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Structural optimization in truth dimension

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Abstract Optimization theory has advanced considerably during the last three decades as is illustrated by a vast number of published books, surveys, and papers concerning this subject. An optimal decision under uncertainty conditions is dependent on the Engineer's objectives, which may be not known with certainty or represented by natural language. To deal with this problem, in this paper, a new approach based on coupling the existing knowledge of experts and numerical results obtained from traditional optimization techniques is presented using non-conventional logic. ***To cite this article: C. Tran, C. R. Mecanique 330 (2002) 609–614.***

analytical mechanics / multi-objective optimization / fuzzy logic / modal logic / multi-valued logic

Optimisation structurale dans la dimension de vérité

Résumé La théorie d'optimisation a fait des progrès considérables pendant les trois dernières décennies, comme en témoigne un nombre important de livres, articles et articles de revues consacrés à ce sujet. Une décision optimale dans des conditions d'incertitude dépend des objectifs fixés par l'Ingénieur ; ces objectifs peuvent ne pas être totalement connus ou exprimés en langage naturel. Afin de traiter ce problème, nous proposons dans cette Note une approche nouvelle basée sur le couplage de la connaissance existante des experts avec des résultats numériques obtenus à partir de techniques d'optimisation usuelles, en appliquant la logique non-conventionelle. ***Pour citer cet article : C. Tran, C. R. Mecanique 330 (2002) 609–614.***

mécanique analytique / optimisation multi-objet / logique floue / logique modale / logique à valeurs multiples

1. Introduction

Although engineer's objectives are usually very general or ambiguous and their decisions are dependent on many factors not known with certainty, the success of the engineering approach is, in fact, evident from the history of technology and the applied science. It reflects the correctness of the logical principles of engineers in relation to the real world. However, without fundamentals of formal logic for reasoning, decision-makers might, in many cases, use mythical logic. It may result in a set of incomplete decisions, which are sometimes contrary to solutions resulting from mathematical models. To solve this problem, an

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approach based on the fuzzy set theory was proposed by Bellman and Zadeh [1], in which, a decision-making process is a restriction of objectives and constraints to the values, x, to be chosen by decision-maker. The restriction is given by two fuzzy subsets A and B, their intersection giving the range of the values x satisfying both the objectives and the constraints, i.e., it gives the subset C of decisions through the membership function:

$$\mu_C(x) = \min\{\mu_A(x), \mu_B(x)\}$$

Much literature has been devoted to this subject using this definition of decision. However, an a priori justification of union and intersection operators (presented in Bellman and Zadeh's definition) suited to each specific real-world problem is highly problematic in practice, as pointed out by Sakawa [2]. In this work, a process of mathematical reasoning for dealing with human experiences through different truth values will be presented.

2. Multi-objective optimization of structures

Generally, the optimal design of any structure has been restricted to the mathematical programming model:

$$\underset{\forall \mathbf{x}\in E, \mathbf{y}\in R}{\text{maximum(minimum)}} \left\{\mathbf{F} = \left(f_1(\mathbf{x},\mathbf{y}), f_2(\mathbf{x},\mathbf{y}), \ldots, f_k(\mathbf{x},\mathbf{y})\right)\right\} \tag{1}$$

where: $\mathbf{x} = (x_1, x_2, \ldots, x_n)$ is a vector of design variables, $\mathbf{y} = (y_1, \underline{y}_2, \ldots, y_r)$ is a vector of random variables, $f_i(\mathbf{x},\mathbf{y})$, $i = 1, 2, \ldots, k$, are objective functions, k denotes the number of objective functions, $R = \{y \mid u(\mathbf{y})\}$, where $u(\mathbf{y})$ is any probability distribution of random variables, E denotes the permissible space, which is expressed as follows:

$$E = \left\{\mathbf{x} \mid g_j(\mathbf{x},\mathbf{y}) \geqslant 0,\ j = 1, 2, \ldots, m\right\} \tag{2}$$

where: $g(\mathbf{x},\mathbf{y})$ denotes the optimal conditions, m is number of the optimal conditions, $\mathbf{y}$ denotes a vector of random variables.

