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3D elastic neutral inclusions with imperfect interfaces

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Abstract Using a physically-based constitutive law for imperfect interfaces, this work deals with the problem of embedding a three-dimensional elastic inclusion in a uniformly stressed elastic matrix without changing the initial stress field of the latter. Necessary and sufficient conditions are deduced for the existence of such a neutral inclusion. When the constituent materials of the matrix and inclusion are isotropic, these conditions are firstly specified and then illustrated by two simple examples. ***To cite this article: Q.-C. He, C. R. Mecanique 330 (2002) 691–696.***

analytical mechanics / inhomogeneities / neutral inclusions / interfaces / three-dimensional elasticity

Inclusions élastiques neutres 3D avec interfaces imparfaites

Résumé En utilisant une loi de comportement physiquement fondée pour des interfaces imparfaites, ce travail traite le problème d'obtention d'une inclusion élastique insérée dans une matrice élastique soumise à un champ de contraintes uniforme, et préservant ce champ de contraintes. Des conditions nécessaires et suffisantes pour l'existence d'une telle inclusion élastique neutre 3D sont établies. Quand les matériaux constituant la matrice et l'inclusion sont isotropes, ces conditions sont d'abord explicitées et ensuite illustrées par deux exemples simples. ***Pour citer cet article : Q.-C. He, C. R. Mecanique 330 (2002) 691–696.***

mécanique analytique / hétérogénéités / inclusions neutres / interfaces / élasticité tridimensionnelle

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Une inclusion insérée dans une matrice soumise à un champ de contraintes est dite neutre si la présence de l'inclusion ne perturbe pas le champ de contraintes initial de la matrice. La notion d'inclusion neutre, initialement introduite par Mansfield [1], a récemment été appliquée et étendue à d'autres phénomènes physiques [1–6]. Jusqu'à présent, les résultats relatifs aux inclusions élastiques neutres semblent limités à l'élasticité bidimensionnelle (2D). En se basant sur une loi de comportement physiquement fondée et mathématiquement cohérente pour des interfaces imparfaites [7–9], ce travail présente une approche générale pour obtenir des inclusions élastiques neutres tridimensionnelles (3D).

Le matériau élastique linéaire homogène constituant la matrice $\Omega^{(1)}$ est défini par le tenseur de complaisance $\mathbf{C}^{(1)}$ et celui constituant l'inclusion $\Omega^{(2)}$ est décrit par le tenseur de complaisance $\mathbf{C}^{(2)}$. L'interface Γ entre $\Omega^{(1)}$ et $\Omega^{(2)}$, supposée imparfaite, est modélisée par l'équation (2) traduisant la continuité des contraintes et par les équations (6) formulant la proportionnalité du saut $[\![\mathbf{u}_{\mathrm{n}}]\!]$ de

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déplacements normaux au vecteur de contraintes normales $\mathbf{p}_n$ et la proportionnalité du saut $[\![\mathbf{u}_t]\!]$ de déplacements tangentiels au vecteur de contraintes tangentielles $\mathbf{p}_t$. La constante k_n caractérise la rigidité normale de Γ et la constante k_t sa rigidité tangentielle. Pour que k_n et k_t soient physiquement admissibles, les inégalités (7) doivent être satisfaites.

Le champ de contraintes $S^{(1)}$ et le champ de déplacements $\mathbf{u}^{(1)}$ dans la matrice $\Omega^{(1)}$ sont supposés donnés. La condition de continuité (2) implique que l'inclusion $\Omega^{(2)}$ est soumise au vecteur de contraintes $\mathbf{p}$ sur sa frontière Γ. Donc, le champ de contraintes $S^{(2)}$ et le champ de déplacements $\mathbf{u}^{(2)}$ dans $\Omega^{(2)}$ sont déterminés par la résolution du problème (8). En utilisant $\mathbf{u}^{(1)}$, $\mathbf{p}$ et $\mathbf{u}^{(2)}$ ainsi obtenus, nous déduisons de (6) et (7) les conditions nécessaires et suffisantes (9) pour la neutralité de $\Omega^{(2)}$. Dans le cas où le champ $S^{(1)}$ est uniforme, l'homogénéité du matériau constituant $\Omega^{(2)}$ nous conduit aux conclusions (10)–(12). Les conditions (9) peuvent donc être explicitées à l'aide de (13)–(15).

