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Yield design of thin periodic plates by a homogenization technique and an application to masonry walls

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Abstract

A homogenization method for determining overall yield strength properties of thin periodic plates from their local strength properties is proposed within the framework of the yield design theory. The proposed method is applied to the determination of the in-plane and out of plane strength criterion for masonry described as a regular assemblage of infinitely resistant bricks separated by Coulomb interfaces. ***To cite this article: K. Sab, C. R. Mecanique 331 (2003).***

Résumé

Calcul à la rupture des plaques minces périodiques par une technique d'homogénéisation et une application aux murs en maçonnerie. On propose dans cette Note une technique d'homogénéisation pour le calcul à la rupture des plaques minces périodiques. On applique cette technique à la détermination d'un critère de rupture portant sur les efforts membranaires et les moments fléchissants dans une maçonnerie décrite comme un assemblage régulier de briques infiniment résistantes, séparées par des interfaces de Coulomb. ***Pour citer cet article : K. Sab, C. R. Mecanique 331 (2003).***

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Mots-clés : Mécanique des solides numérique ; Homogénéisation ; Maçonnerie ; Hors plan ; Plaque ; Calcul à la rupture

Version française abrégée

On considère une plaque périodique dans les directions 1 et 2 occupant le domaine $\Omega = \omega \times]-\frac{t}{2}, \frac{t}{2}[$ de $\Re^3$. On note $Y = A \times]-\frac{t}{2}, \frac{t}{2}[$ une cellule de base. On suppose que l'épaisseur t de la plaque est comparable à la taille des hétérogénéités et qu'elle est très petite devant la taille de ω. On propose dans cette Note une technique d'homogénéisation en calcul à la rupture [1] conduisant à un modèle de plaque homogène de Love–Kirchhoff, et on applique cette technique aux murs de maçonnerie.

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En un point $\mathbf{x} = (x_\alpha)$ de ω, on note $\mathbf{V} = (V_i)$ une vitesse virtuelle de la plaque, $\mathbf{D} = (D_{\alpha\beta})$ un tenseur taux de déformation membranaire, $\boldsymbol{\chi} = (\chi_{\alpha\beta})$ un tenseur taux de courbure, $\mathbf{N} = (N_{\alpha\beta})$ un tenseur des efforts membranaires, et $\mathbf{M} = (M_{\alpha\beta})$ un tenseur des moments fléchissants ; avec les indices grecs $\alpha, \beta = 1, 2$ et les indices latins $i, j = 1, 2, 3$. $\mathbf{D}$ et $\boldsymbol{\chi}$ sont reliés à $\mathbf{V}$ par (1).

L'espace $SA(\mathbf{N}, \mathbf{M})$ des champs de contrainte $\boldsymbol{\sigma} = (\sigma_{ij})$ statiquement compatibles sur Y est défini par (2) avec : $\partial Y_3^{\pm} = A \times \{\pm\frac{t}{2}\}$, $\partial Y_l = \partial A \times]-\frac{t}{2}, \frac{t}{2}[$ et $\langle f \rangle$ moyenne volumique de f sur Y. On définit par (3) le convexe de résistance G_p^{hom} de la plaque homogénéisée, $G(\mathbf{y})$ étant le convexe de résistance au point $\mathbf{y}$ de la cellule de base. Sous les conditions mathématiques [2,3], une définition cinématique équivalente est donnée par (7), (8). La fonction d'appui π du convexe $G(\mathbf{y})$ est définie par (6) où $\mathbf{d} = (d_{ij})$ désigne un tenseur taux de déformation. L'espace $KA(\mathbf{D}, \boldsymbol{\chi})$ des champs de vitesse virtuelle $\mathbf{v} = (v_i(\mathbf{y}))$ cinématiquement compatibles sur Y est donné par (4) avec : grad^s la partie symétrique du gradient, $\widetilde{D}_{\alpha\beta} = D_{\alpha\beta}$, $\widetilde{D}_{i3} = 0$, $\widetilde{\chi}_{\alpha\beta} = \chi_{\alpha\beta}$ et $\widetilde{\chi}_{i3} = 0$. Les espaces $KA(\mathbf{D}, \boldsymbol{\chi})$ et $SA(\mathbf{N}, \mathbf{M})$ ont été introduits dans [5] dans le cadre de l'élasticité linéaire. Ils sont en dualité (5) au sens du principe des puissances virtuelles sur Y.

