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C. R. Mecanique 331 (2003) 481–487

COMPTES RENDUS MECANIQUE

Transient gas flow simulation using an Adaptive Method of Lines

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Received 10 July 2002; accepted after revision 12 May 2003

Presented by Évariste Sanchez-Palencia

Abstract

Designing natural gas pipelines to safely and efficiently handle unsteady flows, requires knowledge of pressure drop, flowrate and temperature distribution throughout the system. The accurate prediction of these parameters is essential in order to achieve optimum cumulative deliverability, and safe and reliable operation. An Adaptive Method of Lines algorithm is formulated for the solution of Euler system of equations, which fully simulates slow and fast transients. Two test cases present the improvement of the numerical solution from grid adaptation. Good results are obtained both for slow and fast transients simulations proving that the suggested numerical procedure is appropriate for such predictions. ***To cite this article: E. Tentis et al., C. R. Mecanique 331 (2003).***

Résumé

Simulation d'écoulements dans un gazoduc par la méthode adaptative de lignes. La conception des gazoducs pour gaz naturels permettant de maîtriser de manière sûre et efficace les écoulements non-uniformes nécessite une bonne connaissance de la chute de pression, du débit et de la distribution des températures à travers le système. Une prédiction correcte de ces paramètres est essentielle pour obtenir une livraison optimale ainsi que sécurité et fiabilité maximales. L'algorithme contenu dans la Méthode Adaptative de Lignes est formulée ici pour la solution du système d'équations d'Euler, qui décrit très bien les flux gazeux lents ou rapides. On présente deux cas particuliers illustrant l'amélioration considérable des solutions numériques grâce au choix de réseau adapté au problème. De bons résultats obtenus pour des simulations d'écoulements lents et rapides montrent que la procédure numérique suggérée est bien appropriée pour les prédictions de ce type. ***Pour citer cet article : E. Tentis et al., C. R. Mecanique 331 (2003).***

Keywords: Computational fluid mechanics; Natural gas; Transient flow; Method of lines; Adaptive grid

Mots-clés : Mécanique des fluides numérique ; Écoulements gazeux ; Simulation numérique ; Solutions numériques d'équations d'Euler ; Modélisation des gazoducs

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doi:10.1016/S1631-0721(03)00106-2

1. Introduction

It is a well established fact that flow in gas pipelines is unsteady. Conditions are always changing with time, no matter how small the changes might be. Most times, transmission and distribution systems for the natural gas pipeline conveyance, are designed purely for constant supply and demand flow conditions. Nevertheless, time-varying flow processes occur, due to the starting and stopping of compression stations, the use of gas storage facilities and the fluctuating consumer demands. The investigation of such transient phenomena leads to optimum design methods and economic, online network control and monitoring.

The dynamic behavior of long pipelines is characterized by large time constants, sometimes of as much as several hours, due to the resistance to flow in pipes and the large storage capacity of the pipelines. Transients in such complex and large scale systems can be satisfactorily described by the non-homogeneous, non-linear hyperbolic, inviscid Euler system of conservation laws in one dimensional form. The appropriate method for the solution of that system must be accurate but also with low computational cost.

Traditional methods for the numerical analysis of that system are the Method of Characteristics (MOC) [1] and several finite difference schemes such as explicit finite differences [2] and fully implicit schemes, i.e., Crank–Nikolson method [3]. Recent relevant studies used higher resolution explicit TVD Methods for the solution of sharp discontinuities fronts [4]. An alternative computational approach is the Method of Lines (MOL) [5]. This technique involves reducing an initial boundary value problem to a system of ordinary differential equations (ODE) in time through the use of a discretization in space. The resulting ODE system can be solved using variable step/variable order methods of numerical integration. The main advantage of the method is that by separating the problem of space and time discretization, it becomes easy to establish stability and convergence for a wide variety of problems.

According to previous studies [6,7] the MOL technique has been proved a reliable method for the solution of transients in gas pipeline systems. A common feature of these approaches is the use of a fixed spatial mesh. However, recent studies [8,9], which deal with the improvement of the Method of Lines, have reported that adapting the spatial mesh offers important advantages with regards to efficiency and accuracy of the solution process, particularly for problems with moving or highly localized features. This is a fact which has been neglected until now by researchers in the area of dynamic pipeline behavior.

