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Simultaneous identification of stiffness and damping properties of isotropic materials from forced vibrating plates

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Abstract

This paper presents a novel methodology for the identification of damping of isotropic plates. It relies on forced inertial excitation of a clamped plate and full-field curvature measurements using a suitable optical technique. Using the Virtual Fields Method, it is shown that the damping parameter is easily related to the curvature field, even on a non-resonant plate. This paper opens a totally new field of investigation for damping identification. ***To cite this article: A. Giraudeau, F. Pierron, C. R. Mecanique 331 (2003).***

Résumé

Identification simultanée de rigidités et d'amortissements de matériaux isotropes à partir de plaques en vibrations forcées. Ce papier présente une méthodologie originale d'identification de l'amortissement matériau d'une plaque mince. L'excitation choisie est inertielle et on suppose que le champs de courbure est mesuré en surface de la plaque. En utilisant la méthode des champs virtuels, on montre que le paramètre d'amortissement s'exprime assez simplement en fonction du champ de courbure, ceci même lorsqu'on est hors-résonance. Ce papier ouvre une voie nouvelle pour la caractérisation de l'amortissement des matériaux en régime vibratoire. ***Pour citer cet article : A. Giraudeau, F. Pierron, C. R. Mecanique 331 (2003).***

Keywords: Vibrations; Identification; Virtual Fields Method; Damping; Forced vibrations; Plates

Mots-clés : Vibrations ; Identification ; Méthode des champs virtuels ; Amortissement ; Vibrations forcées ; Plaques

1. Introduction

Identification of damping is essential for the computation of stress and strains in vibrating structures. This parameter is often obtained from unsteady vibrations of beams. Nevertheless, intrinsic material behaviour is usually difficult to separate from damping arising from boundary conditions. This paper presents a novel procedure to

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identify stiffness and damping parameters of isotropic thin plates provided that full-field curvature measurements are available at the surface of the plate. It is based on the Virtual Fields Method [1] and is an extension of initial work by Grédiac et al. [2].

2. Theory

For a solid of any shape, in the case of small perturbations, the general expression of the principle of virtual work writes:

$$-\int_V \sigma : \varepsilon^* \,\mathrm{d}V + \int_{\partial V} F \cdot u^* \,\mathrm{d}S = \int_V a \cdot u^* \,\mathrm{d}m \tag{1}$$

where σ is the stress tensor, ε^* is a virtual strain tensor, V is the volume of the solid, ∂V the surface of its boundary, F the surface density of external forces acting on ∂V, u^* is a kinematically admissible virtual displacement field associated to ε^* and a is the acceleration field. $\mathrm{d}V$, $\mathrm{d}S$ and $\mathrm{d}m$ are respectively elementary volume, surface and mass.

2.1. *Considered case*

Let us consider now that the solid is a rectangular thin plate (Fig. 1).

The plate is clamped in point O as seen in Fig. 1, which is the origin of the coordinate system. Let us now suppose that this point is translated along the z axis in a sine movement (inertial excitation). If we call $\delta(t)$ the displacement of this point along the z direction, and using usual complex notations, then:

$$\delta(t) = d\cos\omega t = \mathrm{Re}\big(d\exp(j\omega t)\big) \tag{2}$$

where ω is the pulsation of the sine movement and d is the amplitude of the movement. For a homogeneous material, this is a case of pure bending and only out-of-plane displacements will be considered and assuming classical thin plate theory, it is independent on z. The global out-of-plane displacement field $\mu(x, y, t)$ is therefore the superposition of the imposed excitation displacement $\delta(t)$ and the out-of-plane deflection $\lambda(x, y, t)$ due to the deformation of the plate:

$$\mu(x, y, t) = \delta(t) + \lambda(x, y, t) \tag{3}$$

The deflection caused by the deformation of the plate is made up of the combination of the response of each mode:

$$\mu(x, y, t) = \delta(t) + \sum_{k=1}^{\infty} \lambda_k(x, y, t) \tag{4}$$

where $\lambda_k(x, y, t)$ is the response of the k-th mode. Since the excitation has a sine shape and assuming the linear behaviour of the plate, the response is harmonic with the same pulsation ω:

$$\mu(x, y, t) = \delta(t) + \sum_{k=1}^{\infty} \big|\lambda_k(x, y)\big| \cos\big(\omega t - \phi_k(x, y)\big) \tag{5}$$