Quand les matériaux élastiques linéaires constituant la matrice et l'inclusion sont isotropes et caractérisés par (16), le saut de déplacements a l'expression (18). L'inclusion $\Omega^{(2)}$, insérée dans la matrice $\Omega^{(1)}$ soumise à un champ S de contraintes uniforme, est neutre si et seulement si les conditions (20) sont vérifiées. Ces conditions sont appliquées à une inclusion sphérique, de rayon r et centrée à l'origine. Dans un premier temps, le champ de contraintes S est supposé hydrostatique. Dans ce cas, l'inclusion sphérique est neutre si et seulement si k_n satisfait (21). Ensuite, le champ de contraintes S est pris comme étant déviateur. Les conditions nécessaires et suffisantes correspondantes pour la neutralité de l'inclusion sphérique s'expriment par (22). Notons que les hypothèses (17) sont essentielles pour garantir la non négativité de k_n et k_t.

1. Introduction

Consider a solid body consisting of a homogeneous material and undergoing a mechanical or physical field. For some practical or theoretical reasons, a hole is, effectively or virtually, made in the body and filled with another homogeneous material. The latter, acting as inhomogeneity, is called an inclusion, intrusion or extrusion, according to its topological location relative to the intact part referred to as the matrix. An inhomogeneity is said to be neutral if its presence does not disturb the initial field in the matrix.

The concept of neutral inhomogeneities can be traced back to the work of Mansfield [1], who showed that a hole in a uniformly stressed plate can be reinforced without altering the initial stress field over the remaining part of the plate. In different contexts and particularly in relation to the development of micromechanics, the problem of finding out neutral inhomogeneities has recently been investigated and extended to a number of physical phenomena (see, e.g., [1–6]). For thermal, electrical and other steady transport phenomena, three-dimensional (3D) neutral inhomogeneities have been shown to exist. For elasticity, two-dimensional (2D) neutral inhomogeneities have been found, but no 3D neutral inhomogeneities seem to have been reported. The reason is probably that the setting of 3D elastic problems is much more complicated than that of 3D steady transport problems.

In the present work, a physically-based and widely spread interface law is used to model imperfect interfaces, and the problem of finding 3D elastic neutral inclusions is addressed and formulated in a general way. Necessary and sufficient geometrical and material conditions are then deduced for obtaining a 3D elastic neutral inclusion embedded in a uniformly stressed elastic matrix. As examples of application, two spherical neutral inclusions are constructed in the case when the constituent materials of the matrix and inclusion are isotropic and when the initial uniform stress field is hydrostatic or deviatoric.

The notation adopted in this work is as follows. Scalars are denoted by Greek letters, and vectors by bold-face minuscule Latin letters. Second- and fourth-order tensors are designated by light- and bold-face majuscule Latin letters, respectively. The components of a vector, second- or fourth-order tensor are represented by the corresponding light-face letter with a suitable number of subscripts.

2. An imperfect interface law

Let a solid body consist of a linearly elastic homogeneous material characterized by the compliance tensor $\mathbf{C}^{(1)}$, occupy a domain Ω of a 3D Euclidian space $\mathbf{R}^3$ and undergo a stress field $\mathrm{S}^{(1)}$ due to a prescribed external loading. This body is firstly cut out over an inner subdomain $\Omega^{(2)}$ of Ω and secondly reinforced by a linearly elastic homogeneous material defined by the compliance tensor $\mathbf{C}^{(2)}$. The interface between the inclusion $\Omega^{(2)}$ and the matrix $\Omega^{(1)} = \Omega \setminus \Omega^{(2)}$ is denoted by Γ. If this interface is perfect, the traction and displacement vectors must be continuous across it and the initial stress field $\mathrm{S}^{(1)}$ over the matrix $\Omega^{(1)}$ cannot be preserved because $\mathbf{C}^{(2)}$ is different from $\mathbf{C}^{(1)}$.

In this work, the interface Γ is assumed to be imperfect and comply with the spring-layer model. In other words, the tractions are assumed to be continuous across Γ and the displacements to be discontinuous across Γ but proportional to the corresponding tractions. The spring-layer model is widely used in describing imperfect interfaces. For the physical soundness and mathematical coherence of this model, the reader can refer to [7–9] and the cited references.

