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Numerical calculation of wall effect and 'backflow' on the Stokes force

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Abstract We give numerical results on the modification of the drag force F_x exerted on a sphere positioned eccentrically and moving at very low Reynolds number, at constant velocity within and along a cylindrical tube. The numerical results computed by Lattice-Boltzmann method or by finite volume formulation are in good agreement with the experimental results obtained by Ambari et al. (J. Fluid Mech. 149 (1984) 235–253). In particular, they confirm the existence of a minimum of the force F_x away from the axis of the cylinder and a sharp increase when the sphere approaches the sidewall. ***To cite this article: T. Godin et al., C. R. Mecanique 330 (2002) 837–842.***

fluid mechanic / Stokes flow / hydrodynamic interactions / backflow / suspension

Calcul numérique sur l'effet de paroi et du "backflow" sur la force de Stokes

Résumé Nous présentons des résultats numériques sur la modification de la force de Stokes subie par une sphère se déplaçant à très faible nombre de Reynolds, dans un tube rempli d'un fluide newtonien. Les résultats numériques ont été obtenus par une technique de Boltzmann sur réseau et par une méthode des volumes finis. L'ensemble des résultats complète ceux obtenus expérimentalement par Ambari et al. (J. Fluid Mech. 149 (1984) 235–253). Ils confirment l'existence d'un minimum de la force subie par cette sphère en dehors de l'axe du tube et une forte augmentation quand celle-ci se déplace au voisinage des parois. ***Pour citer cet article : T. Godin et al., C. R. Mecanique 330 (2002) 837–842.***

mécanique des fluides / écoulement de Stokes / interactions hydrodynamiques / backflow / suspension

Version française abrégée

Dans le cadre de l'étude des interactions hydrodynamiques en situation confinée, nous avons analysé numériquement les modifications de la force de Stokes subie par une sphère de rayon a se déplaçant à

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un très faible nombre de Reynolds ($Re = \rho U_x a/\mu < 10^{-2}$), dans un tube de rayon R, rempli d'un fluide newtonien de viscosité dynamique μ.

Les calculs numériques ont été obtenus par deux techniques totalement différentes. La première est basée sur l'équation de Boltzmann sur réseau [7,8], l'autre utilise la technique des volumes finis via un code commercial [9].

Dans cette Note, nous présentons uniquement l'évolution du facteur de correction de la force de Stokes : $\lambda(e,k) = F_x(e,k)/6\pi\mu a U_x$ en fonction de l'excentrement $c = eR$, défini par la distance entre l'axe du cylindre et la trajectoire de la sphère, parallèle à l'axe du tube (Fig. 1). La sphère se déplace sans rotation à la vitesse uniforme U_x. Les simulations ont été effectuées pour différents rapport d'aspect $k = a/R$.

L'ensemble de ces résultats numériques confirme et complète l'étude expérimentale menée dans les mêmes conditions par Ambari et al. [4]. Ils corroborent en particulier l'existence du minimum de la force subie par cette sphère en dehors de l'axe du tube, et son augmentation rapide quand celle-ci se déplace au voisinage de la paroi.

Cette étude numérique nous a permis de quantifier la position et la valeur du minimum de cette force pour différentes valeurs de k, dans un domaine ne pouvant pas être traité par les deux approches asymptotiques menées par Brenner [5] et Tözeren [6]. Elle montre notamment une amplification de la valeur du minimum de cette force lorsque $k \to 1$. Sur la Fig. 2 (a)–(d), on peut noter le très bon accord entre les résultats numériques et ceux obtenus dans l'étude expérimentale citée ci-dessus. La Fig. 3 montre également un bon accord entre l'approche numérique et l'approche asymptotique de Tözeren [6].

L'application de ces deux techniques de calcul nous a permis d'obtenir de bons résultats sur l'évolution de la force de traînée subie par une sphère en translation uniforme dans un tube. Les résultats montrent clairement l'effet du « backflow » sur le facteur de correction de la force de Stokes en situation confinée.

