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The generalized-kinetics-based equilibrium distribution function for composite particles

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Abstract

This work is devoted to the equilibrium distribution function for a fluid of mutually non-interacting identical composite point particles in three-dimensional physical space. The distribution function is derived within the generalized-kinetics (GK) vision from the proposed probabilistic model based on quantum-mechanical bosons and fermions. The first GK advantage is that the derivation does not involve any assumption on the interpolation between bosons and fermions whereas the resulting function provides this interpolation. The second GK advantage is that composons, the particles described with the GK-based distribution function, are considerably less schematic and more consistent physically than quons. Composons correspond to a specific case of Isakov's general q-commutation relation involving an infinite number of the q-coefficients. Connection of the composon concept to previous results in the literature is pointed out. A few directions for future research on the topic are formulated. The results of the work can be used in the composite-particle fluid problems where the Maxwell–Boltzmann description is not valid, for instance, in dense populations of not too massive point-like particles of a complex, composite nature at not too high temperatures. ***To cite this article: N. Bellomo et al., C. R. Mecanique 331 (2003).***

Résumé

La fonction de distribution d'équilibre pour des particules composées basée sur la cinétique généralisée. Ce travail s'intéresse à la fonction de distribution d'équilibre pour un fluide mutuellement non agissant, composé de particules points dans un espace de dimension trois. La fonction de distribution provient, d'un point de vue de CG, d'un modèle probabiliste issu de la mécanique quantique des fermions et des bosons. Le premier avantage de CG est que la dérivation ne nécessite aucune hypothèse sur l'interpolation entre les bosons et les fermions alors que la fonction résultante fournit cette interpolation. Le second est que les composons, les particules décrites par ce procédé sont considérablement moins schématiques et plus consistantes, physiquement, que les quons. Les composons correspondent à un cas particulier de la relation générale de q-commutation d'Isakov, pour un nombre infini de q-coefficients. Les résultats antérieurs liés au concept de composon sont signalés et quelques directions de recherches futures sont proposées. Les résultats de ce travail peuvent servir pour l'étude de fluides composés, où la description Maxwell–Boltzmann n'est pas valable, par exemple, pour une dense population de

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particules, pas trop lourdes et a des températures pas trop élevées, et d'une comoposition de nature complexe. ***Pour citer cet article : N. Bellomo et al., C. R. Mecanique 331 (2003).***

Keywords: Fluid mechanics, quantum mechanics; Boson; Fermion; Distribution function; Composite particle; Generalized kinetics; q-commutation relation; Isakov's coefficient; Quon; Composon

Mots-clés : Mécanique des fluides, mécanique quantique ; Boson ; Fermion ; Fonction de distribution ; Particules composées ; Cinetique généralisée ; Relation de q-commutation ; Coefficients d'Isakov ; Quon ; Composon

1. The problem and the purpose of the work

This works is devoted to the equilibrium distribution function for a fluid of mutually non-interacting identical composite point particles in *three*-dimensional physical space. Composite particles are either elementary ones or those formed by the latter.

If the fluid particles are elementary, then, as is well known (e.g., [1, §56]), the distribution function is $f(x, g_e)$ where $f(x, g) = (g/h^3)N(x, g)$, h is Planck's constant, $g_e \in N$ is fixed, $g_e = 2s_e + 1$, $x = \exp[(u - \mu)/(KT)]$, $\mathbb{N} = \{1, 2, 3, \ldots\}$, g_e is the number of the particle–spin orientations, s_e is the particle quantum-spin number (expressing the intrinsic angular momentum of a particle, or its spin, with $s_e h/(2\pi)$), K is Boltzmann's constant, T is the absolute temperature of the fluid, μ is the electrochemical potential, $\mu \in (-\infty, \infty)$, $u \geqslant 0$ is the particle kinetic energy, $N(x, g)$ is the mean occupation number,

$$N(x, g) = \left[x + \iota(g)\right]^{-1}, \quad \exp\left[-\mu/(kT)\right] \geqslant -\iota(g), \quad g \in \mathbb{R} \tag{1}$$

$$\iota(g) = \begin{cases} 0, & g \in \mathbb{R}\backslash\mathbb{N} \\ -1, & g \in \mathbb{N} \text{ and } g \text{ is odd} \\ 1, & g \in \mathbb{N} \text{ and } g \text{ is even} \end{cases} \tag{2}$$