Pour justifier la procédure d'homogénéisation proposée, on traite le cas particulier d'une plaque rectangulaire soumise à un poids spécifique constant γ dans la direction verticale 2, et à un chargement hors plan de la forme $\lambda \times t^2 \times h^{\pm}(\mathbf{x})$ qui s'exerce sur $\omega \times \{\pm\frac{t}{2}\}$; voir Fig. 1. En mettant en œuvre l'approche cinématique du calcul à la rupture, on peut montrer par des arguments similaires à ceux utilisés pour les milieux périodiques [3,4] que la valeur maximale (10) du paramètre de chargement $\lambda \geqslant 0$ calculée sur la plaque homogénéisée est asymptotiquement supérieure ou égale à la valeur maximale (9) calculée sur la plaque hétérogène. La limite dans (11) est obtenue en fixant Y^*, ω, γ et $h^{\pm}$, Y^* étant la cellule de base d'épaisseur unité définie par $Y = tY^*$.

La définition (7), (8) est appliquée à une maçonnerie décrite comme dans [6] comme un assemblage régulier de briques infiniment résistantes, séparées par des interfaces obéissant au critère de Coulomb ; voir Fig. 2. On montre dans ce cas qu'il suffit de se restreindre dans (8) aux vitesses virtuelles qui sont discontinues aux interfaces J de Y, et dont la restriction sur chaque bloc de Y est un champ de vitesse de solide rigide. Pour ces champs, on trouve dans [7] l'expression du saut de vitesse $[\![\mathbf{v}]\!]$ sur J dans le sens de la normale $\mathbf{n}$, en fonction de $\mathbf{D}$, $\boldsymbol{\chi}$ et de 3 constantes indétérminées. De plus, en utilisant la fonction π correspondant au critère de Coulomb [1], on obtient (13), $0 \leqslant c$ étant la cohésion et $0 < \phi < \pi/2$ l'angle de frottement. En effectuant la minimisation dans (8), on trouve que le domaine macroscopique est l'ensemble des $(\mathbf{N}, \mathbf{M})$ qui vérifient (14) pour tout $\mathbf{D}$, $\boldsymbol{\chi}$ et tout réel C qui satisfont (15)–(17) avec $\varepsilon_1 = \pm 1$, $\varepsilon_2 = \pm 1$, $\varepsilon = \pm 1$ et $m = 2a/b$. En se limitant à $\chi_{12} = 0$, on montre que la projection de G_p^{hom} sur le sous-espace $M_{12} = 0$ est donnée par (18), (19) pour $\phi < \pi/4$. On retrouve alors les résultats (18) de l'étude [6] consacrée aux seuls efforts membranaires. La Fig. 3 représente les nouvelles conditions (19) sur les moments fléchissants dans le cas particulier $N_{11}^* = N_{12}^* = 0$. On constate ici le caractère anisotrope du critère de rupture déjà mis en évidence dans [6].

Enfin, dans le cas isostatique d'un mur infini dans la direction 1 avec $h^{\pm}(\mathbf{x}) = h^{\pm}(x_2)$, on obtient (20). On remarque que $\lambda_p^{\max}$ ne dépend pas de ϕ pour $c = 0$. En effet, l'effort tranchant n'est pas pris en compte dans une modélisation de type Love–Kirchhoff.

1. The yield design homogenization method for thin and periodic plates

The heterogeneous plate under consideration occupies a domain $\Omega = \omega \times]-\frac{t}{2}, \frac{t}{2}[$ where $\omega \subset \Re^2$ is the middle surface of the plate and t is its thickness. The plate exhibits a periodic structure in directions 1 and 2 so that it is possible to extract a unit cell, denoted by $Y = A \times]-\frac{t}{2}, \frac{t}{2}[$, which contains all information necessary to completely describe the plate. It is assumed that t is of the same order as the typical size of A and it is very small in comparison with the typical size of ω.