3. Truth values

Let us return to the early 'black or white' reasoning, for example, beginning with Aristotle's syllogistic, i.e., using the two symbols: T (true) and $\neg T = F$ (false) – two-valued logic, we represent 'black' by two possibilities $(B, \neg B)$ and 'white' by, $(W, \neg W)$. Then, we have 4 possible cases:

$$\text{`}\textit{black or white}' = \{BW, B\neg W, \neg BW, \neg B\neg W\} \tag{3}$$

To solve optimization problems we can traditionally use this kind of choice; to select one option from two possibilities according to some criterion on which the two can be compared. Now, from modal logic using box connective, $\Box$, and diamond connective, $\Diamond$, we can represent '*black*' by 4 symbols, ($\Box B$, $\Diamond B$, $\neg\Box B$, $\neg\Diamond B$), where, $\Box B$ and $\Diamond B$ are read: 'B is necessary' and 'B is possible' respectively; '*white*' by ($\Box W$, $\Diamond W$, $\neg\Box W$, $\neg\Diamond W$). Then, we have 16 possible cases:

$$\begin{aligned} &\text{`}\textit{black or white}\text{'} \\ &\quad = \{\Box B\Box W, \Box B\Diamond W, \Box B\neg\Box W, \Box B\neg\Diamond W, \Diamond B\Box W, \Diamond B\Diamond W, \Diamond B\neg\Box W, \Diamond B\neg\Diamond W, \\ &\qquad \neg\Box B\Box W, \neg\Box B\Diamond W, \neg\Box B\neg\Box W, \neg\Box\neg\Diamond W, \neg\Diamond B\Box W, \neg\Diamond B\Diamond W, \\ &\qquad \neg\Diamond B\neg\Box W, \neg\Diamond B\neg\Diamond W\} \end{aligned} \tag{4}$$

Thus, 4 truth-values allow us to enter much possible cases, which are *individually viable in our mind*. Generally, it expresses an effect of multi-valued logic and suggests a new kind of choice for optimization problems – to select one option according to some criterions in truth dimension. Other logics, in which the truth values are labeled by rational numbers in the unit interval $[0, 1]$ or by linguistic variables, are called multi-valued logic or fuzzy logic respectively. Discussion about mentioned logics beyond this note. Using some ideas from them to solve the multi-objective optimization problems is presented below.

4. Optimization with truth measures

Let a set Q be in the domain of the objective function, f, such that:

$$f : Q \to RL \tag{5}$$

where, RL (real line) is an order structure of solution space. It is used to define alternative solutions. Here, the preferences between alternatives are described by objective function. In this approach, some logical operators can be used to observe two directions characterized by two relations: greater or lower. Here, 'true' and 'false' are sufficient to define either existence or non-existence of the optimal solution. In contrast, preferences of the alternatives described in multi-objective optimization are defined using the Pareto-optimal concept as follows: vector $\mathbf{F}^0 = \mathbf{F}(\mathbf{x}^0)$ is the Pareto-optimal solution of the model (1), (2), if subjected to following expression:

$$\left\{\forall i \in (1, 2, \ldots, k)\left\lfloor \neg\left(f_i^0 \geqslant f_i\right)\right\rfloor,\ \forall f \in H\right\} \wedge \left\{\exists l \in (1, 2, \ldots, k)\left\lfloor f_l^0 > f_l\right\rfloor\right\} \tag{6}$$

where H is k-dimension space containing different values of objective functions. Using the first kind of choice mentioned above, according to the Preto-optimal theory, we have a qualitative description based on relations (d_i, $i = 1, 2, \ldots, k$) between any alternative $(\mathbf{x}, \mathbf{F}(\mathbf{x}, \mathbf{y}))$ to the ideal solution $(\mathbf{x}', \mathbf{F}'(\mathbf{x}, \mathbf{y}))$, which is represented as follows:

$$\mathrm{d}_i(\mathbf{x}, \mathbf{y}) = \frac{|f_i(\mathbf{x}, \mathbf{y}) - f_i'(\mathbf{x}, \mathbf{y})|}{f_i'(\mathbf{x}, \mathbf{y})} \tag{7}$$

$$f_i'(\mathbf{x}, \mathbf{y}) = \max_{\mathbf{x}\in E,\ \mathbf{y}\in R}(\min)\left\{f_i(\mathbf{x}, \mathbf{y})\right\} \tag{8}$$

Then the general system of objective functions becomes:

$$\overset{*}{\min_{\mathbf{x}\in E,\ \mathbf{y}\in R}} \left\{\mathbf{D}(\mathbf{x}, \mathbf{y}) = \left[\mathrm{d}_1(\mathbf{x}, \mathbf{y}), \mathrm{d}_2(\mathbf{x}, \mathbf{y}), \ldots, \mathrm{d}_k(\mathbf{x}, \mathbf{y})\right]\right\} \tag{9}$$

Now, we construct the description of value d_i in the real line RL. Each value d_i of the attribute D may be described in the real line as a vector:

$$\mathbf{d}_i \overset{RL}{=} \left(\mathrm{d}_i^1, \mathrm{d}_i^2, \ldots, \mathrm{d}_i^n\right) \tag{10}$$