Let $\mathbf{n}(\mathbf{x})$ designate the unit normal vector to Γ at $\mathbf{x} \in \Gamma$ and from $\Omega^{(2)}$ to $\Omega^{(1)}$. The traction vectors acting at $\mathbf{x} \in \Gamma$ and associated to the stress fields $\mathrm{S}^{(1)}$ over $\Omega^{(1)}$ and $\mathrm{S}^{(2)}$ over $\Omega^{(2)}$ are given by

$$\mathbf{p}^{(i)}(\mathbf{n}) = \mathrm{S}^{(i)}\mathbf{n} \quad (i = 1, 2) \tag{1}$$

According to the spring-layer interface model, the tractions are continuous across Γ, so that

$$\mathbf{p}(\mathbf{n}) = \mathbf{p}^{(1)}(\mathbf{n}) = \mathbf{p}^{(2)}(\mathbf{n}) \tag{2}$$

The vector $\mathbf{p}$ can be decomposed into a normal traction $\mathbf{p}_\mathrm{n}$ and a tangent traction $\mathbf{p}_\mathrm{t}$:

$$\mathbf{p}_\mathrm{n} = (\mathbf{n} \otimes \mathbf{n})\mathbf{p}, \qquad \mathbf{p}_\mathrm{t} = (\mathrm{I} - \mathbf{n} \otimes \mathbf{n})\mathbf{p} \tag{3}$$

with the second-order identity tensor I. Denoting by $\mathbf{u}^{(i)}$ the displacement field over $\Omega^{(i)}$, the displacement jump vector is given by

$$[\![\mathbf{u}]\!] = \mathbf{u}^{(1)} - \mathbf{u}^{(2)} \tag{4}$$

and can also be split into a normal displacement jump vector $[\![\mathbf{u}_\mathrm{n}]\!]$ and a tangent one $[\![\mathbf{u}_\mathrm{t}]\!]$:

$$[\![\mathbf{u}_\mathrm{n}]\!] = (\mathbf{n} \otimes \mathbf{n})[\![\mathbf{u}]\!], \qquad [\![\mathbf{u}_\mathrm{t}]\!] = (\mathrm{I} - \mathbf{n} \otimes \mathbf{n})[\![\mathbf{u}]\!] \tag{5}$$

The interface Γ is assumed to be transversely isotropic everywhere with respect to the normal axis characterized by $\mathbf{n}$. Thus, according to the spring-layer model, the normal and tangent displacement jumps are related to the tractions counterparts as

$$\mathbf{p}_\mathrm{n} = \mathrm{k}_\mathrm{n}[\![\mathbf{u}_\mathrm{n}]\!], \qquad \mathbf{p}_\mathrm{t} = \mathrm{k}_\mathrm{t}[\![\mathbf{u}_\mathrm{t}]\!] \tag{6}$$

Above, k_n characterizes the normal stiffness of Γ and k_t the tangent stiffness of Γ. These material parameters can vary on Γ. In particular, when $\mathrm{k}_\mathrm{n} \to \infty$ and $\mathrm{k}_\mathrm{t} \to \infty$, conditions (2) and (6) describe the perfect interface. To be physically admissible, k_n and k_t must be nonnegative at every point of Γ:

$$\mathrm{k}_\mathrm{n} \geqslant 0, \qquad \mathrm{k}_\mathrm{t} \geqslant 0 \tag{7}$$

Normally, the impenetrability condition $[\![\mathbf{u}_\mathrm{n}]\!] \cdot \mathbf{n} \geqslant 0$ is additionally to be imposed. However, for a spring-layer model, this restrictive condition can be relaxed to tolerate certain interpenetration by using a physical argument of Hashin [7]. According to him, a spring-layer interface is an idealization of a thin soft intermediate phase, and a small interpenetration can in practice be accommodated by such an intermediate phase. This argument is used in our work to circumvent the impenetrability condition.