The present work is performed within the framework of the yield design theory [1]. Its purpose is to provide a homogenization method which leads to a homogeneous Love–Kirchhoff plate model, and to implement this method on masonry walls.

The following notations are used: Greek index $\alpha, \beta = 1, 2$, Latin index $i, j = 1, 2, 3$, $\mathbf{N} = (N_{\alpha\beta})$ is the macroscopic in-plane (membranal) stress field of the homogenized plate, $\mathbf{M} = (M_{\alpha\beta})$ is the macroscopic out of plane (bending) stress field, $\mathbf{V} = (V_i)$ is a virtual velocity field, $\mathbf{D} = (D_{\alpha\beta})$ is the corresponding in-plane strain rate field, and $\boldsymbol{\chi} = (\chi_{\alpha\beta})$ is the corresponding out of plane strain rate field (curvature rate). $\mathbf{N}$, $\mathbf{M}$, $\mathbf{V}$, $\mathbf{D}$, and $\boldsymbol{\chi}$ are defined at every point $\mathbf{x} = (x_\alpha)$ of ω, and

$$D_{\alpha\beta} = \frac{1}{2}(V_{\alpha,\beta} + V_{\beta,\alpha}), \qquad \chi_{\alpha\beta} = -V_{3,\alpha\beta} \tag{1}$$

For every $(\mathbf{N}, \mathbf{M})$, the set of statically compatible 3D stress fields $\boldsymbol{\sigma} = (\sigma_{ij})$ of the unit cell is defined by:

$$\begin{aligned} SA(\mathbf{N}, \mathbf{M}) = \bigl\{\boldsymbol{\sigma} \mid N_{\alpha\beta} = t\langle\sigma_{\alpha\beta}\rangle,\ M_{\alpha\beta} = t\langle y_3\sigma_{\alpha\beta}\rangle,& \\ \operatorname{div}\boldsymbol{\sigma} = 0 \text{ on } Y,\ \boldsymbol{\sigma}\cdot\mathbf{n} \text{ anti-periodic on } \partial Y_l,\ \boldsymbol{\sigma}\cdot\mathbf{e}_3 = 0 \text{ on } \partial Y_3^{\pm}\bigr\}& \end{aligned} \tag{2}$$

where $\partial Y_3^{\pm} = A \times \{\pm\frac{t}{2}\}$ is the upper (lower) boundary of Y, $\partial Y_l = \partial A \times]-\frac{t}{2}, \frac{t}{2}[$ is the lateral boundary of Y and $\langle f\rangle$ is the volume average of f on Y.

The convex domain $G(\mathbf{y})$ characterizing the strength capacities of the constituent material at every point $\mathbf{y}$ of Y is introduced. The strength domain of the homogenized plate is defined as:

$$G_p^{\text{hom}} = \bigl\{(\mathbf{N}, \mathbf{M}) \mid \exists\boldsymbol{\sigma} \in SA(\mathbf{N}, \mathbf{M}),\ \boldsymbol{\sigma}(\mathbf{y}) \in G(\mathbf{y}),\ \forall\mathbf{y} \in Y\bigr\} \tag{3}$$

A kinematic definition of G_p^{hom} can be obtained through the dualization of (3). For every $(\mathbf{D}, \boldsymbol{\chi})$, the set $KA(\mathbf{D}, \boldsymbol{\chi})$ of the velocity fields of the unit cell, $\mathbf{v} = (v_i(\mathbf{y}))$, which are kinematically compatible is:

$$KA(\mathbf{D}, \boldsymbol{\chi}) = \bigl\{\mathbf{v} \mid \operatorname{grad}^s(\mathbf{v}) = \widetilde{\mathbf{D}} + y_3\widetilde{\boldsymbol{\chi}} + \operatorname{grad}^s\bigl(\mathbf{u}^{\text{per}}\bigr),\ \mathbf{u}^{\text{per}}\ A\text{-periodic}\bigr\} \tag{4}$$

where grad^s is the symmetric part of the gradient operator, $\widetilde{D}_{\alpha\beta} = D_{\alpha\beta}$, $\widetilde{D}_{i3} = 0$, $\widetilde{\chi}_{\alpha\beta} = \chi_{\alpha\beta}$ and $\widetilde{\chi}_{i3} = 0$.