The scope of the present paper is the development of an adaptive MOL algorithm and the investigation of its performance for the unsteady gas pipe flow simulation. For that reason the one dimensional Euler system was solved with the use of the MOL technique in connection with an adaptive grid algorithm. A static remeshing procedure based on the equidistribution principle has been developed. Numerical experiments are used to show off the effectiveness of the proposed grid adaptation which, indeed, is reducing the computational time in the calculation of the slow transient phenomena without significant losses in accuracy. The results which were obtained from a coarse adaptive grid solution proved to be equivalent with those obtained from a much finer uniform grid for slow transients propagation. Additionally, excellent improvement to the fast transients calculations was achieved. The proposed method is computationally efficient and is readily applicable as a method for design and control of network systems.

2. Mathematical formulation

The inhomogeneous, one-dimensional inviscid Euler system [10] of equations can describe completely the compressible unsteady gas flow in a horizontal pipeline with length L. It derives from the conservation of mass, momentum, energy for a small control volume. The flow can be adiabatic/isentropic which means flow without heat exchange with the ground outside, or isothermal which implies flow with complete heat exchange with the ground outside. In general matrix representation the system is formulated as:

$$\frac{\partial U}{\partial t} + \frac{\partial F(U)}{\partial x} + S(U) = 0 \tag{1}$$

where U is a vector of dependent variables, F is a non-linear in U matrix and S is a vector accounting for irreversibility:

$$U = \begin{bmatrix} \rho \\ \rho u \\ e_0 \end{bmatrix}, \quad F(U) = \begin{bmatrix} \rho u \\ \rho u^2 + p \\ (e_0 + p)u \end{bmatrix}, \quad S(U) = \begin{bmatrix} \rho u \beta \\ \rho(G + u^2\beta) \\ (e_0 + p)u\beta - \rho(q + uG) \end{bmatrix} \tag{2}$$

where $e_0 = \rho E = p/(Z(\gamma - 1)) + \rho u^2/2$ is the total energy per unit volume, ρ is the density, u is the velocity and p is the pressure. The inhomogeneous term S includes:

$$\beta = \frac{1}{A}\frac{\mathrm{d}A}{\mathrm{d}x}, \quad G = f\frac{u^2}{2}\frac{u}{|u|}\frac{4}{D}, \quad q = \frac{\dot{Q}}{\rho A\,\mathrm{d}x} \tag{3}$$

which represent area A, friction f and heat transfer effect $\dot{Q}$ respectively. For the closure of the system the equation of state $p = Z\rho RT$ is used, where Z is the gas compressibility factor calculating with GERG-88 formula [11]. The friction factor f is calculated with the explicit Colebrook–White formula.

3. Method of Lines

The Method of Lines is a convenient, reliable technique to approximate the solution of initial value problems for systems of linear and nonlinear partial differential equations (PDE) [5]. There are two crucial aspects of MOL which determine the method's success. The first deals with the choice of the spatial derivative's approximation. The second is the integrity of the chosen ODE solver.

For the discretization procedure of the system of Eq. (1), Taylor series approximation needs only first derivative. For higher accuracy a fourth order, five point approximation differentiation formula was used. For the i node:

$$\frac{\partial F(x_i)}{\partial x} = \frac{F(x_{i-2}) - 8F(x_{i-1}) + 8F(x_{i+1}) - F(x_{i+2})}{3(\Delta x_{i-2} + \Delta x_{i-1} + \Delta x_i + \Delta x_{i+1})} + \mathrm{O}\big(\Delta x^4\big) \tag{4}$$

hence the system (1) converted to the following ODE system:

$$\frac{\mathrm{d}U_i}{\mathrm{d}t} = -\frac{F(x_{i-2}) - 8F(x_{i-1}) + 8F(x_{i+1}) - F(x_{i+2})}{3(\Delta x_{i-2} + \Delta x_{i-1} + \Delta x_i + \Delta x_{i+1})} - S_i \tag{5}$$

consisting of a total of $3 * n$ equations where n is the number of computational nodes. Δx_i is the spatial step which is calculated as $\Delta x_i = L/(n-1)$ for the uniform grid and as $\Delta x_i = x_{i+1} - x_i$ for the adaptive grid.

For the time integration of the above ODE system, the implicit Gear's Backward Differentiation Formula (BDF) method [12] was used. The method is appropriate for stiff systems of equations. The integration procedure works with automatic step size control and automatically selects the global error order of a given order bound.