3. General necessary and sufficient conditions for 3D neutral elastic inclusions

Let be given the stress field $\mathrm{S}^{(1)}$ and displacement field $\mathbf{u}^{(1)}$ over the matrix $\Omega^{(1)}$. Owing to the traction continuity condition (2), the inclusion $\Omega^{(2)}$ is subjected to the tractions $\mathbf{p}$ on its boundary $\partial\Omega^{(2)} = \Gamma$. Thus, the stress field $\mathrm{S}^{(2)}$ over $\Omega^{(2)}$ is entirely determined, and the displacement field $\mathbf{u}^{(2)}$ over $\Omega^{(2)}$ is determined to within a rigid motion, by solving the classical elastic traction boundary value problem:

$$\mathrm{K}^{(2)}_{\mathrm{ijkl}}\mathrm{u}^{(2)}_{\mathrm{k,lj}}(\mathbf{x}) = 0 \quad \text{for } \mathbf{x} \in \Omega^{(2)} \quad \text{and} \quad \mathrm{K}^{(2)}_{\mathrm{ijkl}}\mathrm{u}^{(2)}_{\mathrm{k,l}}(\mathbf{x})\mathrm{n}_{\mathrm{j}}(\mathbf{x}) = \mathrm{p}_{\mathrm{i}}(\mathbf{x}) \quad \text{for } \mathbf{x} \in \partial\Omega^{(2)} \tag{8}$$

with $\mathrm{K}^{(2)}_{\mathrm{ijkl}}$ being the components of the elastic stiffness tensor $\mathbf{K}^{(2)} = (\mathbf{C}^{(2)})^{-1}$. Further, for the conditions (6) and (7) to be satisfied, the geometrical form and material properties of the interface Γ must be such that

$$\mathrm{k}_{\mathrm{n}} = \frac{\mathbf{p}_{\mathrm{n}} \cdot [\![\mathbf{u}_{\mathrm{n}}]\!]}{[\![\mathbf{u}_{\mathrm{n}}]\!] \cdot [\![\mathbf{u}_{\mathrm{n}}]\!]} \geqslant 0, \qquad k_{\mathrm{t}} = \frac{\mathbf{p}_{\mathrm{t}} \cdot [\![\mathbf{u}_{\mathrm{t}}]\!]}{[\![\mathbf{u}_{\mathrm{t}}]\!] \cdot [\![\mathbf{u}_{\mathrm{t}}]\!]} \geqslant 0 \tag{9}$$

These are the *necessary and sufficient conditions for the inclusion* $\Omega^{(2)}$ *to be neutral*.

In the following, we are interested in the case where the stress field $\mathrm{S}^{(1)}$ over $\Omega^{(1)}$ is uniform. Then, the continuity condition (2) implies that $\Omega^{(2)}$ undergoes the uniform tractions $\mathbf{p}(\mathbf{x}) = \mathrm{S}^{(1)}\mathrm{n}(\mathbf{x})$ on the boundary $\partial\Omega^{(2)}$. As the material constituting $\Omega^{(2)}$ is homogeneous, it is directly inferred from (8) that the stress field $\mathrm{S}^{(2)}$ over $\Omega^{(2)}$ is uniform and equal to $\mathrm{S}^{(1)}$. Hence, we can write

$$\mathrm{S} = \mathrm{S}^{(1)} = \mathrm{S}^{(2)} \tag{10}$$

The strain fields over $\Omega^{(i)}$ $(i = 1, 2)$ is also uniform and given by

$$\mathrm{E}^{(i)} = \mathbf{C}^{(i)}\mathrm{S} \tag{11}$$

To within a rotation, the displacement field over $\Omega^{(\mathrm{i})}$ takes the form

$$\mathbf{u}^{(i)}(\mathbf{x}) = \mathbf{C}^{(i)}\mathrm{S}\mathbf{x} + \mathbf{b}^{(i)} \tag{12}$$

with $\mathbf{b}^{(i)}$ being a translation vector. Using the results (10)–(12) and definitions (1)–(5), we obtain

$$\mathbf{p}_{\mathrm{n}} = (\mathbf{n} \cdot \mathrm{S}\mathbf{n})\mathbf{n}, \qquad \mathbf{p}_{\mathrm{t}} = \mathrm{S}\mathbf{n} - (\mathbf{n} \cdot \mathrm{S}\mathbf{n})\mathbf{n} \tag{13}$$

$$[\![\mathbf{u}_{\mathrm{n}}]\!] = \left[\mathbf{n} \cdot \left(\left(\mathbf{C}^{(1)} - \mathbf{C}^{(2)}\right)\mathrm{S}\right)\mathbf{x} + \mathbf{n} \cdot \mathbf{b}\right]\mathbf{n} \tag{14}$$