Actually, $KA(\mathbf{D}, \boldsymbol{\chi})$ and $SA(\mathbf{N}, \mathbf{M})$ are dual sets in the sense of the principle of virtual work on the unit cell:

$$\forall\boldsymbol{\sigma} \in SA(\mathbf{N}, \mathbf{M}),\ \forall\mathbf{v} \in KA(\mathbf{D}, \boldsymbol{\chi}), \quad \mathbf{N} : \mathbf{D} + \mathbf{M} : \boldsymbol{\chi} = t\bigl\langle\boldsymbol{\sigma} : \operatorname{grad}^s(\mathbf{v})\bigr\rangle \tag{5}$$

The support function $\pi(\mathbf{d})$ of $G(\mathbf{y})$ is defined by:

$$\pi(\mathbf{d}) = \sup\bigl\{\boldsymbol{\sigma} : \mathbf{d};\ \boldsymbol{\sigma} \in G(\mathbf{y})\bigr\}\ G(\mathbf{y}) = \bigl\{\boldsymbol{\sigma} \mid \boldsymbol{\sigma} : \mathbf{d} \leqslant \pi(\mathbf{d}),\ \forall\mathbf{d}\bigr\} \tag{6}$$

where $\mathbf{d} = (d_{ij})$ denotes a strain rate second order tensor. Using (3), (5) and (6), it can be shown under the assumption of uniformly bounded strength domains $G(\mathbf{y})$, [2,3], that G_p^{hom} is equivalently defined by:

$$G_p^{\text{hom}} = \bigl\{(\mathbf{N}, \mathbf{M}) \mid \mathbf{N} : \mathbf{D} + \mathbf{M} : \boldsymbol{\chi} \leqslant \pi_p^{\text{hom}}(\mathbf{D}, \boldsymbol{\chi}),\ \forall(\mathbf{D}, \boldsymbol{\chi})\bigr\} \tag{7}$$

with

$$\pi_p^{\text{hom}}(\mathbf{D}, \boldsymbol{\chi}) = \inf\bigl\{t\bigl\langle\pi\bigl(\operatorname{grad}^s(\mathbf{v})\bigr)\bigr\rangle;\ \mathbf{v} \in KA(\mathbf{D}, \boldsymbol{\chi})\bigr\} \tag{8}$$

2. Justification of the proposed homogenization method

An upper-bound estimate for the ultimate failure of the heterogeneous plate can be asymptotically obtained through the yield design approach implemented on the homogenized plate. For the sake of simplicity, this general statement is specified in the following case: a rectangular plate with thickness t is submitted to a constant vertical specific weight γ in direction 2, and to out of plane distributed forces $\lambda \times t^2 \times h^{\pm}(\mathbf{x})$ on $\omega \times \{\pm\frac{t}{2}\}$ (Fig. 1).

The positive parameter λ is gradually increased from 0. The critical value of λ for which the ultimate failure occurs can be estimated through the yield design kinematic approach implemented either on the heterogeneous plate:

$$\lambda^{\max} = \inf_{\substack{\{w \mid w_i(x_1, 0, z) = 0,\\ t\int_\omega h^+(x)w_3(x, +\frac{t}{2}) + h^-(x)w_3(x, -\frac{t}{2})\,\mathrm{d}\omega = 1\}}} t^{-1}\int_\Omega \pi\bigl(\operatorname{grad}^s(\mathbf{w})\bigr) + \gamma w_2(\mathbf{x}, z)\,\mathrm{d}\omega\,\mathrm{d}z \tag{9}$$

or, on the homogenized plate:

$$\lambda_p^{\max} = \inf_{\substack{\{V \mid V_i(x_1,0)=0,\ V_{3,2}(x_1,0)=0, \\ t\int_\omega (h^+(x)+h^-(x))V_3(x)\,\mathrm{d}\omega = 1\}}} t^{-1}\int_\omega \pi_p^{\mathrm{hom}}(\mathbf{D},\boldsymbol{\chi})\,\mathrm{d}\omega + \gamma \int_\omega V_2(\mathbf{x})\,\mathrm{d}\omega \tag{10}$$