1. Introduction

The theoretical approach of the dynamic of the fluid containing suspended particles presents a great difficulty because of hydrodynamic interactions between these particles. Those are of long range (velocity generated by each moving particle $u \sim 1/r$). They control the distribution of the particles. In turn, the distribution itself determines the dynamical behaviour of each individual particle. The continual change of the microstructure of the suspension and the hydrodynamic interactions which results from it induce fluctuations of the movement of each particle within the suspension [1]. Certain complex situations are due to these long range interactions. For example, we can quote the Boycott effect [2,3] which consists in an increase of the sedimentation rate of the particles in a tube maintained in a tilted position, compared to that reached in this same vertical tube. Another practical example in the usual method to measure the viscosity ν of a liquid is from the velocity U_x of a sphere of radius a falling under gravity in a vertical cylindrical tube of radius R filled with the liquid at low Reynolds number. Because of the finite value of the ratio $k = a/R$, the Stokes force expressing the hydrodynamic drag of the sphere must be corrected for the effect of the lateral cylindrical wall.

In the study of these interactions, we have focused on the situation described in the experiment carried out by Ambari and al. [4]. This one relates to a spherical particle of radius a, moving at very low Reynolds number $Re = \rho U_x a/\mu$ and constant velocity U_x, in a circular cylinder of radius R, filled with a newtonian fluid of viscosity μ and density ρ. This work has consisted in measuring the hydrodynamic force exerted by the sphere translating without any rotation. In fact the used sphere is equivalent to a magnetic moment attached to this sphere which aligns along the induction field B under the influence of a magnetic torque. This one is sufficiently high to equilibrate the hydrodynamic torque avoiding any rotation of the sphere as justified in Section 2.3 of [4].

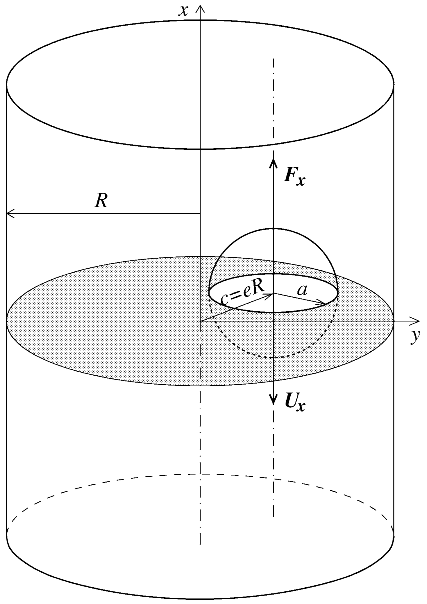

Figure 1. Sketch of the problem for the off-axis motion of a sphere along a fluid-filled cylindrical tube of radius R.

Figure 1. *Schéma décrivant la translation de la sphère en position excentrée dans un tube cylindrique.*

Experimental results are presented in term of the correction factor normalized by its value in the axis of the tube $\lambda(e,k)/\lambda(e=0,k) = F_x(e,k)/F_x(e=0,k)$ where $k = a/R$ represents the aspect ratio and $\lambda(e,k) = F_x/F_s$ in which $F_s = 6\pi\mu a U_x$ represents the Stokes force. The experimental results are reported in Fig. 2 (a)–(d). The eccentricity of the sphere is defined by the distance between the axis of cylinder and the trajectory of the sphere parallel with this axis $c = eR$ as described in Fig. 1.

The originality of the effect described in this work lies in the fact that, contrary to what we physically expected, it is observed a no monotonous variation of the drag force exerted by the sphere in the off-axis position.

It should be noted that this variation of the correction factor has been obtained using an asymptotic technique by Brenner and Happel [5] in the case of very low interactions ($k \to 0$). For higher interactions ($k \gg 0$), Tözeren [6], using an asymptotic development, has obtained results for the correction factor for low eccentricity ($e \simeq 0$) and for $k \leqslant 0.7$. His results can be summarized by the following expression: $F_x(e,k) = F_s[\lambda_0(k) + \lambda_2(k)e^2 + \cdots]$.

In the present study concerned with high interactions, we have chosen a numerical approach of this three-dimensional problem. We used the same parameters than those used experimentally and described above; e.g., for the following aspect ratio: $k = 0.12$; 0.17; 0.29; 0.44. This approach is justified by the absence of theoretical results concerning all the range of variation of eccentricity, in order to quantify the influence of the aspect ratio on the minimum amplitude of the force.