$$[\![\mathbf{u}_{\mathrm{t}}]\!] = \left(\left(\mathbf{C}^{(1)} - \mathbf{C}^{(2)}\right)\mathrm{S}\right)\mathbf{x} + \mathbf{b} - \left[\mathbf{n} \cdot \left(\left(\mathbf{C}^{(1)} - \mathbf{C}^{(2)}\right)\mathrm{S}\right)\mathbf{x} + \mathbf{n} \cdot \mathbf{b}\right]\mathbf{n} \tag{15}$$

where $\mathbf{b} = \mathbf{b}^{(1)} - \mathbf{b}^{(2)}$. Introducing (13)–(15) into (9) results in the necessary and sufficient conditions to be satisfied in designing 3D neutral elastic inclusions embedded in a uniformly stressed elastic body. As will be seen from examples, for the resulting conditions to be effectively verified, the requirement is needed that the compliance tensors $\mathbf{C}^{(1)}$ and $\mathbf{C}^{(2)}$ be well-ordered.

4. Elastic isotropic matrix and inclusion

Now we consider the important simple case where both the matrix and inclusion are made of elastic isotropic homogeneous materials. Precisely, the compliance tensors $\mathbf{C}^{(i)}$ take the forms:

$$\mathbf{C}^{(i)} = \frac{1}{9\kappa^{(i)}}\mathrm{I} \otimes \mathrm{I} + \frac{1}{2\mu^{(i)}}\left(\mathbf{1} - \frac{1}{3}\mathrm{I} \otimes \mathrm{I}\right) \tag{16}$$

where $\kappa^{(i)}$ and $\mu^{(i)}$ are the bulk and shear moduli and $\mathbf{1}$ is the fourth-order identity tensor. In addition, the material constituting the inclusion is assumed to be stiffer than the material constituting the matrix, so that

$$\kappa^{(2)} \geqslant \kappa^{(1)}, \qquad \mu^{(2)} \geqslant \mu^{(1)} \tag{17}$$

This amounts to demanding that the difference tensor $\mathbf{C}^{(1)} - \mathbf{C}^{(2)}$ be semi-definite positive. This order relationship is essential.

Inserting (16) into (12), yields

$$[\![\mathbf{u}]\!] = \left(\frac{1}{9\kappa^{(1)}} - \frac{1}{9\kappa^{(2)}}\right)(\operatorname{tr}\mathrm{S})\mathbf{x} + \left(\frac{1}{2\mu^{(1)}} - \frac{1}{2\mu^{(2)}}\right)\left[\mathrm{S} - \frac{1}{3}(\operatorname{tr}\mathrm{S})\mathrm{I}\right]\mathbf{x} + \mathbf{b} \tag{18}$$

Then, it is immediate that

$$\mathbf{n} \cdot [\![\mathbf{u}]\!] = \left(\frac{1}{9\kappa^{(1)}} - \frac{1}{9\kappa^{(2)}}\right)(\operatorname{tr}\mathrm{S})\mathbf{n} \cdot \mathbf{x} + \left(\frac{1}{2\mu^{(1)}} - \frac{1}{2\mu^{(2)}}\right)\left[\mathbf{n} \cdot \mathrm{S}\mathbf{x} - \frac{1}{3}(\operatorname{tr}\mathrm{S})\mathbf{n} \cdot \mathbf{x}\right] + \mathbf{n} \cdot \mathbf{b} \tag{19}$$

Accounting for the definitions (3) and (5), the conditions (9) can equivalently be written as

$$\mathrm{k_n} = \frac{\mathbf{n}\cdot \mathrm{S}\mathbf{n}}{\mathbf{n}\cdot [\![\mathbf{u}]\!]} \geqslant 0, \qquad \mathrm{k_t} = \frac{[\![\mathbf{u}]\!]\cdot \mathrm{S}\mathbf{n} - (\mathbf{n}\cdot[\![\mathbf{u}]\!])\mathbf{n}\cdot \mathrm{S}\mathbf{n}}{[\![\mathbf{u}]\!]\cdot[\![\mathbf{u}]\!] + (\mathbf{n}\cdot[\![\mathbf{u}]\!])^2} \geqslant 0 \tag{20}$$

Introducing (18), (19) into (20) gives the necessary and sufficient conditions for neutrality of an elastic isotropic inclusion embedded in an elastic isotropic matrix.