and $\mathbb{R} = (-\infty, \infty)$. Quantity g_e is also known as the particle g-factor, or spin degeneracy, and, because of multiplier g in $f(x, g)$, can be regarded as the particle *multiplicity*. In what follows, we for brevity use this term for g_e. Also, for brevity, we shall call s_e the particle spin. Relations $g_e \in \mathbb{N}$ and $g_e = 2s_e + 1$ correspond to

$$s_e \in \{0, 1/2, 1, 3/2, 2, \ldots\} \tag{3}$$

Number $g_e \in N$ is odd or even depending on if the particles are bosons or fermions, respectively. Quantities $f(x, g)$ and (1) at $g = g_e$ when g_e is odd (even) are called the Bose–Einstein (BE) (Fermi–Dirac (FD)) distribution function and mean occupation number respectively.

Not only elementary particles but also composite particles, even very big ones, manifest their quantum nature. This is emphasized by recent experimental results (e.g., [2]). However, common quantum mechanics does not provide distribution functions for general composite particles. The rules to determine if a composite particle is boson or fermion (or, in terms of $f(x, g)$, what is a specific value of $\iota(g)$) are known only in certain simple cases (e.g., [3, p. 25], [4]). Modern research works in the field are beyond the standard, either-boson-or-fermion prescription. They are summarized, for example, in [5, Section 1] and the references therein. All these works *assume* interpolations between bosons and fermions (see also Remark 5 below for the details).

The present work approaches the problem from an entirely different point of view. Section 2 proposes the spin-mixture model and a derivation of the equilibrium distribution function within the generalized-kinetics (GK) vision [6, Section 10.3], [7,8]. The composite particles described in this way are called 'composons'. Section 3 shows that the resulting expression corresponds to a specific case of Isakov's general q-commutation relation and points out how composons differ from quons. The work is summed up in Section 4. (All the remarks are formulated in Section 2.)

2. Probabilistic derivation of the boson–fermion interpolation: the GK-based distribution function for composite particles

Since the fluid particles are composite, we generally do not know the corresponding specific value of the particle multiplicity g_e. It is natural to consider g in $f(x,g)$ as a value of a random variable, say, G and set $\mathbb{N}$ as the set of the values of G. This means that G is a function of elementary event $\xi \in \Xi$, Ξ is the space of elementary events, and random variable $G(\cdot)$ is discrete. It presents the *random* multiplicity of the fluid particles. More specifically, the random multiplicity takes value (cf. $g_e \in \mathbb{N}$) $j \in \mathbb{N}$ with probability

$$p_j \geqslant 0, \quad j \in \mathbb{N} \tag{4}$$

where

$$\sum_{j=1}^{\infty} p_j = 1 \tag{5}$$

We assume that expectation $\bar{g}$ of random multiplicity G exists, i.e.,

$$\bar{g} = \int_{\mathbb{R}} g\rho(g)\,\mathrm{d}g < \infty \tag{6}$$

The spin value corresponding to p_j is $(j-1)/2$. The probability density ρ of random variable G is

$$\rho(g) = \sum_{j\in\mathbb{N}} p_j \delta(g-j), \quad g \in \mathbb{R} \tag{7}$$

where δ is the Dirac delta-function.

Subsequently, distribution function $f(x,g)$ can be regarded as the conditional distribution function under the condition that $G(\xi) = g$. The corresponding overall, joint distribution function is

$$f(x,g)\rho(g), \quad \exp\bigl[-\mu/(KT)\bigr] \geqslant -\iota(g), \quad g \in \mathbb{R} \tag{8}$$

In fact, this is a particular case of the generalized distribution function introduced in the GK theory [6, Section 10.3], [7] (see also [8]). The key feature of function (8) is that it takes into account the randomness in the involved *parameter*, multiplicity g in the present case, of the fluid particles. Common, i.e., averaged in the multiplicity, version $\overline{F}(x)$ of $f(x,g)$ is determined by means of (8) as the well-known expectation [7, Section 2], [8, (5.1)], i.e.,

$$\overline{F}(x) = \int_{\mathbb{R}} f(x,g)\rho(g)\,\mathrm{d}g \tag{9}$$

It follows from (9), the f-function form, (1), (2), and (7) that (see also the text above (1) for x)