2. Numerical approach

We solve numerically the Navier–Stokes equations and the continuity equation at very low Reynolds number ($Re < 10^{-2}$). Thus parameters are used for this study: the aspect ratio $k = a/R$ and the dimensionless eccentricity $e = c/R$. The flow simulation were performed using two different codes based on totally different techniques. The first one used the three-dimensional lattice-Boltzmann equation (LBE) in the simple 13 velocity case [7] for computing the flow and the hydrodynamic force acting on the solid sphere that settles in a viscous fluid contained in the cylindrical enclosure. The moving boundary conditions on a surface are applied using the combination of the bounce-back scheme and spatial interpolations of first

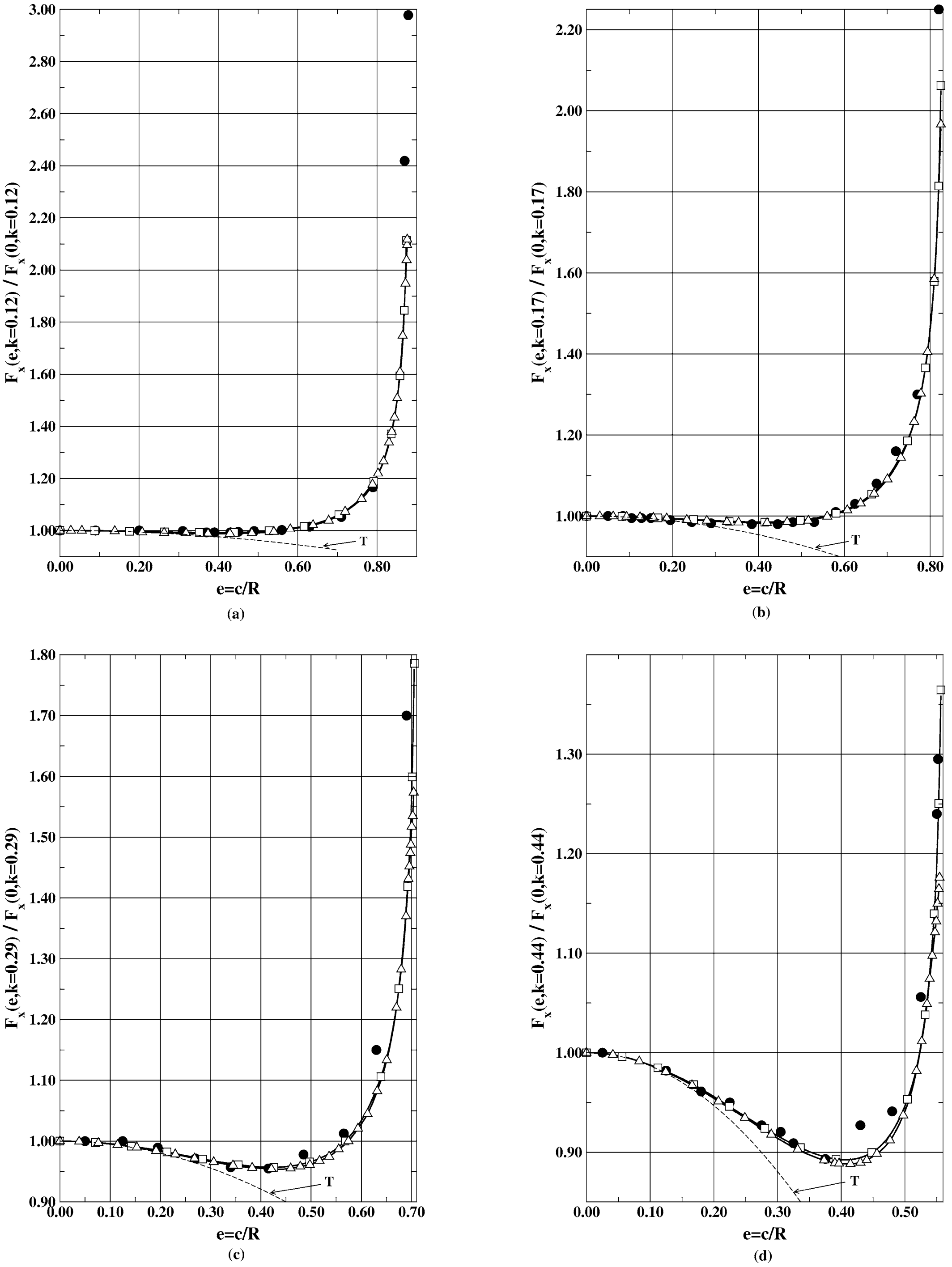

Figure 2. Variation of the normalized drag force versus eccentricity for different aspect ratio: (a) $k = 0.12$, (b) $k = 0.17$, (c) $k = 0.29$, (d) $k = 0.44$. ●, experimental results. □, finite volume method. △, lattice gas method. - - -, theoretical curve by Tözeren (T).