Now we proceed to give two simple examples. Let the inclusion $\Omega^{(2)}$ be prescribed as a spherical one of radius r, with the center coinciding with the origin. Then, the unit outward normal $\mathbf{n}(\mathbf{x})$ at $\mathbf{x}\in\Gamma$ has the simple expression $\mathbf{n}(\mathbf{x}) = \mathbf{x}/\|\mathbf{x}\| = \mathbf{x}/r$. Regarding the initial uniform stress tensor field S of the matrix, two situations are considered as follows.

Example 1. – The stress tensor S is hydrostatic, i.e., $\mathrm{S} = \sigma \mathrm{I}$ with σ being a scalar. Introducing this expression together with $\mathbf{n}(\mathbf{x}) = \mathbf{x}/r$ into (18)–(20), it is concluded that the spherical inclusion is neutral if and only if the normal stiffness of the interface has the following expression:

$$\mathrm{k_n} = \frac{3\kappa^{(2)}\kappa^{(1)}}{r(\kappa^{(2)} - \kappa^{(1)})} \tag{21}$$

The condition $\mathrm{k_n} \geqslant 0$ is verified owing to the hypothesis (17). Note that the requirement $\mathbf{b} = \mathbf{0}$ is implicitly involved in (21). Indeed, if $\mathbf{b} \neq \mathbf{0}$, $\mathbf{n}\cdot\mathbf{b}$ can be positive and negative when $\mathbf{n}$ varies, and the condition $\mathrm{k_n} \geqslant 0$ cannot be ensured. When $\kappa^{(2)} = \kappa^{(1)}$, then $\mathrm{k_n} \to \infty$ and the normal displacement continuity is needed. Further, observe that $\mathrm{k_t}$ can take any nonnegative value without affecting the neutrality of the inclusion.

Example 2. – The stress tensor S is deviatoric, i.e., $\mathrm{tr}\,\mathrm{S} = 0$. In this case, it is deduced from (18)–(20) that the spherical inclusion is neutral if and only if

$$\mathrm{k_n} = \frac{2\mu^{(2)}\mu^{(1)}}{r(\mu^{(2)} - \mu^{(1)})}, \qquad \mathrm{k_t} = \frac{2\mu^{(2)}\mu^{(1)}[\mathbf{n}\cdot \mathrm{S}^2\mathbf{n} - (\mathbf{n}\cdot \mathrm{S}\mathbf{n})^2]}{r(\mu^{(2)} - \mu^{(1)})[\mathbf{n}\cdot \mathrm{S}^2\mathbf{n} + (\mathbf{n}\cdot \mathrm{S}\mathbf{n})^2]} \tag{22}$$

Remark that the conditions $\mathrm{k_n} \geqslant 0$ and $\mathrm{k_t} \geqslant 0$ are ensured due to the hypotheses (17) and to the fact that $\mathbf{n}\cdot \mathrm{S}^2\mathbf{n} - (\mathbf{n}\cdot \mathrm{S}\mathbf{n})^2 = \|(\mathrm{I} - \mathbf{n}\otimes\mathbf{n})\mathrm{S}\mathbf{n}\|^2 \geqslant 0$. It is interesting to note that the bulk moduli $\kappa^{(1)}$ and $\kappa^{(1)}$ do not affect the interface parameters but $\mathrm{k_n}$ cannot be arbitrary. Observe also that, if $\mu^{(2)} = \mu^{(1)}$, then $\mathrm{k_n} \to \infty$ and $\mathrm{k_t} \to \infty$. This means that the perfect interface is necessary. The simple shear and anti-plane shear are two important particular cases of the example treated here.

Exploring necessary and sufficient neutrality conditions (9), we have recently found out 3D elastic neutral inclusions having more general geometric shapes than the spherical one and embedded in anisotropic matrices. These results will be reported in another paper [10].

5. Concluding remarks

Using the spring-layer interface model, this work presents an approach to constructing 3D elastic neutral inclusions. This approach can be viewed as a 3D generalization of the relevant 2D elastic one or an elastic extension of the relevant 3D one developed for steady transport phenomena (see [2,3]). It is applicable to the case when the stress field in the matrix is not uniform. In such a situation, solution of an elastic non-uniform traction boundary value problem as formulated by (8) is entailed. Benveniste and Chen [4,5] has treated this situation for the torsion problem.

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