Let Y^* denote the cell with unit thickness defined by $Y = tY^*$. Actually, for fixed Y^*, ω, γ and $h^\pm$, the critical value $\lambda_p^{\max}$ does not depend on t; so, it can be calculated on a homogenized plate with unit thickness. On the other hand, $\lambda^{\max}$ depends on t and it can be proved under the assumption of uniformly bounded strength domains $G(\mathbf{y})$ that:

$$\lim_{t\to 0} \lambda^{\max} \leqslant \lambda_p^{\max}. \tag{11}$$

The proof is formally based on the use of the following virtual velocity field in (9):

$$\mathbf{w}(\mathbf{x},z) = \begin{pmatrix} V_1^*(\mathbf{x}) - t^{-1}zV_{3,1}^*(\mathbf{x}) + tu_1^{*\mathrm{per}}(\mathbf{D}^*(\mathbf{x}),\boldsymbol{\chi}^*(\mathbf{x});\mathbf{y}^*) \\ V_2^*(\mathbf{x}) - t^{-1}zV_{3,2}^*(\mathbf{x}) + tu_2^{*\mathrm{per}}(\mathbf{D}^*(\mathbf{x}),\boldsymbol{\chi}^*(\mathbf{x});\mathbf{y}^*) \\ t^{-1}V_3^*(\mathbf{x}) + tu_3^{*\mathrm{per}}(\mathbf{D}^*(\mathbf{x}),\boldsymbol{\chi}^*(\mathbf{x});\mathbf{y}^*) \end{pmatrix}, \quad \mathbf{y}^* = t^{-1}(\mathbf{x},z) \tag{12}$$

Here, $\mathbf{u}^{*\mathrm{per}}(\mathbf{D}^*,\boldsymbol{\chi}^*;\mathbf{y}^*)$ is a solution of the minimization problem (8) associated to Y^* and $\mathbf{V}^*$ is a virtual velocity field of the homogenized plate with unit thickness. For more details, see the similar proofs by Bouchitté [3] and de Buhan [4] for periodic media.

Actually, stress boundary conditions on $\omega \times \{\pm\frac{t}{2}\}$ are not exactly fulfilled in the homogenized plate model and transverse shear effects are not taken into account. Therefore, inequality (11) is most likely to be strict in the general case. An illustrative example is given in Section 3. See also the example by de Buhan ([4], pp. 296–302) for periodic media.

Note that compatibility conditions (2) and (4) used in the definition of the strength domain of the homogenized plate (3) and (7), (8) have been initially introduced by Caillerie [5] within the framework of linear elasticity.

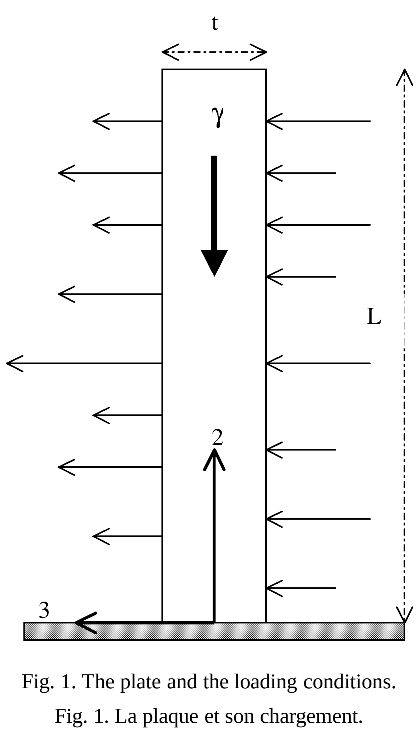

Fig. 1. The plate and the loading conditions.

Fig. 1. La plaque et son chargement.