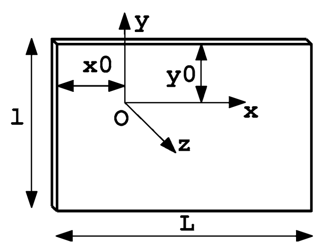

Fig. 1. Rectangular plate under study.

where $|\lambda_k(x,y)|$ and $\phi_k(x,y)$ are respectively the amplitude and the phase of the response of the k-th mode. Because of the damping, the latter can take all the values between 0 and π, depending on the distance between the excitation and the k-th mode frequencies. It is easier to write:

$$\mathrm{Re}\big(u(x,y)\exp(j\omega t)\big) = \mathrm{Re}\big(\big(d + w(x,y)\big)\exp(j\omega t)\big), \quad u(x,y) \in \mathbb{C},\ w(x,y) \in \mathbb{C} \tag{6}$$

where $u(x,y) = u_r(x,y) + ju_i(x,y)$ and $w(x,y) = w_r(x,y) + jw_i(x,y)$. In the case of a plate with low damping, having sufficiently separated modes and excited close to its k-th mode frequency, the response is dominated by that of the resonant mode which amplitude becomes predominant and which phase comes close to $\pi/2$. The response linked to the other modes have relatively small amplitudes and their phases are close to 0 or π. It is to be noted that it is commonly assumed for sufficiently separated modes and small damping that the contribution of $w_r(x,y)$ can be neglected in Eq. (6).

2.2. *Choice of the virtual fields*

The virtual displacement fields must be kinematically admissible. In particular, it means that they should be such that the virtual movement of point O must match its true movement. For instance, one can be written as:

$$u^*(x,y)\exp(j\omega t) = \big(d + w^*(x,y)\big)\exp(j\omega t), \quad u^*(x,y) \in \mathbb{C},\ w^*(x,y) \in \mathbb{C} \tag{7}$$

where $w^*(x,y) = w_r^*(x,y) + jw_i^*(x,y)$ is a virtual deflection field, with $w^*(0,0) = 0$. As mentioned in Section 2.3, u^* must be such that the clamping moments do not work virtually. Therefore, one must have:

$$\frac{\partial w^*}{\partial x}(0,0) = 0 \quad \text{and} \quad \frac{\partial w^*}{\partial y}(0,0) = 0 \tag{8}$$

2.3. *Virtual work of external forces*

The only external forces acting on the plate are that introduced by the clamping at point O (see Fig. 1). If there is a chance to measure experimentally the force acting in the z direction, the two bending moments M_x and M_y will be unknown and their contribution to the virtual work of the external forces will have to be zeroed by choosing appropriate virtual fields (see Section 2.2). Let us consider now only F, the force in the z direction. It can be written as:

$$F\exp(j\omega t) = (F_r + jF_i)\exp(j\omega t) \tag{9}$$

Therefore, the virtual work of the external forces *VWEF* simply writes:

$$VWEF = \mathrm{Re}\big(F\exp(j\omega t)\big)\,\mathrm{Re}\big(d\exp(j\omega t)\big) \tag{10}$$

Developing the above, it can be shown that:

$$VWEF = \frac{1}{2}F_r d + \frac{1}{2}\mathrm{Re}\big(Fd\exp(j2\omega t)\big) \tag{11}$$

Finally, it comes:

$$VWEF = \frac{1}{2}F_r d + \frac{1}{2}F_r d\cos(2\omega t) - \frac{1}{2}F_i d\sin(2\omega t) \tag{12}$$

2.4. *Virtual work of the internal forces*

The virtual work of the internal forces (*VWIF*) is written according to the Love–Kirchhoff theory of thin plates. In the case of dissipative materials, *VWIF* can be written as the sum of an elastic and a dissipative contributions.

One has:

$$VWIF = -\int_S \big(\{m^e\} + \{m^d\}\big)\{\kappa\}^* \, \mathrm{d}S \tag{13}$$

where $\{m^e\}$ and $\{m^d\}$ are respectively the elastic and dissipative moments and $\{\kappa\}^*$ is the virtual curvature field resulting from the virtual displacement field described in Section 2.2. It is important to note at this stage that the three above quantities are complex.