$$\overline{F}(x) = h^{-3}\bigl[\bar{g}_b/(x-1) + \bar{g}_f/(x+1)\bigr], \quad \mu \leqslant 0 \quad \text{if } \bar{g}_b > 0 \tag{10}$$

where numbers

$$\bar{g}_b = \int_{\mathbb{R}} \sum_{j\in\mathbb{N}:\ j \text{ is odd}} g p_j \delta(g-j)\,\mathrm{d}g, \qquad \bar{g}_f = \int_{\mathbb{R}} \sum_{j\in\mathbb{N}:\ j \text{ is even}} g p_j \delta(g-j)\,\mathrm{d}g \tag{11}$$

are the expectations of the bosonic and fermionic multiplicities of the fluid particles. Obviously, expectation (6) of the total particle multiplicity is coupled with these numbers by equality

$$\bar{g} = \bar{g}_b + \bar{g}_f \tag{12}$$

In view of relations $g_e = 2s_e + 1$ and (7), the particle–spin-number expectation $\bar{s}$ corresponding to (6) is $\bar{s} = \int_{\mathbb{R}} [(g-1)/2]\rho(g)\,\mathrm{d}g$ and hence $\bar{g}$ can also be expressed as follows

$$\bar{g} = 2\bar{s} + 1 \tag{13}$$

Remark 1. The above random model describes the composite-particle multiplicity and spin with the infinitely countable sets of the values corresponding to probabilities (4). If one needs to assign a *single* value to the multiplicity or spin, expectations $\bar{g}$ and $\bar{s}$ are the very numbers to be used in this capacity. Note that they are generally real and hence need not be in set $\mathbb{N}$ and the set displayed in (3). The physical picture for the latter is pointed out in Remark 2 whereas Remark 6 discusses the corresponding examples.

Numbers $\bar{g}$ and $\bar{s}$ coincide with g_e and s_e respectively if the fluid particles are elementary, i.e., $p_j = 0$ for all $j \in \mathbb{N}$ such that $j \neq g_e$ and $p_{g_e} = 1$. This also agrees with that (13) is the composite-particle generalization of equality $g_e = 2s_e + 1$.

Remark 2. Comparison of the GK model (10), (11) based on (9) with the well-known mixed-state representations [9, (14.4), (14.6), (14.10)] shows the following. The proposed model describes a composite particle as a particle at the *mixed* quantum state composed of a countable set of the mixing states (pure or also mixed) where the jth mixing state, $j \in \mathbb{N}$, has multiplicity j (and spin $(j-1)/2$). In so doing, probabilities p_j are similar to the ones used in the mixed-state density matrix (cf. [10, Chapter 3]). In other words, a composite particle is regarded as a particle with the *multiplicity* (or *spin*) *mixture*. This in particular leads to quantum correlations between *different* mixing states of the *same* composite particle, new phenomena in quantum mechanics.

Relations (4), (5), (11), and (12) imply that and $\bar{g}_b \geqslant P_b$, $\bar{g}_f \geqslant 2P_f$, and

$$\bar{g} \geqslant 1 + P_f \tag{14}$$

where numbers

$$P_b = \int\limits_{\mathbb{R}} \sum_{j \in \mathbb{N}:\ j \text{ is odd}} p_j \delta(g-j)\,\mathrm{d}g \geqslant 0, \qquad P_f = \int\limits_{\mathbb{R}} \sum_{j \in \mathbb{N}:\ j \text{ is even}} p_j \delta(g-j)\,\mathrm{d}g \geqslant 0 \tag{15}$$

are the probabilities of that the fluid particles are bosons and fermions respectively. They are such that

$$P_b + P_f = 1 \tag{16}$$

Equality (16) follows from (15) and the fact that function ρ (see (7)) is a probability density.

Remark 3. Relations (15), (16), and (11) indicate that the fluid particles are bosons (fermions) if and only if $P_b = 1$ ($P_f = 1$). Importantly, equality $P_b = 1$ ($P_f = 1$) is equivalent to equality $\bar{g}_f = 0$ ($\bar{g}_b = 0$). Subsequently (see also (14) and the inequalities above it), $\overline{Q} \in [-1, 1]$ where

$$\overline{Q} = (\bar{g}_b - \bar{g}_f)/(\bar{g}_b + \bar{g}_f) \tag{17}$$

$\overline{Q} = 1$ and $\overline{Q} = -1$ for bosons and fermions respectively. Since the boson-like condition $\mu \leqslant 0$ is necessary as far as $\bar{g}_b > 0$ (or $P_b > 0$), one can regard the particle nature to be mostly bosonic.