Evaluation of the decision-maker, usually through truth measures, maps a concrete value d_i^n from a semantic scale (measuring in various traditional units) onto a universal scale (using truth measures). On this scale, the linguistic estimates of the decision-maker may be formalized as a fuzzy subset independent on the semantic of the objective functions.

$$\tau : \mathbf{D} \to [0, 1] \tag{11}$$

It represents the degree of the decision-maker's aspiration according to each objective function. Let ${}^0\mathrm{d}_i$ be totally required level for f_i and ${}_0\mathrm{d}_i$ is an unacceptable level for f_i. The truth function $\tau(\mathbf{x}, \mathbf{y})$, which is a strictly monotone and continuous function with respect to $\mathrm{d}_i(\mathbf{x}, \mathbf{y})$ will be determined as truth measure as follows:

$$\tau(\mathbf{x}, \mathbf{y}) = \begin{cases} 0 & \text{for } \mathrm{d}_i(\mathbf{x}, \mathbf{y}) = {}_0\mathrm{d}_i(\mathbf{x}, \mathbf{y}) \\ 1 - \mathrm{d}_i(\mathbf{x}, \mathbf{y}) & \text{for } \mathrm{d}_i(\mathbf{x}, \mathbf{y}) \in \left({}^0\mathrm{d}_i(\mathbf{x}, \mathbf{y}), {}_0\mathrm{d}_i(\mathbf{x}, \mathbf{y})\right) \\ 1 & \text{for } \mathrm{d}_i(\mathbf{x}, \mathbf{y}) = {}^0\mathrm{d}_i(\mathbf{x}, \mathbf{y}) \end{cases} \tag{12}$$

It makes it possible to transfer, through truth measures, the values of the objective functions from the semantic scale to linguistic estimates on the universal scale. Then, for each value d_i we have correspondingly a fuzzy truth value (in sense of multi-valued logic). It can be expressed as:

$$\mathrm{d}_i \to \mathrm{d}_i|\tau_i \tag{13}$$

An ideal solution $\mathbf{x}^*$ is perceived as the preferred solution in respect of truth τ.

The decision-maker's required solution d_i should be in the left vicinity of ${}^0\mathrm{d}_i$. To express the experiences extracted from the decision-maker, three hedges: T, FT, VT (true, fairly true, very true) are often used. In logic term, 'linguistic hedges' (or simply hedges) are special linguistic terms by which other linguistic terms are modified. Hedges are used for modifying fuzzy predicates, fuzzy truth-values. We can construct a modified proposition using linguistic hedge, H, represented a decision-maker's judgment to restrict $\tau(\mathbf{x},\mathbf{y})$ to which s is assigned. The preferred degree of the decision-maker's estimates is presented by $\tau^*(\mathrm{d})$:

$$\tau^*(\mathrm{d}) = \mathit{TFM}\big\{\mathsf{H}\big(\tau(\mathbf{x},\mathbf{y})\big) = s,\ s \in (T, FT, VT)\big\} \tag{14}$$

where $\mathit{TFM}\{\cdot\}$ is Zadeh–Baldwin's logic operation called the truth function modification (see Baldwin [3])'. The TFM of $\mathrm{d} - \tau^*(\mathrm{d})$ is graphically represented in Fig. 1, in which LT, LVT denote 'Logic True', 'Logic Very True' or in short: 'True' and 'very True' respectively.

In the truth dimension, by the verbalization procedure, the decision-maker can change one knowledge in respect of truth as well as integration with another.

In multi-optimization problems, let $\tau^{\mathrm{imp}}(f_i(\mathbf{x},\mathbf{y})) \in [0,1]$, determined by the designer, be an important degree of the objective function f_i. Then, each Pareto-optimal solution must be subjected to the condition: "All of preferred degrees of the decision-maker $\tau^*(\mathrm{d}_i)$, $i = 1, 2, \ldots, k$, are satisfied with important degrees of all the corresponding objective function $\tau^{\mathrm{imp}}(f_i)$". That is to say: "*It is not possible that both important degrees of all the corresponding objective functions are true and decision-maker's estimates are false*."

