fields corresponding to the homogeneous solution. This solution is such that $\mathbf{w}^0 = \nabla p^0 = \mathbf{0}$. To investigate its stability we superpose to it at a generic instant an infinitesimal perturbation denoted by δ and we analyze the behaviour of the perturbed fields $\mathbf{u} = \mathbf{u}^0 + \delta\mathbf{u}$, $\boldsymbol{\sigma} = \boldsymbol{\sigma}^0 + \delta\boldsymbol{\sigma}$, etc. Because the perturbation is small, this nonlinear problem is linearized around the reference solution (superscript 0). The linearization procedure and the derivation of the eigenvalue problem are too long to be reported here. The details may be found in [3]. We seek for the linearized problem (assuming total loading everywhere and neglecting therefore unloadings) solutions in the form

$$\delta\mathbf{X} = \widetilde{\mathbf{X}}\exp\big(\mathrm{i}\xi\mathbf{n}\cdot\mathbf{x} + \eta(t - t_0)\big) \tag{16}$$

where $\mathbf{n}$ is a polarization direction, ξ the wave number of the perturbation mode and η may be related to the local rate of growth (in time) of the perturbation. The derivation is carried out here for the general class of constitutive equations described in Section 1.

The general condition for growth of perturbations can be written

$$\det\left\{\xi^2\left[\mathbf{E}-\frac{\mathbf{P}\otimes\mathbf{Q}}{H}\right]+\left((1-\phi)\rho_s+\frac{\phi^2\rho_f}{k\rho_f\eta+\phi}\right)\mathbf{1}\bar{\otimes}\mathbf{1}\right\}=0 \tag{17}$$

with the following notations (the tensor product $\bar{\otimes}$ is such that $[\mathbf{1}\bar{\otimes}\mathbf{1}]_{ijhk}=\delta_{ik}\delta_{jh}$)

$$\mathbf{E}=\mathbf{G}+\frac{1}{r}\left(\frac{b}{3K^d}\mathbf{G}:\mathbf{1}-\phi\Omega\mathbf{1}\right)\otimes\left(\frac{b}{3K^d}\mathbf{G}:\mathbf{1}-\phi\Omega\mathbf{1}\right) \tag{18}$$

$$\mathbf{P}=\mathbf{G}:\frac{\partial F}{\partial\boldsymbol{\sigma}}-\frac{1}{r}\left(\frac{\partial F}{\partial p}-\frac{b}{3K^d}\frac{\partial F}{\partial\boldsymbol{\sigma}}:\mathbf{G}:\mathbf{1}\right)\left(\frac{b}{3K^d}\mathbf{G}:\mathbf{1}-\phi\Omega\mathbf{1}\right) \tag{19}$$

$$\mathbf{Q}=\frac{\partial f}{\partial\boldsymbol{\sigma}}:\mathbf{G}-\frac{1}{r}\left(\frac{\partial f}{\partial p}-\frac{b}{3K^d}\frac{\partial f}{\partial\boldsymbol{\sigma}}:\mathbf{G}:\mathbf{1}\right)\left(\frac{b}{3K^d}\mathbf{G}:\mathbf{1}-\phi\Omega\mathbf{1}\right) \tag{20}$$

$$H=h^d+\frac{\partial f}{\partial\boldsymbol{\sigma}}:G:\frac{\partial F}{\partial\boldsymbol{\sigma}}+\frac{1}{r}\left(\frac{\partial f}{\partial p}-\frac{b}{3K^d}\frac{\partial f}{\partial\boldsymbol{\sigma}}:\mathbf{G}:\mathbf{1}\right)\left(\frac{\partial F}{\partial p}-\frac{b}{3K^d}\frac{\partial F}{\partial\boldsymbol{\sigma}}:\mathbf{G}:\mathbf{1}\right) \tag{21}$$

$$\mathbf{G}=\left[\left(\mathbf{E}^d\right)^{-1}+\frac{\dot{\lambda}}{\eta}\frac{\partial^2F}{\partial\boldsymbol{\sigma}\partial\boldsymbol{\sigma}}\right]^{-1} \tag{22}$$

$$\omega=\frac{Mk\xi^2}{\eta}\frac{\phi}{k\rho_f\eta+\phi},\quad \Omega=\frac{k\eta\rho_f}{k\rho_f\eta+\phi},\quad s=\frac{1+\omega}{M}+\frac{b^2}{K^d}+\frac{\dot{\lambda}}{\eta}\frac{\partial^2F}{\partial p^2},\quad r=s-\frac{b^2\mathbf{1}:\mathbf{G}:\mathbf{1}}{9(K^d)^2} \tag{23}$$

3. Unbounded growth under dynamic conditions

We are interested only in the critical conditions for unbounded growth of perturbations. This corresponds to $\mathrm{Re}(\eta)\to\infty$. However, the analysis is performed for $|\eta|\to\infty$. This includes beside unbounded growth, unbounded decay ($\mathrm{Re}(\eta)\to-\infty$) and highly oscillating perturbations ($\mathrm{Re}(\eta)$ finite and $\mathrm{Im}(\eta)\to\infty$). In these situations, we have $\omega\approx M\xi^2\phi/(\rho_f\eta^2)$ and condition (17) reduces to

$$\det\left[\xi^2\mathbf{n}\cdot\left(\mathbf{H}^d+\frac{(\mathbf{K}-\phi\mathbf{1})\otimes(\mathbf{L}-\phi\mathbf{1})}{N+\xi^2\phi/(\rho_f\eta^2)}\right)\cdot\mathbf{n}+(1-\phi)\rho_s\eta^2\mathbf{1}\right]=0 \tag{24}$$

and this is exactly the characteristic equation for the speeds c of acceleration waves if one sets $\eta=-\mathrm{i}\xi c$ (see [4]). Using the relation giving the determinant of a rank-one update of a 3×3 matrix, Eq. (24) is recast to the following form

$$\rho_f c^2 N\det\left[\mathbf{n}\cdot\mathbf{H}^a\cdot\mathbf{n}-(1-\phi)\rho_s c^2\right]-k\det\left[\mathbf{n}\cdot\mathbf{H}^d\cdot\mathbf{n}-(1-\phi)\rho_s c^2\right]=0 \tag{25}$$

$$\mathbf{H}^a=\mathbf{H}^d+\frac{(\mathbf{K}-\phi\mathbf{1})\otimes(\mathbf{L}-\phi\mathbf{1})}{N} \tag{26}$$

