Available online at www.sciencedirect.com

SCIENCE DIRECT®

C. R. Mecanique 331 (2003) 55–60

COMPTES RENDUS MECANIQUE

Direct numerical simulation of transition to turbulence in an oscillatory channel flow

Simulation numérique de la transition vers la turbulence d'un ecoulement oscillant dans une conduite bi-dimensionnelle

L. Héctor Juárez [a], Eduardo Ramos [b]

[a] *Departamento de Matemáticas UAM-I, AP55-534, 09340 México D.F., Mexico*
[b] *Centro de Investigación en Energía, UNAM, AP34, 62580 Temixco Mor., Mexico*

Received 20 December 2002; accepted 6 January 2003

Presented by Roland Glowinski

Abstract

In this Note, we present results of the numerical simulation of transition to turbulence for a purely oscillatory channel flow. These simulations were performed for various values of the Reynolds number, the so-called Stokes parameter being equal to 4. The methodology used for the flow simulation relies on a combination of finite element space approximations with time-discretization by operator splitting; it has shown to be very effective, even when it is applied to relatively complex domains with strong expansions at the inlet and outlet of the channel. The numerical results obtained agree qualitatively well with previous experiments by other investigators. ***To cite this article: L.H. Juárez, E. Ramos, C. R. Mecanique 331 (2003).***

Résumé

Dans cette Note, on présente les résultats de la simulation numérique de la transition vers la turbulence d'un écoulement oscillant dans une conduite bi-dimensionnelle. Ces simulations ont été effectuées pour diverses valeurs du nombre de Reynolds, la valeur du paramètre de Stokes restant fixée à 4. La méthodologie utilisée pour ces calculs combine approximation en espace par éléments finis et discrétisation en temps par décomposition d'opérateurs ; au vu des résultats obtenus, elle semble très efficace, en particulier pour le cas où la conduite présente de forts expansions, en entrée et en sortie. Les résultats numériques obtenus sont qualitativement en bon accord avec ceux d'autres auteurs. ***Pour citer cet article : L.H. Juárez, E. Ramos, C. R. Mecanique 331 (2003).***

Keywords: Computational fluid mechanics; Turbulence transition; Oscillatory channel flow

Mots-clés : Mécanique des fluides numérique ; Transition à la turbulence ; Écoulement oscillant

E-mail addresses: hect@xanum.uam.mx (L.H. Juárez), ramos@chaos.ph.utexas.edu (E. Ramos).

1631-0721/03/$ – see front matter
doi:10.1016/S1631-0721(03)00023-8

1. Introduction

Oscillatory flows arise in a variety of applications in many important fields such as offshore engineering, industrial processes, biomedical sciences, pulmonary, circulatory and auditive flows. Although in most practical cases of interest the geometry of the ducts is very complex, most studies have been concentrated in oscillatory flows in straight ducts, with rigid walls. These flows are described by two non-dimensional parameters: the oscillatory Reynolds number R_δ, and the Stokes parameter Λ, defined in Section 3. Experimental observations for oscillatory flows with zero mean indicate that three distinct qualitative behaviors can be identified in the R_δ–Λ parameter space: (a) laminar flow; (b) laminar flow with a superimposed turbulent perturbation of small amplitude; and (c) laminar flow with turbulent bursts starting at the end of the acceleration phase of the cycle and relaminarization in the deceleration phase (see [1–3]). The portion of the cycle where the flow is turbulent increases with the Reynolds number but a pure turbulent flow in the whole cycle has not been observed. It is now generally agreed that the transition to type (c) flows occurs at $R_\delta \approx 550$ as long as $\Lambda > 2$.

Several theories have appeared in the literature aiming to predict the laminar to turbulent transition. Several authors (e.g., [4,5]) have developed a quasi-steady linear stability and also have used a Floquet theory to calculate the critical Reynolds number, but their results do not agree with observations. Akhavan et al. [6] present a numerical calculation based on spectral methods. They assume periodic boundary conditions and parallel walls along the whole length of the duct. This means that no entrance or exit effects are considered in the model. The laminar and turbulent results obtained are in agreement with experimental results.

Here we present the numerical solution for Navier–Stokes equations for oscillatory flows with zero mean in 2D ducts with expansions at the ends (see Fig. 1). The qualitative features obtained for laminar and turbulent flows are similar than those of straight tubes, but in our study, due to the non-parallel walls at the ends of the duct, we can analyze the end effects of the oscillatory flow. In particular, we display the spatio-temporal distribution of vortices inside the duct as a function of the oscillatory Reynolds number.