In logic term of modal logic, we can describe this condition as follows:

$$\neg\Diamond\big\{\big(\tau^{\mathrm{imp}}(f_i) = \mathrm{true}\big) \wedge \neg\big(\tau^*(\mathrm{d}_i) = \mathrm{true}\big)\big\} \tag{15}$$

It is the interpretation of logic implication "⇒" of modal logic, i.e.,

$$\big\{\tau^{\mathrm{imp}}(f_i) = \mathrm{true}\big\} \Rightarrow \big\{\tau^*(\mathrm{d}_i) = \mathrm{true}\big\} \equiv I\big\{\big[\tau^{\mathrm{imp}}(f_i) = \mathrm{true}\big], \big[\tau^*(\mathrm{d}_i) = \mathrm{true}\big]\big\} \tag{16}$$

where, I denotes a fuzzy implication, which is a function expressed in the form:

$$I : [0,1] \times [0,1] \to [0,1] \tag{17}$$

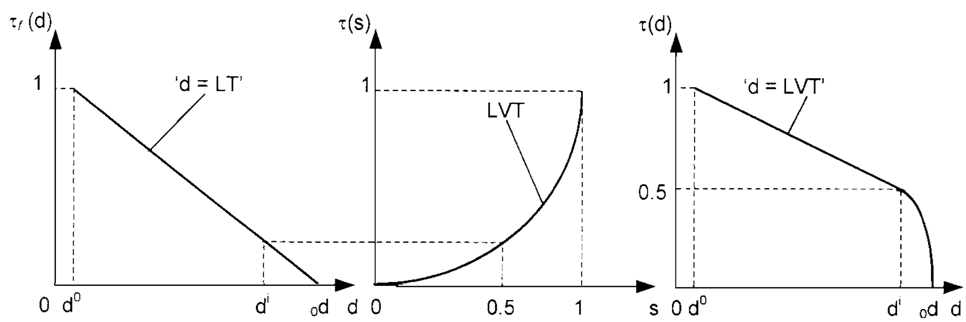

Figure 1. The truth modification of expert's estimation 'very true'.

We can perform the quantification of this fuzzy implication using Łukasiewicz's multi-valued logic.

$$I\{\tau^{\mathrm{imp}}(f_i), \tau^*(\mathrm{d}_i)\} = \min\{1, 1 - \tau^{\mathrm{imp}}(f_i) + \tau^*(\mathrm{d}_i)\} \tag{18}$$

Thus, we obtain new condition for which an 'optimistic' possibility of the decision-maker is formulated as:

$$\operatorname*{minimum}_i I\{\tau^{\mathrm{imp}}(f_i(\mathbf{x}, \mathbf{y})), \tau^*(\mathrm{d}_i)\}, \quad i = 1, 2, 3, \ldots, k \tag{19}$$

The optimal solution, SF, of the fuzzy optimization problem can be defined by maximizing the 'optimistic' possibility of the decision-maker. It is represented by:

$$SF = \max_j \operatorname*{minimum}_i I\{\tau^{\mathrm{imp}}(f_i(\mathbf{x}, \mathbf{y})_j), \tau^*(\mathrm{d}_i)_j\}, \quad i = 1, 2, 3, \ldots, k \tag{20}$$

where, $j = 1, 2, \ldots, m$, which denotes jth solution being in Pareto-solution set.

5. Example calculation

In Fig. 2, a model and numerical data used for optimization of a 'retaining wall structure' are presented. For the mathematical model, vector of objectives and technical conditions are defined as follows:

$$F = \{f_1(\mathbf{x}), f_2(\mathbf{x}, \mathbf{y}), f_3(\mathbf{x}, \mathbf{y}), f_4(\mathbf{x}, \mathbf{y})\};$$
$$g_1(\mathbf{x}, \mathbf{y}) = \sum H^p / \sum H^g(\tan\phi), \qquad g_2(\mathbf{x}, \mathbf{y}) = \sum M_0 / \sum M_n,$$
$$g_3(\mathbf{x}, \mathbf{y}) = QI/R_v(\cos\delta), \qquad g_4(\mathbf{x}, \mathbf{y}) = M_1/M_2,$$

where, $\sum M_0$ is the sum of anti-clockwise moments. $\sum M_n$ is the sum of clockwise moments. QI is the vertical component of the bearing capacity of subsoil under the retaining wall. R_v is the vertical component of the resultant load, δ is the inclination of the resultant load with respect to the normal to the foundation plane. H^p is the horizontal component of the resultant load acting onto the foundation of the wall. M_1 is the resisting moment of the 'soil-wall' mass as a whole, M_2 is the overturning moment of the 'soil-wall' system. In the objective functions, $f_1(\mathbf{x})$ denotes the total weight of the wall; $f_2(\mathbf{x}, \mathbf{y})$ is the stability of the wall; $f_3(\mathbf{x}, \mathbf{y})$ denotes the volume of earthworks; $f_4(\mathbf{x}, \mathbf{y})$ is the safety index (β) of the 'soil-wall' system.