As $\eta=-\mathrm{i}\xi c$, $|\eta|\to\infty$ corresponds either to $\xi\to\infty$ or $|c|\to\infty$. From (25) it is easily seen that the second situation is excluded and one concludes that $|\eta|\to\infty$ corresponds necessarily to the shortwavelength limit $\xi\to\infty$. Unbounded growth of perturbations corresponds therefore to $c^2\leqslant0$ or c^2 complex. Inception of unbounded growth is related to stationary discontinuities or to flutter instabilities. A similar result was also obtained in [5] for growth of harmonic waves. As in [4], putting $c=0$ in (24), one gets the condition for stationary discontinuities

$$\det\left[\mathbf{n}\cdot\mathbf{H}^d\cdot\mathbf{n}\right]=0 \tag{27}$$

For slow dynamic processes as those occurring in seismic analyses, the inertial terms of the fluid can be neglected. This is obtained by setting $\rho_f = 0$. The eigenvalue equation (24) becomes here

$$\det\left[\xi^2 \mathbf{n}\cdot\left(\mathbf{H}^d + \frac{\mathbf{K}\otimes\mathbf{L}}{N + k\xi^2/\eta}\right)\cdot\mathbf{n} + (1-\phi)\rho_s\eta^2\mathbf{1}\right] = 0 \tag{28}$$

4. Unbounded growth under quasi-static conditions

The quasi-static case is recovered by putting $\rho_s = \rho_f = 0$ in all the equations. Condition (17) is recast to

$$\det\left[\xi^2 \mathbf{n}\cdot\left(\mathbf{H}^d + \frac{\mathbf{K}\otimes\mathbf{L}}{N + k\xi^2/\eta}\right)\cdot\mathbf{n}\right] = 0 \tag{29}$$

Now using the relation giving the determinant of a rank-one update of a 3×3 matrix, from (29) one obtains

$$\eta N \det\left[\mathbf{n}\cdot\mathbf{H}^u\cdot\mathbf{n}\right] + k\xi^2 \det\left[\mathbf{n}\cdot\mathbf{H}^d\cdot\mathbf{n}\right] = 0 \tag{30}$$

involving both the drained and the undrained acoustic tensors of the saturated porous medium. The following conclusions are immediate from (30):

- If for a given direction $\mathbf{n}$, $\det(\mathbf{n}\cdot\mathbf{H}^u\cdot\mathbf{n}) = 0$ and $\det(\mathbf{n}\cdot\mathbf{H}^d\cdot\mathbf{n}) \neq 0$, then from (30) $\xi = 0$ and η is arbitrary and therefore can be unbounded.
- In the special case where for a given direction $\mathbf{n}$, $\det(\mathbf{n}\cdot\mathbf{H}^d\cdot\mathbf{n}) = 0$ and $\det(\mathbf{n}\cdot\mathbf{H}^u\cdot\mathbf{n}) = 0$, then from (30) both η and ξ are arbitrary. All wavelengths grow at arbitrary rates and particularly at unbounded rates. This is the situation that prevails for one-phase rate-independent materials.
- When $\det(\mathbf{n}\cdot\mathbf{H}^u\cdot\mathbf{n}) \neq 0$, the rate of growth η is given by

$$\eta = -\frac{k\xi^2}{N}\frac{\det[\mathbf{n}\cdot\mathbf{H}^d\cdot\mathbf{n}]}{\det[\mathbf{n}\cdot\mathbf{H}^u\cdot\mathbf{n}]} \tag{31}$$

  and one sees that unbounded growth occurs for $\xi\to\infty$ when

$$\det\left(\mathbf{n}\cdot\mathbf{H}^d\cdot\mathbf{n}\right)\det\left(\mathbf{n}\cdot\mathbf{H}^u\cdot\mathbf{n}\right) < 0 \tag{32}$$

The above discussion allows us to highlight some features and peculiarities of failure modes in porous media under quasi-static conditions.

First of all, when the drained acoustic tensor becomes singular before the undrained one during a loading process, infinitely shortly after the critical condition is passed, unbounded rates of growth are available and these rates are associated to infinitely small wavelengths ($\xi\to\infty$). This suggests that the failure mode is a localized one.

Secondly, when the undrained acoustic tensor becomes singular before the drained one, and this may occur for non-associative materials, as discussed in [6], unlimited rates of growth are available for any ξ. This suggests that the failure mode may be either a diffuse or a localized one.

5. Conclusions

In this work the conditions for unbounded growth of perturbation in poroplastic media, under dynamic and under quasi-static conditions have been studied and the differences between the two cases highlighted in particular. While in dynamics perturbations growing unboundedly are always associated to the shortwavelength regime and the failure mode expected is localized, under quasi-static conditions it may be associated to the full range of wavelengths and the failure mode may be diffuse or localized. While the results obtained for the shorwavelength regime apply for finite bodies, the longwavelength regime need to be reexamined in the light of boundary conditions.

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