In the case of an isotropic and homogeneous material:

$$\{m^e\} = \begin{bmatrix} m_x^e \\ m_y^e \\ m_s^e \end{bmatrix} = \begin{bmatrix} D_{xx} & D_{xy} & 0 \\ D_{xy} & D_{xx} & 0 \\ 0 & 0 & (D_{xx} - D_{xy})/2 \end{bmatrix} \begin{bmatrix} k_x \\ k_y \\ k_s \end{bmatrix} \tag{14}$$

where D_{xx} and D_{xy} are the isotropic bending stiffness components of the plate.

Assuming that the dissipation is viscous and that the dissipative moment is proportional to the elastic one, one can write:

$$\{m^d\} = \beta \frac{\partial \{m^e\}}{\partial t} = j\beta\omega\{m^e\} \tag{15}$$

It is important to note that the method could be used with other formulations of damping. The one chosen here is kept as simple as possible to demonstrate the method. It can be shown that Eq. (13) can be written as:

$$VWIF = \frac{1}{2}\mathrm{Re}\left(\int_S \big(\{m^e\} + \{m^d\}\big)\{\bar{\kappa}\}^* \, \mathrm{d}S\right) + \frac{1}{2}\mathrm{Re}\left(\int_S \big(\{m^e\} + \{m^d\}\big)\{\kappa\}^* \exp(j2\omega t) \, \mathrm{d}S\right) \tag{16}$$

where $\{\bar{\kappa}\}^*$ indicates the conjugate of the virtual curvatures.

Using Eqs. (14) and (15) to replace in Eq. (16), one has:

$$\begin{aligned} H_{p,q}(x,y) = {} & -D_{xx} \int_S \left(k_x(x,y)k_x^* + k_y(x,y)k_y^* + \frac{1}{2}k_s(x,y)k_s^*\right) \mathrm{d}S \\ & - D_{xy} \int_S \left(k_x(x,y)k_y^* + k_x(x,y)k_y^* - \frac{1}{2}k_s(x,y)k_s^*\right) \mathrm{d}S \end{aligned} \tag{17}$$

where p is either r or i, indicating respectively real or imaginary parts of the true curvatures and q is either r or i, indicating respectively real or imaginary parts of the virtual curvatures. Therefore, there are four H functions: $H_{r,r}(x,y)$, $H_{i,i}(x,y)$, $H_{i,r}(x,y)$, $H_{r,i}(x,y)$.

Finally, one has:

$$\begin{aligned} VWIF = {} & \frac{1}{2}\big[(H_{r,r} - \beta\omega H_{i,r}) + (H_{i,i} + \beta\omega H_{r,i})\big] + \frac{1}{2}\big[(H_{r,r} - \beta\omega H_{i,r}) + (H_{i,i} + \beta\omega H_{r,i})\big]\cos(2\omega t) \\ & - \frac{1}{2}\big[(H_{i,r} + \beta\omega H_{r,r}) + (H_{r,i} + \beta\omega H_{i,i})\big]\sin(2\omega t) \end{aligned} \tag{18}$$

2.5. Virtual work of the inertial forces

Taking into account Eq. (6), the acceleration a writes:

$$a(x,y)\exp(j\omega t) = \frac{\partial^2}{\partial t^2}\big(u(x,y)\exp(j\omega t)\big) = -d\omega^2\exp(j\omega t) - w(x,y)\omega^2\exp(j\omega t) \tag{19}$$

Moreover, $dm = \rho h\,\mathrm{d}S$ where ρ is the density of the material. Therefore, the virtual work of the inertial forces *VWAC* writes:

$$VWAC = -\frac{\rho h\omega^2}{2}\,\mathrm{Re}\left(\int_S u(x,y)\bar{u}^*(x,y)\,\mathrm{d}S\right) - \frac{\rho h\omega^2}{2}\,\mathrm{Re}\left(\int_S u(x,y)u^*(x,y)\exp(j2\omega t)\,\mathrm{d}S\right) \tag{20}$$

where $\bar{u}^*(x,y)$ indicates the conjugate of the virtual displacement field. Finally, it comes:

$$\begin{aligned} VWAC \;=\; & -\frac{\rho h\omega^2}{2}\int_S (u_r u_r^* + u_i u_i^*)\,\mathrm{d}S - \frac{\rho h\omega^2}{2}\int_S (u_r u_r^* - u_i u_i^*)\,\mathrm{d}S\cdot\cos(2\omega t) \\ & + \frac{\rho h\omega^2}{2}\int_S (u_r u_i^* + u_i u_r^*)\,\mathrm{d}S\cdot\sin(2\omega t) \end{aligned} \tag{21}$$