4. Adaptive grid strategy

In order to reduce the truncation error of the solution in a coarse grid, an adaptive grid algorithm is developed [8]. The discrete remeshing procedure consists of three main components. The first component is the algorithm for the construction of the new mesh of M points X_M, which derives from the existing mesh of N points X_N. The second component is the interpolation operator for the definition of the solution in the new mesh in terms of the solution of the old mesh. The third component of the remeshing algorithm is the means of restarting the time integrator.

The spatial node movement is based on the equidistribution principle. For the description of the algorithm consider the vector of the coordinates of the grid nodes $\underline{x} = [x_1, x_2, x_3, \ldots, x_n]$ with $x_{i-1} < x_i < x_{i+1}$ and having known locations at a time level t_j. Consider the corresponding solution as the vector $\underline{v} = [v_1, v_2, v_3, \ldots, v_n]$ for one of the primitive variables ρ, u, p. The grid movement is based on the discrete function:

$$m_i = \sqrt{\sigma + \alpha \cdot \|v_x\|^\kappa} \tag{6}$$

The parameters σ, α, κ are normally adjusted by solving a part of the problem. By the parameter $\sigma > 0$ the maximum possible mesh-size can be controlled and a common value for κ is 2. The discrete function m_i is converted to a continuous one with cubic spline interpolation. Integrating numerically the m_i with the trapezoidal rule from 0 to L the following area is calculated:

$$M_0 = \frac{\int_0^L m_i \, \mathrm{d}x}{n-1} \tag{7}$$

The new x_i values are computed from the numerical solution with the Newton–Raphson method of the equation:

$$\frac{m_i + m_{i-1}}{2}(x_i - x_{i-1}) - M_0 = 0 \tag{8}$$

The values of the primitive variables are recalculated using polynomial interpolation for the updated values of the grid. The mesh is changed only at discrete times, interpolates the old mesh to the new mesh and then restarts the time integration. This approach is called static rezoning or discrete remeshing. Alternatively, as interpolation operator a cubic-spline approximation may be used instead of polynomial interpolation so that the produced derivatives behave more smoothly.

5. Results and discussion

For the evaluation of the adaptive grid model two representative test cases are examined. The gas velocity is used as the monitoring function for the grid adaptation model. The CPU time is benchmarked in a 433 MHz Pentium personal computer. The bound for absolute and relative error of the Gear's BDF method is set to 10^{-5}. So long as the grid adaptation movement is static the ODE's integrator restarts 1000 times for case 1 and 100 times for case 2.

5.1. Test case 1: Slow transient propagation in an actual transmission pipeline

The system consists of a 72259.5 m long and 0.2 m diameter pipeline which transports natural gas of 0.675 specific gravity and 10 °C temperature. The gas viscosity μ_g is $11.84 \times 10^{-6}\ \mathrm{kg \cdot m^{-1} \cdot s^{-1}}$ while the pipeline wall roughness is 0.617 mm. The same problem has been simulated from Taylor et al. [13]. As concerns the boundary conditions, at the pipeline's inlet the gas pressure and density are kept constant whereas the pipe's mass flow rate at the outlet varies with a 24-hour cycle, corresponding to changes in consumer demand within a day (Fig. 1).

The numerical experiment was carried out as follows. A very coarse, 11 nodes adaptive grid, is compared with a uniform one of the same size and with a finer one of 101 nodes. The Fig. 2 shows the comparison between pressure histories at pipe's outlet for the three different grids. The coarse adaptive grid gave good results in comparison to the finer grid, with a maximum deviation of 2.1%, only in a small section at the end of the experiment. On the contrary the uniform coarse grid presents significant deviation especially after 14 h, with maximum value of 13.28%.

Fig. 3 depicts the comparison of volume flow rate at pipe's inlet. Again the adaptive grid gave better results than the uniform one. Fig. 4 shows the predicted pressure distributions along the pipeline at three times ($t = 4.84$, 12.95, 23.00 h). In all times, the pressure decreases monotonically from inlet to outlet, due to pressure drop in the flow direction along the pipeline. In Fig. 5 the time dependent adaptive grid movement along the pipeline is depicted. Table 1 is the summary of the numerical experiments' parameters and computational costs. That is the number of steps that ODE's integrator used as well as the computational time. Δt is the mean value of the numerical integration step. Grid adaptation improves the accuracy in comparison with the coarse uniform grid.