2. Formulation and discretization of the problem

The flow region Ω chosen for the numerical simulations is a two-dimensional channel with expansions at both ends, as shown in Fig. 1. The conservation equations that describe the oscillatory flow in this region are:

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\frac{1}{\rho} \nabla p + \nu \nabla^2 \mathbf{u} + \mathbf{f} \quad \text{in } \Omega \tag{1}$$

$$\nabla \cdot \mathbf{u} = 0 \quad \text{in } \Omega \tag{2}$$

Here $\mathbf{u} = (u, v)$ is the velocity vector where u and v are the axial and transversal velocity components respectively; ν and ρ are the kinematic viscosity and density respectively. If we denote by $\mathbf{U}$ the characteristic velocity of the flow, and by ω the frequency of oscillations, then the external force $\mathbf{f} = \mathbf{U}\omega \sin(\omega t)/\rho$ in Eq. (1) represents the mean pressure gradient. Thus, ∇p in Eq. (1) denotes only the departure from the mean pressure gradient. Eqs. (1)

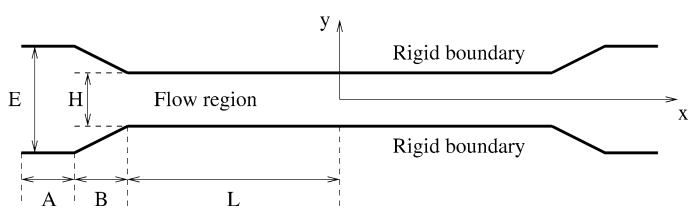

Fig. 1. Two dimensional duct with expansions at the ends. L = 28.125, H = 3, E = 6, A = 11.25, B = 5.625.

Fig. 1. Conduite bi-dimensionnelle avec expansions aux extrèmes. L = 28.125, H = 3, E = 6, A = 11.25, B = 5.625.

and (2) are complemented by the initial condition $\mathbf{u}(x, y, 0) = 0$. The boundary conditions are $\mathbf{u} = \mathbf{0}$ on channel walls, and $\mathbf{u}$ *periodic* at channel ends.

Concerning the *space approximation* of the problem, we introduce a triangulation $\mathcal{T}_h$ of the flow region Ω, and a triangulation twice finer $\mathcal{T}_{h/2}$, where h is the *space discretization step*. We denote by P_1 the space of polynomials of degree less or equal to one. The functional spaces for velocity and pressure are approximated by the following finite dimensional spaces: $V_h = \{\mathbf{v} \in (C^0(\overline{\Omega}))^2\colon \mathbf{v}|_T \in P_1 \times P_1,\ \forall T \in \mathcal{T}_{h/2},\ \mathbf{v} = \mathbf{0}$ on channel walls, and $\mathbf{v}$ periodic at channel ends$\}$, and $L_h = \{q \in C^0(\overline{\Omega})\colon q|_T \in P_1,\ \forall T \in \mathcal{T}_h,\ \int_\Omega q\,\mathrm{d}\mathbf{x} = 0,\ q$ periodic at channel ends$\}$, respectively. The discrete version of the *variational formulation* of problem (1), (2) is: for $t > 0$ find $\mathbf{u}_h(t) \in V_h$, and $p_h(t) \in L_h$, such that for all $\mathbf{v} \in V_h$, and for all $q \in L_h^2$

$$\int_\Omega \left[\frac{\partial \mathbf{u}_h}{\partial t} + (\mathbf{u}_h \cdot \nabla)\mathbf{u}_h\right] \cdot \mathbf{v}\,\mathrm{d}\mathbf{x} + \nu \int_\Omega \nabla \mathbf{u}_h : \nabla \mathbf{v}\,\mathrm{d}\mathbf{x} - \frac{1}{\rho}\int_\Omega p_h \nabla \cdot \mathbf{v}\,\mathrm{d}\mathbf{x} = \int_\Omega \mathbf{f}_h \cdot \mathbf{v}\,\mathrm{d}\mathbf{x} \tag{3}$$

$$\int_\Omega q \nabla \cdot \mathbf{u}_h(t)\,\mathrm{d}x = 0 \tag{4}$$

This problem is an *initial value problem* which contain three numerical difficulties each of which can be associated a specific operator: (i) the incompressibility condition and the related unknown pressure; (ii) an advection problem; and (iii) a diffusion term. Given a time discretization step Δt, the following fractional step scheme *à la* Marchuk–Yanenko [7] was used to solve the problem (3), (4): given $\mathbf{u}_h^0 = \mathbf{0}$, for $n \geqslant 0$, knowing $\mathbf{u}_h^n$, compute $\mathbf{u}_h^{n+1/3} \in V_h$, and $p_h^{n+1} \in L_h$ via the solution of