2.6. Summary

Following the above calculations, the principle of virtual work writes:

$$VWIF + VWEF = VWAC \tag{22}$$

with the expressions for *VWIF*, *VWEF* and *VWAC* given in the previous sections. As shown previously, each contribution is written as the sum of a term independent on time and two terms dependent on sine and cosine of twice the excitation pulsation. Since the above equality is verified for any time, then it can be split into three separate equations. Moreover, the expressions are valid for any combination of the real and imaginary parts of the virtual fields. As a consequence, each of the three equations can again be split into two equations, leading finally to a system of six equations, from which only four are independent. These equations are given below.

$$H_{r,r} - \beta\omega H_{i,r} + F_r d = -\rho h\omega^2 \int_S u_r u_r^*\,\mathrm{d}S \tag{23}$$

$$H_{i,i} + \beta\omega H_{r,i} = -\rho h\omega^2 \int_S u_i u_i^*\,\mathrm{d}S \tag{24}$$

$$H_{i,r} + \beta\omega H_{r,r} + F_i d = -\rho h\omega^2 \int_S u_i u_r^*\,\mathrm{d}S \tag{25}$$

$$H_{r,i} + \beta\omega H_{i,i} = -\rho h\omega^2 \int_S u_r u_i^*\,\mathrm{d}S \tag{26}$$

Since $u(x,y) = d + w_r(x,y) + jw_i(x,y)$ and $u^*(x,y) = d + w_r^*(x,y) + jw_i^*(x,y)$, the above system can be rewritten as:

$$H_{i,i} + \beta\omega H_{r,i} = -\rho h\omega^2 \int_S w_i w_i^*\,\mathrm{d}S \tag{27}$$

$$H_{r,r} - \beta\omega H_{i,r} = -\rho h\omega^2 \int_S (d + w_r)w_r^*\,\mathrm{d}S \tag{28}$$

$$F_r = -\rho h\omega^2 \int_S (d + w_r)\,\mathrm{d}S, \qquad F_i = -\rho h\omega^2 \int_S w_i\,\mathrm{d}S \tag{29}$$

3. Validation

Forced vibrations of a plate excited at its centre have been simulated using finite element analysis. The model consists in 800 shell elements. The numerical parameters of the model are reported in Table 1.

Table 1
Parameters of the FE model

L (mm)	l (mm)	h (mm)	d	ρ (kg·m^{-3})	E (GPa)	ν	β (s)	f_1 (Hz)	f_2 (Hz)
200	100	1	0.1	7800	210	0.3	10^{-4}	85.4	337

From the FE model, the element curvatures and deflections are extracted, together with the coordinates of the elements centroids, for the two frequencies of Table 1 corresponding to the first (bending along x) and third (torsion) modes of the plate respectively. Two virtual deflection fields are used: $w_1^*(x, y) = x^2(1 + j)$ and $w_2^*(x, y) = y^2(1 + j)$ (uniform virtual curvature fields).

To identify the three unknown from Eqs. (27) and (28), the following least-square objective function is built up:

$$f(D_{xx}, D_{xy}, \beta) = \left(H_{i,i} + \beta\omega H_{r,i} + \rho h\omega^2 \int_S w_i w_i^* \, \mathrm{d}S\right)^2 + \left(H_{r,r} - \beta\omega H_{i,r} + \rho h\omega^2 \int_S (d + w_r) w_r^* \, \mathrm{d}S\right)^2 \tag{30}$$

The integrals in $H_{p,q}$ are discretized using the finite element results and a simplex method is used to solve $f = 0$. The results are reported in Table 2.

Table 2
Results from identification

Relative error (%)	D_{xx}	D_{xy}	β
Mode 1	−0.3	−0.4	0.2
Mode 3	−0.2	2.0	0.8

4. Conclusion

The main advantages of the method are:

- the method provides simultaneous identification of both stiffness and damping parameters;
- material damping is obtained, not modal damping;
- the plate can be of any shape;
- provided that the virtual fields are well chosen, the method will be insensitive to the clamping conditions;
- although the validation results have been obtained from a plate excited at resonance, this assumption is not used in the method and therefore, it could be used on non-resonant response.

The present method would certainly be a useful tool to investigate frequency dependence of damping. Moreover, future applications to anisotropic materials (composites) can be considered. Nevertheless, further investigation will be necessary to fully validate the method.

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