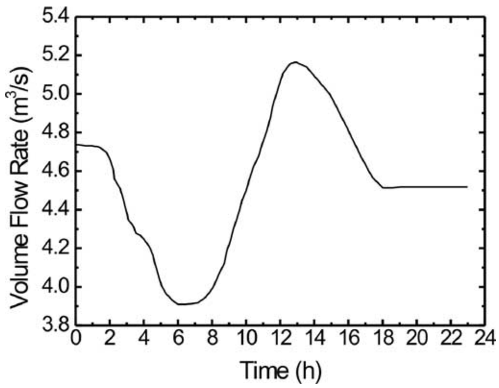

Fig. 1. Prescribed outlet volume flow rate for test case 1 (boundary condition).

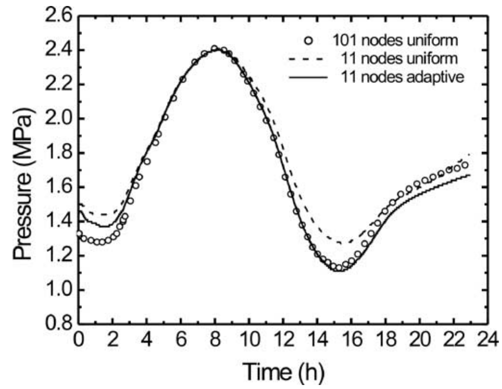

Fig. 2. Comparison between adaptive and uniform grids for pipe's outlet pressure history.

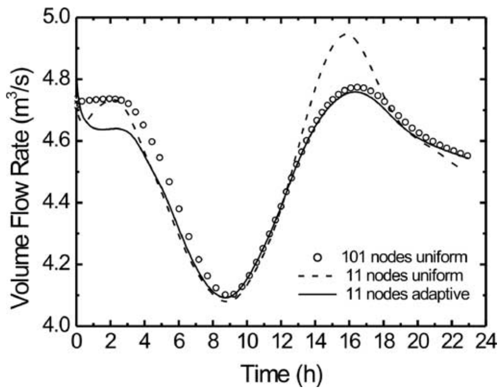

Fig. 3. Comparison between adaptive and uniform grids for inlet volume flow rate history.

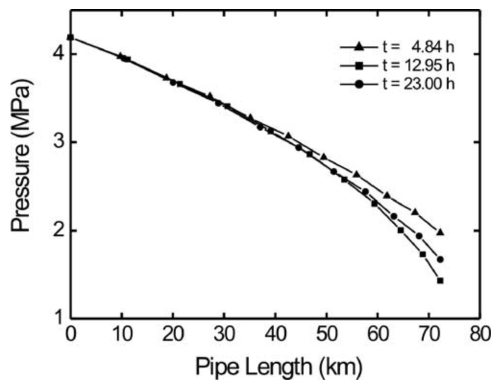

Fig. 4. Pressure distribution along pipeline at three instants for test case 1.

Table 1
Computational statistics for test case 1

Grid	Nodes	Steps	CPU (s)	Δt (s)
Uniform	101	1264905	5422.748	58.587
Uniform	11	391019	56.892	28.434
Adaptive	11	411617	69.710	28.682

5.2. Test case 2: Instantaneous closure of downstream valve

Severe transients conditions are created from a downstream valve's stroking. At $t = 0^-$, steady state flow is established (pipeline length and diameter are 30 m and 0.1 m respectively) with gas mass flux of 20 $\text{kg}\cdot\text{m}^{-2}\cdot\text{s}$ and inlet gas density of 2 $\text{kg}\cdot\text{m}^{-3}$; at $t = 0^+$, the downstream valve is closed instantaneously, thus setting the gas mass flux at the outlet equal to zero, while the inlet mass flux is maintained at its original value. The pipe roughness is set to 0.03 mm, gas density is 0.795 $\text{kg}\cdot\text{m}^{-3}$ hence the duration of the numerical experiment is 0.08 s. Figs. 6 and 7 depict a comparison between the predicted wave fronts of velocity for uniform and adaptive grid of 101 nodes. Multiple lines show the velocity waves while travelling backwards along the pipeline after the valve's stroking. The time interval between two neighboring wave fronts is 8 ms. In Fig. 8 the adaptive grid movement with time