$$\begin{cases} \displaystyle\int_\Omega \frac{\mathbf{u}_h^{n+1/3} - \mathbf{u}_h^n}{\Delta t} \cdot \mathbf{v}\,\mathrm{d}\mathbf{x} - \frac{1}{\rho}\int_\Omega p_h^{n+1} \nabla \cdot \mathbf{v}\,\mathrm{d}\mathbf{x} = \int_\Omega \mathbf{f}_h^{n+1} \cdot \mathbf{v}\,\mathrm{d}\mathbf{x}, \quad \forall \mathbf{v} \in V_h \\ \displaystyle\int_\Omega q \nabla \cdot \mathbf{u}_h^{n+1/3}\,\mathrm{d}\mathbf{x} = 0, \quad \forall q \in L_h^2 \end{cases} \tag{5}$$

Then, compute $\mathbf{u}_h^{n+2/3} = \mathbf{u}_h(t^{n+1}) \in V_h$, where $\mathbf{u}_h(t)$ is the discrete solution of the following pure advection problem on (t^n, t^{n+1})

$$\begin{cases} \displaystyle\int_\Omega \frac{\partial \mathbf{u}_h}{\partial t} \cdot \mathbf{v}\,\mathrm{d}\mathbf{x} + \int_\Omega \big(\mathbf{u}_h^{n+1/3} \cdot \nabla\big)\mathbf{u}_h \cdot \mathbf{v}\,\mathrm{d}\mathbf{x} = 0, \quad \forall \mathbf{v} \in V_h \\ \mathbf{u}_h\big(t^n\big) = \mathbf{u}_h^{n+1/3} \end{cases} \tag{6}$$

Next, find $\mathbf{u}^{n+1} \in V_h$ by solving the diffusion problem.

$$\int_\Omega \frac{\mathbf{u}_h^{n+1} - \mathbf{u}_h^{n+2/3}}{\Delta t} \cdot \mathbf{v}\,\mathrm{d}\mathbf{x} + \nu \int_\Omega \nabla \mathbf{u}_h^{n+1} : \nabla \mathbf{v}\,\mathrm{d}\mathbf{x} = 0, \quad \forall \mathbf{v} \in V_h \tag{7}$$

Problem (5) is a finite dimensional linear saddle-point problems which is solved by an *Uzawa/conjugate gradient algorithm* [8]. The pure advection problem (6) is solved by the wave-like equation method discussed in [9] and [10]. Problem (7) is a discrete elliptic system whose iterative or direct solution is a quite classical problem. In this work all the linear systems are solved by a sparse matrix algorithm based on Markowitz' method [11].

3. Numerical experiments and conclusions

The results are be given in terms of the Reynolds number $R_\delta = U\delta/\nu$, where U is a characteristic velocity, $\delta = \sqrt{2\nu/\omega}$ is the Stokes layer thickness, and ν is the kinematic viscosity. We have calculated the oscillatory