Remark 4. The probability $(1 - P_b)(1 - P_f)$ of that a particle of the fluid is neither boson nor fermion is, by virtue of (16), equal to $P_b P_f$, i.e., the probability of that the particle is both boson and fermion. This is the GK-based generalization of the common, either-boson-or-fermion prescription mentioned in Section 1. In this sense, the present approach inherits the cornerstone rule of standard quantum distributions.

Remark 5. It follows from Remark 3 (see also (12)) that the GK-based model (10) describes a composite particle by means of an interpolation between bosons and fermions. This interpolation is *derived* from the more basic, spin-mixture model (see Remark 2), as opposed to many works (e.g., [5,11–15]) which do assume the boson–fermion interpolations. These interpolations are usually linear and implemented with the interpolating parameters (e.g., [11, 'q' on p. 705], [12, 'q' in (1)], [13, 'g_{ab}' in (2)], [5, 'q_1', 'q_2', $\ldots$ in (81)–(83)]], [14, 'g_{ab}' in (24) for (23)], [15, 'α' in (2) and 'α_{ij}' in (3), (4)]). The fact that the present work is free from any interpolation assumption is one of the key advantages of the underlying GK approach.

We for brevity call the particles described with the GK-based distribution function (10), (11) *composons* (due to French 'composons' which means 'let us compose'). This term is thought to emphasize that the particles are naturally composed of bosons and fermions (cf. Remark 4) within the GK vision of the proposed spin-mixture model with no formalistic assumptions on the boson-fermion interpolation.

Remark 6. Experiments show that composons, composite particles with the spin (or multiplicity) mixture, exist. One of many examples of composons is the ^{165}Ho(n,γ) nucleus where [16] (about) 39% of the compound states are with spin 4 whereas 61% of the states are with spin 3. Subsequently, all p_j, except p_7 and p_9, are zero, $p_7 = 0.61$, $p_9 = 0.39$, $P_b = 1$, $P_f = 0$, $\bar{g}_f = 0$, $\bar{g} = \bar{g}_b = 9 \times 0.39 + 7 \times 0.61 = 7.78$, $\bar{s} = (7.78 - 1)/2 = 3.39$. Thus, the ^{165}Ho(n,$\gamma$) nucleus is boson with *expected* spin 3.39. This illustrates Remarks 1 and 2 as well. Also rather simple but dissimilar examples of composons are associated with experiments in nuclear science [17, Table 3], electronics [18], relativistic gravity [19], biology [20–22], and the Fermi-gas theory [23,24].

The GK-based model (10) is fully determined if parameters $\bar{g}_b$ and $\bar{g}_f$ or, equivalently (see (11)), probabilities (4) of properties (5), (6) are available. They can be obtained by means of the experimental or theoretical studies. However, model (10) also admits another representation which is the topic of the next section.

3. Connection of the GK-based distribution function to Isakov's general q-commutation quantum-mechanical relation: composons and quons

The GK-based distribution function (10) is described with the help of two parameters $\bar{g}_b$ and $\bar{g}_f$. It can equivalently be formulated in terms of $\bar{g}$ (see (12)) and $\overline{Q}$ (see (17)) where $\overline{Q}$, as we shall see below, turns out to be equal to parameter q_1 in the general q-commutation quantum-mechanical (QM) relation by Isakov [5]. We first consider a series of the related issues.

As is well known (e.g., [5, Section 2]), mean occupation number $N(x)$ (e.g., see (1)) involved in $f(x, g)$ is a fundamental notion which is introduced for fluids of rather general, composite particles, not only in the either-boson-or-fermion case ($g_e \in \mathbb{N}$, (1)). For any fluid, the following features (e.g., [5, p. 743]) hold

$$N\bigl(1/x^{-1}\bigr) = 0 \quad \text{and} \quad \mathrm{d}N\bigl(1/x^{-1}\bigr)/\mathrm{d}\bigl(x^{-1}\bigr) = 1 \quad \text{in the limit case as } x \to \infty \tag{18}$$