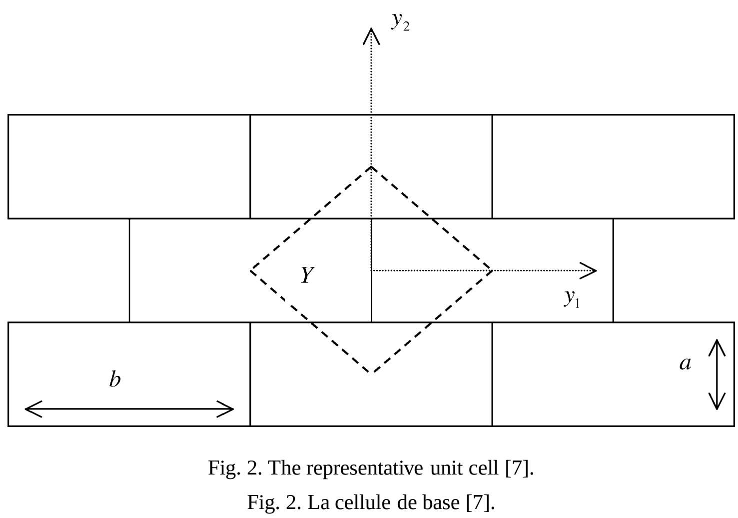

Fig. 2. The representative unit cell [7].

Fig. 2. La cellule de base [7].

3. Application to block masonry

The structure under consideration is a panel made of identical parallelepipedic bricks separated by horizontal continuous bed joints and alternate vertical head joints. Fig. 2 represents the corresponding diamond-shape unit cell associated with the periodicity of the structure in the (1,2) plane [6]. The brick material is assumed to be infinitely resistant and the joints are modelled as Coulomb interfaces. The result of this is to limit the minimization in (8) to those virtual velocity fields $\mathbf{v}$ in $KA(\mathbf{D}, \boldsymbol{\chi})$ whose restriction to each of the four blocks making up the unit cell is the blocks own rigid body motion. Making use of the π-function corresponding to the Coulomb failure condition [1], the maximum resisting work in (8) is:

$$t\langle\pi\left(\mathrm{grad}^s(\mathbf{v})\right)\rangle = \begin{cases} \frac{1}{ab}\int_J (c/\tan\phi)[\![\mathbf{v}]\!]\cdot\mathbf{n}\,\mathrm{d}J & \text{if } [\![\mathbf{v}]\!]\cdot\mathbf{n} \geqslant \left|[\![\mathbf{v}]\!]\right|\sin\phi \\ +\infty & \text{otherwise} \end{cases} \tag{13}$$

Here, $[\![\mathbf{v}]\!]$ denotes the velocity jump across the joint interface J when following its normal $\mathbf{n}$, $0 \leqslant c$ denotes the cohesion and $0 < \phi < \pi/2$ the friction angle.

As shown in [7], $[\![\mathbf{v}]\!]$ is a piecewise-linear field on J which can be expressed in terms of $\mathbf{D}$, $\boldsymbol{\chi}$ and 3 undetermined constants. With this expression and (13), it can be proved that G_p^{hom} is the anisotropic domain made of all the stress states such that

$$\mathbf{N}:\mathbf{D} + \mathbf{M}:\boldsymbol{\chi} \leqslant t\,(c/\tan\phi)\,\mathrm{tr}(\mathbf{D}) \tag{14}$$

for any $\mathbf{D}$, $\boldsymbol{\chi}$ and any real C satisfying condition (15) with $\varepsilon_1 = \pm 1$, $\varepsilon_2 = \pm 1$ (4 conditions) and condition (16) with $\varepsilon = \pm 1$ (2 conditions),

$$\tan\phi\sqrt{(a\chi_{12})^2 + \left(D_{11}^{\varepsilon_1} + m\left(D_{12}^{\varepsilon_1} + C\right)\varepsilon_2\right)^2} \leqslant \left(D_{12}^{\varepsilon_1} - C\right)\varepsilon_2 + m D_{22}^{\varepsilon_1} \tag{15}$$

$$\tan\phi\sqrt{\left(\frac{a}{2}\chi_{12}\right)^2 + \left(D_{12}^{\varepsilon} - C\right)^2} \leqslant D_{11}^{\varepsilon} \tag{16}$$

where $m = 2a/b$ and

$$D_{\alpha\beta}^{\varepsilon} = D_{\alpha\beta} + \varepsilon\frac{t}{2}\chi_{\alpha\beta} \tag{17}$$