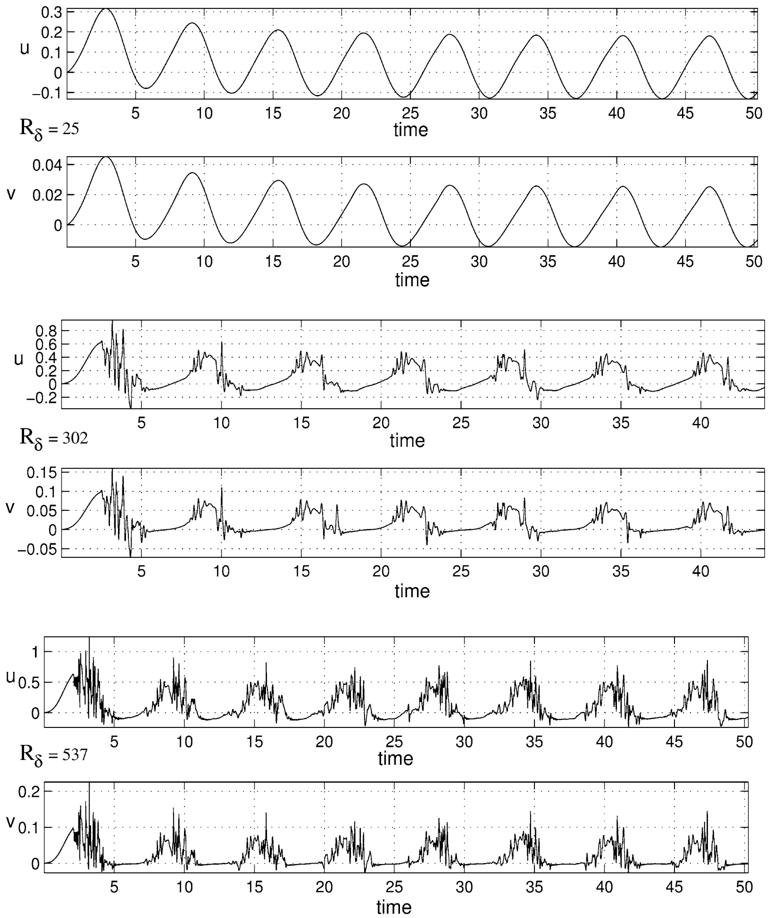

Fig. 2. Time history of the axial and transversal velocities at point $(x, y) = (-28, -1.47)$ for $R_\delta = 25, 302$ and 537.

Fig. 2. Comportement au temps des vitesses transversales et axiales au point $(x, y) = (-28, -1, 47)$ pour $R_\delta = 25, 302$ et 537.

flow in a duct considering $25 \leqslant R_\delta \leqslant 1521$ and Stokes parameter $\Lambda = \frac{H/2}{\delta} = 4$. The discretization values are $h = 1/320$ for velocity, and $\Delta t = 0.00025$. Fig. 2 shows the time history of the axial and transversal velocities at point $(x, y) = (-28, -1.47)$ for $R_\delta = 25, 302$, and 537. In all cases, there is a transient at the beginning where the influence of the initial conditions is important. Typically this time is of the order of three cycles. At $R_\delta = 25$ the flow is laminar, with the motion of the fluid synchronized with the pressure oscillation. At $R_\delta = 302$, bursts of small amplitude and high frequency oscillations appear at the end of the acceleration phase. At $R_\delta = 537$, a turbulent-like flow appears in almost the total duration of the cycle, except when the velocity is nearly zero. In order to give a more global picture of the flow, Fig. 3 shows the velocity field for two times in the cycle, $\phi = 0$ and $\phi = 2\pi/5$ for $R_\delta = 25, 302$, and 537. $\phi = 0$ corresponds to zero pressure gradient, while $\phi = \pi/4, 3\pi/4$ correspond to maximum and minimum axial pressure gradient respectively. Vortices develop near the expansions at the ends of the duct, and at $R_\delta = 25$, the dissipate inside the expansions. For $R_\delta = 302$, the vortices penetrate up to the middle of the channel. Finally, for $R_\delta = 537$ at certain parts of the cycle the whole of the duct is filled with vortical structures.

In conclusion, we have analyzed the oscillatory flow in a duct with expansions at the ends. In contrast with the studies of oscillatory flow stability that have appeared in the literature, this geometry allows us to study the end effects which include the generation and spatio-temporal distribution of vortices. In agreement with information