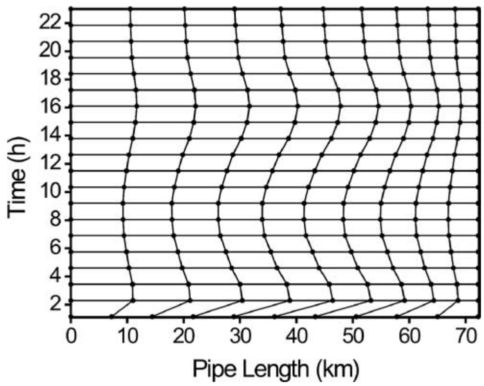

Fig. 5. Adaptive grid movement with time along pipeline for test case 1.

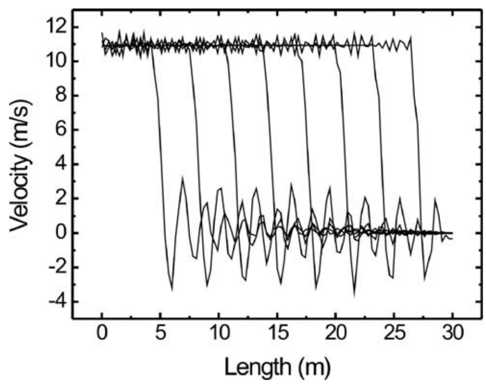

Fig. 6. Velocity wave propagation with uniform grid solution for test case 2.

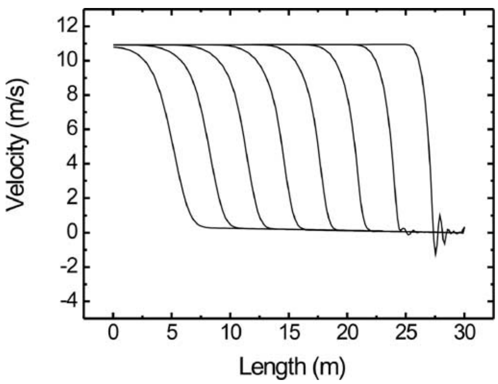

Fig. 7. Velocity wave propagation with adaptive grid solution for test case 2.

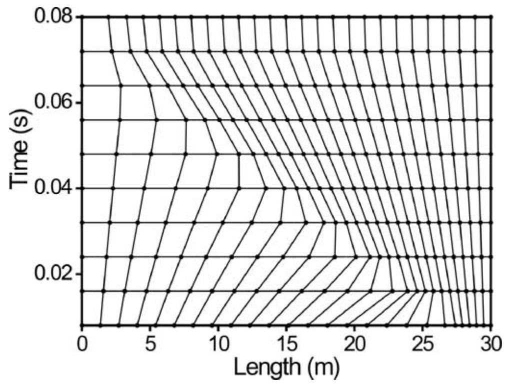

Fig. 8. Grid adaptive distribution with time along pipeline for test case 2.

is shown. To ensure the clarity of the diagram only every fourth node is depicted. The spurious oscillations are very strong for the uniform grid solution. Using adaptive grid calculations the spurious oscillations are damping significantly especially after the first 0.01 s.

6. Conclusions

An adaptive MOL grid algorithm is combined with the full one-dimensional Euler equations for the simulation of unsteady flow in gas transmission systems. Two numerical experiments are used for the validation of the proposed method. The first simulates a slow transient propagation at a 24 h cycle, in a 72259.5 m long and 0.2 m diameter pipeline. The numerical results of the adaptive coarse grid are in good agreement with those of the finer uniform one. On the contrary the uniform coarse grid gave poor results. From Table 1 we observe that solution by the adaptive MOL as compared to that with fine uniform grid is obtained significantly more efficiently, in terms of CPU time for numerical results obtained with almost the same accuracy.

In the second test case fast transients are created from the instantaneous closure of a downstream valve in a short pipe. Intense spurious oscillations are present in the uniform MOL solution. On the other hand, using grid adaptation, the results are improved dramatically without the use of any artificial viscosity terms or flux limiters. The solution captures and maintains the integrity of the wave fronts satisfactorily.

The numerical results of this study prove that the proposed methodology is appropriate for the acceleration of slow transient gas pipe predictions, and the error reduction in fast transient predictions. The investigation is continuing and the optimization of the model and of the whole numerical procedure will be presented in future papers.

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