This means that the mean occupation number for *any* fluid in the limit case in (18) coincides with the Maxwell–Boltzmann (MB) one, i.e., $N(x) = x^{-1}$. (Note that the MB version of distribution function (see (10)) includes $\bar{g}$ (see (12)), namely, $\overline{F}(x) = (\bar{g}/h^3)x^{-1}$.) Mean occupation number $N(x)$ is coupled with the corresponding partition function $\Pi(x^{-1})$ as follows (e.g., [5, (14), (15)])

$$N\bigl(1/x^{-1}\bigr) = x^{-1}\bigl[d\Pi\bigl(x^{-1}\bigr)/d\bigl(x^{-1}\bigr)\bigr]/\bigl[\Pi\bigl(x^{-1}\bigr)\bigr], \qquad \lim_{x^{-1}\to 0}\bigl[d\Pi\bigl(x^{-1}\bigr)/d\bigl(x^{-1}\bigr)\bigr]/\bigl[\Pi\bigl(x^{-1}\bigr)\bigr] = 1 \tag{19}$$

The GK-based distribution function (10) is (see (12)) equivalent to $\overline{F}(x) = (\bar{g}/h^3)[x - (\bar{g}_b - \bar{g}_f)/(\bar{g}_b + \bar{g}_f)]/(x^2 - 1)$ where $\mu \leqslant 0$ if $\bar{g}_b > 0$. This can, after accounting Remark 3, be represented in the form similar to that of function f, namely, $\overline{F}(x) = \bar{f}(x, \bar{g}, \overline{Q})$ where

$$\bar{f}\bigl(x, \bar{g}, \overline{Q}\bigr) = \bigl(\bar{g}/h^3\bigr)\overline{N}\bigl(x, \overline{Q}\bigr) \tag{20}$$

and mean occupation number $\overline{N}(x, \overline{Q})$ differs from (1) since

$$\overline{N}\bigl(x, \overline{Q}\bigr) = \bigl[\bigl(1 + \overline{Q}\bigr)/2\bigr](x-1)^{-1} + \bigl[\bigl(1 - \overline{Q}\bigr)/2\bigr](x+1)^{-1} = \bigl(x + \overline{Q}\bigr)/\bigl(x^2 - 1\bigr), \ \mu \leqslant 0 \quad \text{if } \overline{Q} > -1 \tag{21}$$

It follows from Remark 3 that both coefficients $(1 + \overline{Q})/2 = \bar{g}_b/\bar{g}$ and $(1 - \overline{Q})/2 = \bar{g}_f/\bar{g}$ in (21) are in interval [0,1]. They present the probabilities with which total multiplicity $\bar{g}$ (see (12)) is formed by bosonic and fermionic multiplicities $\bar{g}_b$ and $\bar{g}_f$, respectively. (Note that these probabilities generally differ from $\overline{P}_b$ and $\overline{P}_f$.) One can readily check that $\overline{N}(x,\overline{Q})$ has properties (18) as it should. The partition function $\Pi(x^{-1},\overline{Q})$(see (19)) is (e.g., [25, 140 and 141.1.]) $\sqrt{(1+x^{-1})^{1-\overline{Q}}/(1-x^{-1})^{1+\overline{Q}}}$ where $\mu\leqslant 0$ if $\overline{Q}>-1$. It is reduced to the FD and BE cases (e.g., [5, p. 743]) $(1+x^{-1})$ and $1/(1-x^{-1})$ at $\overline{Q}=-1$ and $\overline{Q}=1$, respectively.

The general q-commutation QM relation was proposed and analyzed by Isakov [5]. It is [5, (83)]

$$aa^{\dagger}=1+q_1a^{\dagger}a+q_2\left(a^{\dagger}\right)^2a^2+\cdots=1+\sum_{i=1}^{\infty}q_i\left(a^{\dagger}\right)^ia^i \tag{22}$$

where a and $a^{\dagger}$ are the QM annihilation and creation operators, $q_1\in[-1,1]$, and $q_i=0$, $i=2,3,\ldots$, at $q_1=\pm1$, i.e., for bosons or fermions. If the particles are not fermions, i.e., $q_1>-1$, then [5, Section 5.2 and the text above (69)] coefficients $q_1,q_2,q_3,\ldots$ in (22) can be obtained from the expansion of mean occupation number in the powers of x^{-1}. This expansion for composons follows from (21) and is of the form below

$$\overline{N}\left(x,\overline{Q}\right)=x^{-1}+\overline{Q}x^{-2}+x^{-3}+\overline{Q}x^{-4}+\cdots,\quad x>1,\ \mu\leqslant 0\quad\text{if }\overline{Q}>-1 \tag{23}$$