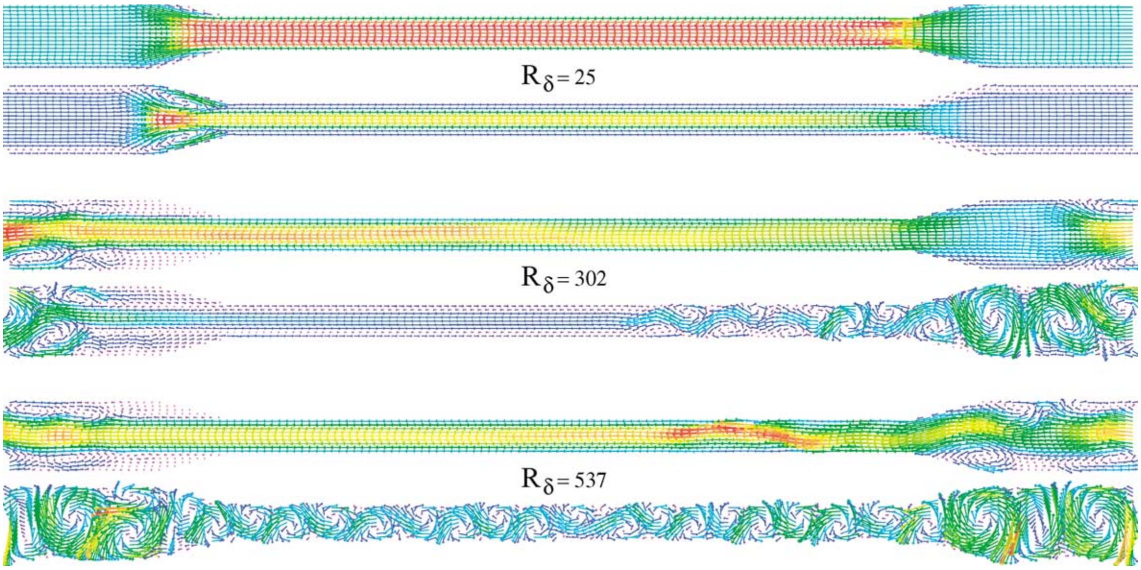

Fig. 3. Velocity flow field for two times in the cycle, $\phi = 0$ and $\phi = 2\pi/5$ for $R_\delta = 25$, 302 and 537.

Fig. 3. Champ de vitesse pour deux temps dans le cycle, $\phi = 0$ et $\phi = 2\pi/5$ pour $R_\delta = 25$, 302 et 537.

available in the literature, three qualitatively different regimes have been observed: (a) laminar flow; (b) flow with the duct partly filled with vortices; and (c) flow where the vortices fully fill the duct at the decelerating phase of the cycle. The critical Reynolds number at which vortices fully fill the duct is approximately 537, which coincides with experimental observations reported in [1] and [2]. It may be possible to have a geometry and dynamical conditions such that the vortices generated at the ends dissipate at a certain distance from the ends, but a spontaneous instability manifests itself in the central region of the duct. We consider this an important question to be answered in a future investigation.

Acknowledgements

Helpful comments and suggestions of Prof. R. Glowinski are greatly appreciated. Part of this work was done while ER was on sabbatical leave at the Center for Nonlinear Dynamics of the University of Texas at Austin, and simultaneously LHJ was visiting the Department of Mathematics at the University of Houston. The hospitality of both Prof. Harry Swinney and Prof. Roland Glowinski is gratefully acknowledged, as is the financial support from the University of Texas, University of Houston and CONACyT (México).

References

[1] R. Akhavan, R.D. Kamm, A.H. Shapiro, An investigation of transition to turbulence in bounded oscillatory Stokes flows. Part 1. Experiments, J. Fluid Mech. 225 (1991) 395–422.
[2] M. Hino, M. Sawamoto, S. Takasu, Experiments on transition to turbulence in an oscillating pipe flow, J. Fluid Mech. 75 (1976) 193–207.
[3] S.I. Sergeev, Fluid oscillations in pipes at moderate Reynolds numbers, Fluid Dynamics 1 (1966) 121–122.
[4] P. Hall, The linear stability of flat Stokes layers, Proc. Roy. Soc. London Ser. A 359 (1978) 151–166.
[5] C. von Kerczek, S.H. Davis, Linear stability theory of oscillatory Stokes layers, J. Fluid Mech. 62 (1974) 753–773.
[6] R. Akhavan, R.D. Kamm, A.H. Shapiro, An investigation of transition to turbulence in bounded oscillatory Stokes flows. Part 2. Numerical simulations, J. Fluid Mech. 225 (1991) 423–444.

[7] G.I. Marchuk, Splitting and alternating direction methods, in: P.G. Ciarlet, J.-L. Lions (Eds.), in: Handbook of Numerical Analysis, Vol. 1, North-Holland, Amsterdam, 1990, pp. 197–462.
[8] R. Glowinski, P. Le Tallec, Augmented Lagrangians and Operator Splitting Methods in Nonlinear Mechanics, SIAM, Philadelphia, PA, 1989.
[9] E.J. Dean, R. Glowinski, A wave equation approach to the numerical simulation of the Navier–Stokes equations for incompressible viscous flows, C. R. Acad. Sci. Paris Sér. I 325 (1997) 789–791.
[10] E.J. Dean, R. Glowinski, T.W. Pan, A wave equation approach to the numerical simulation of incompressible viscous flow modeled by the Navier–Stokes equations, in: J.A. De Santo (Ed.), Mathematical and Numerical Aspects of Wave Propagation, SIAM, Philadelphia, PA, 1998, pp. 65–74.
[11] S. Pissanetzky, Sparse Matrix Technology, Academic Press, 1984.