Figure 2. *Variation de la force de traînée normalisée en fonction de l'excentrement pour différents rapports d'aspect.*

or second order [8]. Numerical grids with sizes from 97 × 97 × 460 to 145 × 145 × 220 were used in all simulations.

In the second technique, the flow simulations were performed using a general-purpose computational fluid dynamics code [9]. This software employs a finite volume method with a second order scheme. The grid consists of a spherical boundary fitted around the sphere and contains 125,000 cells. For the calculations, we fixed the ratio L/a to be 10, where L is the length of the cylindrical tube (Fig. 1). It corresponded to a good compromise between computing time, accuracy and minimization of the end effect.

3. Results and discussion

Fig. 2 (a)–(d) show the evolution of the drag force $F_x(e,k)$ exerted on the sphere, normalized by its value measured on the axis of the tube $F_x(e=0,k)$ for the different aspect ratios $k = 0.12$ (a), $k = 0.17$ (b), $k = 0.29$ (c), $k = 0.44$ (d). They present all the numerical results computed by the two techniques described above as well as the experimental results obtained by Ambari et al. [4].

These numerical results clearly show, in all cases, the no monotonous character of the evolution of the drag force as a function of eccentricity and the existence of a minimum located far from the axis of the tube. In addition, this minimum is amplified when the aspect ratio increases. The results obtained by the two numerical approaches seem to be in a very good agreement between them and perfectly corroborate the experimental results.

The comparison between the numerical results and the asymptotic analysis by Tözeren [6] is made through the following parabolic behavior near of the axis: $F_x(e,k)/F_x(e=0,k) = 1 - (\lambda_2(k)/\lambda_0(k))e^2$.

Fig. 3 shows the various values of λ_2/λ_0, obtained numerically, as a function of aspect ratio. The results seem in good agreement with those obtained by Tözeren [6].

These results enable us to suggest that the Boycott effect is partially due to the reduction in hydrodynamic resistance undergone by the spheres in the vicinity of the tilted sedimentation interface. Indeed the reduction

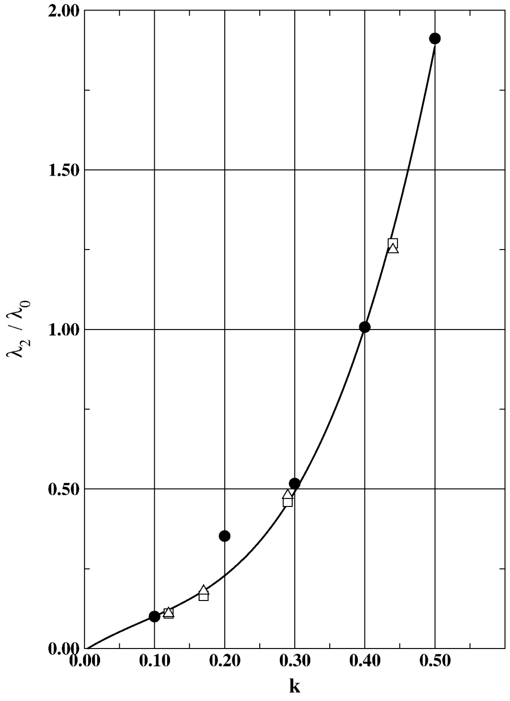

Figure 3. Variation of the function $\lambda_2(k)/\lambda_0(k)$ versus aspect ratio k. ●, Tözeren results. □, finite volume method. △, lattice gas method.

Figure 3. *Variation de $\lambda_2(k)/\lambda_0(k)$ en fonction du rapport d'aspect k.*

of the concentration of the particles of only one side of this interface facilitates the flow of the 'backflow' particle being in the vicinity of this interface tilted.

4. Conclusion

The numerical results obtained by these two techniques of calculation seem to be well adapted to the study of this type of three-dimensional hydrodynamic interactions. A first application of these techniques enabled us to obtain good results on the evolution of the drag force exerted on a sphere at uniform translation in a tube. These results clearly show the effect of the 'backflow' on the correction factor of the Stokes force in a confined situation. It is significant to note that the existance of the minimum of the force related to the 'backflow', persists if this one is free to turn as we could numerically check it in another study which is in progress in our laboratory. Indeed, the rotation of the sphere affects this minimum very slightly.

This approach has enabled us to treat situations difficult to achieve in experiments and to reach parameters such as the torque undergone by the sphere or the separate measurement of the force due to viscosity and that due to the pressure.

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