To solve the non-fuzzy multi-objective optimization problem, a Monte Carlo simulation is firstly applied. The non-fuzzy Pareto solution, F^*, obtained from a Pareto solution set is:

$$F^* = [154.7579, \ 2.7996, \ 102.3521, \ 2.6680]$$

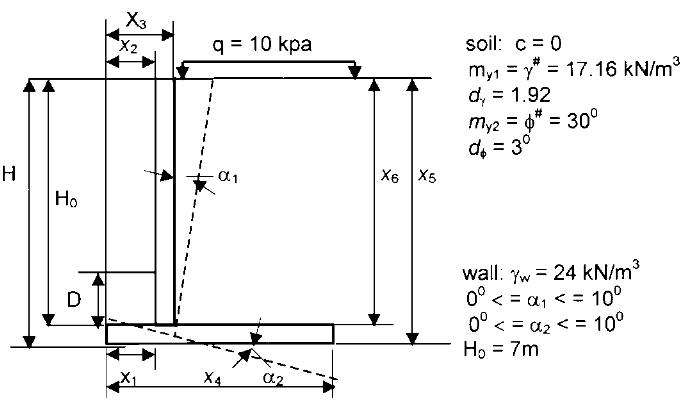

Figure 2. Design variables of *retaining wall structure*, where $\alpha_1\alpha_2$ denote inclinations of the wall and the foundation plane.

Next, assessment of the Pareto-solution set (obtained from traditional method) using linguistic estimates is described in the matrix $\mathbf{L}_1$, according to the important degree of each objective function, $\tau^{\mathrm{imp1}}(F(\mathbf{x},\mathbf{y}))$, (in first case) and in second case: $\mathbf{L}_2$, $\tau^{\mathrm{imp2}}(F(\mathbf{x},\mathbf{y}))$,

$$\mathbf{L}_1 = \begin{bmatrix} T & FT & FT & T \\ FT & FT & T & FT \\ FT & FT & FT & T \\ FT & FT & FT & T \\ FT & FT & FT & FT \\ FT & FT & FT & T \\ FT & FT & FT & T \\ FT & FT & FT & FT \end{bmatrix}, \qquad \mathbf{L}_2 = \begin{bmatrix} T & T & T & T \\ T & T & T & T \\ T & T & T & T \\ T & T & T & T \\ T & T & T & T \\ T & T & T & T \\ T & T & T & T \\ T & T & T & T \end{bmatrix}$$

$$\tau^{\mathrm{imp1}}\big(F(\mathbf{x},\mathbf{y})\big) = \big\{1.0|f_1(\mathbf{x},\mathbf{y}), 1.0|f_2(\mathbf{x},\mathbf{y}), 0.5|f_3(\mathbf{x},\mathbf{y}), 0.8|f_4(\mathbf{x},\mathbf{y})\big\}$$

$$\tau^{\mathrm{imp2}}\big(\mathbf{F}(\mathbf{x},\mathbf{y})\big) = \big\{1.0|f_1(\mathbf{x},\mathbf{y}), 1.0|f_2(\mathbf{x},\mathbf{y})|, 1.0|f_3(\mathbf{x},\mathbf{y})|, 1.0|f_4(\mathbf{x},\mathbf{y})\big\}$$

it resulted in the fuzzy-optimal solutions F_1^{**} and F_2^{**}, respectively:

$$F_1^{**} = [153.8382,\ 2.5276,\ 94.8450,\ 1.9758] \qquad \text{with } SF_1 = 0.8211$$

$$F_2^{**} = [154.7579,\ 2.7996,\ 102.3521,\ 2.6680] \qquad \text{with } SF_2 = 0.8391$$

It is interesting to note that when linguistic estimates, 'true', of the decision-maker are used for all of Pareto-solution set (second case), we obtain $F_2^{**} = F^*$. It indicates that result F_2^{**} obtained, based on fuzzy logic (in second case too), is equivalent to result F^*, obtained from a classical optimization problem, based on two-valued logic. We would like to emphases in this case that binary logic is, from a quantitative point of view, a particular reduction of multi-valued logic. Moreover, this result reflects the consistency of the proposed method.

6. Conclusions

To solve the optimization problems we are, at the moment, confined by two-valued logic. Although we would be able to find out, in the framework of this logic, the optimal solution in respect of different measures, which are characterized by assigning numbers to lengths, volumes, money etc., we would lose an ability to find out an optimal solution in respect of truth. Logic is product of our mind; it enables us to think more soundly about solved problems. It indicates that modal logic, multi-value logic including two-valued logic, and fuzzy logic are necessary for dealing, in truth dimensions, with insufficient and incomplete information of optimization problems, in which engineering experiences play an important role.

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