This is a specific case of the series [5, (90)] corresponding to (22). Comparison of (23) and [5, (90)] points out that all the Isakov coefficients $\{q_i\}_{i\in\mathbb{N}}$ for composons are determined solely by $\overline{Q}$. In particular, $q_1=\overline{Q}$, $q_2=0$ if $\overline{Q}=-1$, and $q_2=1-\overline{Q}$ if $\overline{Q}\in(-1,1]$. Note that q_2 is discontinuous in the limit case as $\overline{Q}\to-1+0$ since in this very case composons become fermions making the boson-like limitation $\mu\leqslant 0$ which is presumed for all $\overline{Q}\in(-1,1]$ no longer necessary.

Composons are not quons. Compared to composons, quons [12] (see also [11]), [5, Section 5.1] are of a little oversimplified nature and considerably more schematic. Indeed, all Isakov's coefficients in (22) except q_1 for quons are identically zero. This peculiar construction of quons does not protect them against the Gibbs paradox (e.g., [11, p. 707], [26, pp. 40–41]) associated with the so-called 'quantum' MB distribution at $q_1=0$, i.e., the MB one that holds for *all* $x\in\mathbb{R}$ rather than as the second limit equality in (18). Unlike this, composons at $\overline{Q}=0$ have (see (21)) $\overline{N}(x,\overline{Q})=x/(x^2-1)$, $\mu\leqslant 0$, that is the same as [5, (21)] and can be regarded as the *quasi*-MB behavior, somewhat similar to that stressed in [27, pp. 933, 934]. Also note that there is no derivation of quons from a more basic model (like that in Section 2 for composons) either.

4. Summing up

The main results of this work are the following.

- The GK-based equilibrium distribution function (10) (or (20), (21), (12), (17)) and (11) (see also the text above (1) on x) of a three-dimensional fluid of mutually non-interacting composite identical point particles is obtained. Composons, the particles described with it, are composite particles with the mixture of the multiplicity (or spin) of a particle (see Remark 2). Rather simple examples of composons are shown in various experiments (see Remark 6). Composons include both bosons and fermions as quite particular cases (see Remarks 3 and 4). The above distribution fucntion is derived without assumptions on the boson–fermion interpolation (see Remark 5).
- The composon model stemming from the GK vision corresponds to a specific case of the Isakov general q-commutation relation (22) involving coefficients $\{q_1,q_2,\ldots\}$. For composons, the work obtains $q_1=\overline{Q}$ and determines Isakov's coefficient $q_2\neq 0$. Composons are not quons. Compared to quons, composons are considerably less schematic and more consistent physically. Connection of the composon concept to some other results in the q-commutation relations is pointed out.

The results of the work can be especially useful in the composite-particle fluid problems where the Maxwell–Boltzmann description is not valid, for instance, in dense populations of not too massive point-like particles of a complex, composite nature at not too high temperatures. Even if the Maxwell–Boltzmann description is valid, the present results are important since the corresponding distribution function depends (see the text between (18) and (19)) on parameter (12). We also emphasize the following three problems for future research on the topic: (i) evaluation of Isakov's coefficients $\{q_3, q_4, \ldots\}$ in (22) for composons, (ii) theoretical and experimental studies on how to determine either probabilities (4), (5) or parameters (12) and (17), and (iii) development of the detailed QM *mixed*-state treatment outlined in Remark 2.

Of a special interest are applications of the generalized-kinetics theory [6–8] to other problems in classical and quantum mechanics. This can noticeably facilitate solving the problems difficult to approach with the help of common techniques.

References

[1] E.M. Lifshitz, L.P. Pitaevskii, Statistical Physics, Part 1, Pergamon Press, Oxford, 1980.
[2] O. Nairz, M. Arndt, A. Zeilinger, Experimental verification of the Heisenberg uncertainty principle for fullerence molecules, Phys. Rev. A 65 (2002) 032109/1–032109/4.
[3] H.A. Bethe, R. Jackiw, Intermediate Quantum Mechanics, Benjamin–Cummings, Menlo Park, 1986.
[4] P. Ehrenfest, J.R. Oppenheimer, Notes on the statistics of nuclei, Phys. Rev. 37 (4) (1931) 333–338.
[5] S.B. Isakov, Generalization of quantum statistics in statistical mechanics, Int. J. Theor. Phys. 32 (5) (1993) 737–767.
[6] N. Bellomo, M. Lo Schiavo, Lecture Notes on the Mathematical Theory of Generalized Boltzmann Models, World Scientific, Singapore, 2000.
[7] L. Arlotti, N. Bellomo, E. De Angelis, Generalized kinetic (Boltzmann) models: Mathematical structures and applications, Math. Models Methods Appl. Sci. 12 (4) (2002) 567–591.
[8] N. Bellomo, E. Mamontov, M. Willander, On the generalized kinetic modelling of a multicomponent "real-life" fluid by means of a single distribution function, Math. Comput. Modelling (2003) 21 pp., accepted.
[9] I. Bialynicki-Birula, M. Cieplak, J. Kaminski, Theory of Quanta, Oxford University Press, New York–Oxford, 1992.
[10] E. Leader, Spin in Particle Physics, Cambridge University Press, Cambridge, 2001.
[11] O.W. Greenberg, Example of infinite statistics, Phys. Rev. Lett. 64 (7) (1990) 705–708.
[12] O.W. Greenberg, Particles with small violations of Fermi or Bose statistics, Phys. Rev. D 43 (12) (1991) 4111–4120.
[13] F.D.M. Haldane, "Fractional statistics" in arbitrary dimensions: a generalization of the Pauli principle, Phys. Rev. Lett. 67 (8) (1991) 937–940.
[14] S.B. Isakov, Generalization of statistics for several species of identical particles, Mod. Phys. Lett. B 8 (5) (1994) 319–327.
[15] Y.-S. Wu, Statistical distribution for generalized ideal gas of fractional-statistics particles, Phys. Rev. Lett. 73 (7) (1994) 922–925.
[16] R.I. Schermer, Spin dependence of the thermal neutron cross section of Ho_{165}, Phys. Rev. 136 (5B) (1964) B1285–B1288.
[17] J. Eichler, F. Djadali, Measurement of the average circular γ-polarization and determination of spins for compound states formed in thermal neutron capture, Nucl. Phys. A 147 (1) (1970) 150–160.
[18] K. Bärner, U. Neitzel, Scattering of charge carriers on a high spin-low spin mixture in $MnAs_{1-x}P_x$ crystals, Phys. Lett. A 91 (7) (1982) 361–364.
[19] L.S. Finn, Gravitational waves from solar oscillations: Proposal for a transition-zone test of general relativity, Class. Quantum Grav. 2 (3) (1985) 381–402.
[20] P. George, J. Beetlestone, J.S. Griffith, Ferrihemoprotein hydroxides: a correlation between magnetic and spectroscopic properties, Rev. Mod. Phys. 36 (1) (1964) 441–458.
[21] A. Feis, M.P. Marzocchi, M. Paoli, G. Smulevich, Spin state and axial ligand bonding in the hydroxide complexes of metmyoglobin, methemoglobin, and horseradish peroxidase at room and low temperatures, Biochemistry 33 (15) (1994) 4577–4583.
[22] M. Czerwinski, J. Dabrowski, Spin–spin interactions in the reduced $[Fe_6S_6]^{5+}$ cluster, Chem. Phys. 213 (1–3) (1996) 45–62.
[23] B. DeMarco, D.S. Jin, Onset of Fermi degeneracy in a trapped atomic gas, Science 285 (1999) 1703–1706.
[24] B. DeMarco, S.B. Papp, D.S. Jin, Pauli blocking of collisions in a quantum degenerate atomic Fermi gas, Phys. Rev. Lett. 86 (24) (2001) 5409–5412.
[25] H.B. Dwight, Tables of Integrals and Other Mathematical Data, Macmillan, New York, 1961.
[26] J.W. Goodison, D.J. Toms, The canonical partition function for quons, Phys. Lett. A 195 (1) (1994) 38–42.
[27] A. Inomata, Quon gas with the boson, fermion, and near-classical limits, Phys. Rev. A 52 (2) (1995) 932–935.