Restricting (14)–(17) to the particular case $\chi_{12} = 0$, it can be shown that, for $\phi < \pi/4$, the set of $(N_{11}, N_{12}, N_{22}, M_{11}, M_{22})$ for which there exists M_{12} such that $(\mathbf{N}, \mathbf{M}) \in G_p^{\mathrm{hom}}$ is given by conditions (18), (19).

$$\begin{cases} N_{\alpha\beta}^* = N_{\alpha\beta} - t\,(c/\tan\phi)\delta_{\alpha\beta} \\ |N_{12}^*| \leqslant -\tan\phi N_{22}^* \\ (1 + m\tan\phi)|N_{12}^*| \leqslant -mN_{11}^* - \tan\phi N_{22}^* \\ (m + \tan\phi)|N_{12}^*| \leqslant -m\tan\phi N_{11}^* - N_{22}^* \quad \text{for } m\tan\phi > 1 \end{cases} \tag{18}$$

$$\begin{cases} M_{\alpha\beta}^* = \frac{2}{t}M_{\alpha\beta} \\ |M_{22}^*| \leqslant -N_{22}^* \\ |mM_{11}^* + \tan\phi M_{22}^*| \leqslant -mN_{11}^* - \tan\phi N_{22}^* \\ |mM_{11}^* - \tan\phi M_{22}^*| \leqslant -pmN_{11}^* - 2q(1 + m\tan\phi)|N_{12}^*| - r\tan\phi N_{22}^* \\ q = 1/\left(1 - (\tan\phi)^2\right), \quad p = \left(1 + (\tan\phi)^2\right)q, \quad r = \left(3 - (\tan\phi)^2\right)q \end{cases} \tag{19}$$

The above results correspond with those obtained in the particular case of in-plane loadings. Indeed, the projection of G_p^{hom} on the sub-space $\mathbf{M} = \mathbf{0}$ given by (18) coincides with the anisotropic macroscopic strength domain in the space of plane stresses [6]. Fig. 3 represents the new conditions (19) in the special case $N_{11}^* = N_{12}^* = 0$.

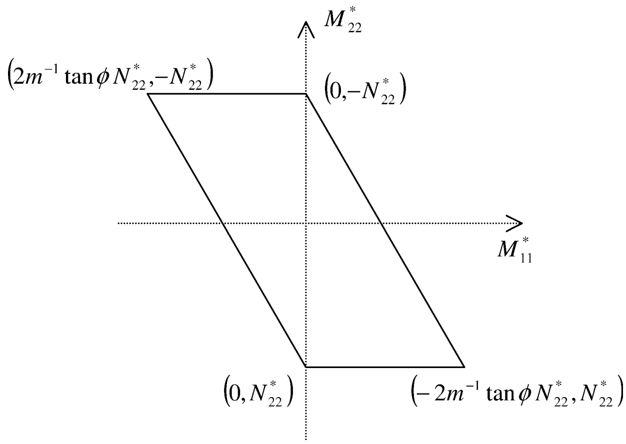

Fig. 3. The domain (19) in the special case $N_{12}^* = N_{11}^* = 0$.
Fig. 3. Le domaine (19) dans le cas particulier $N_{12}^* = N_{11}^* = 0$.

In the simple case of an infinite wall in direction 1 with $h^{\pm}(\mathbf{x}) = h^{\pm}(x_2)$, it can be proved that the critical load (10) is given by:

$$\lambda_p^{\max} = \inf_{0 \leqslant x_2 \leqslant L} \frac{\gamma(L - x_2) + (c/\tan\phi)}{2|\int_{x_2}^{L}(h^+(\xi) + h^-(\xi))(L - \xi)\,\mathrm{d}\xi|} \tag{20}$$

For $c = 0$, $\lambda_p^{\max}$ does not depend on ϕ because transverse shear effects are not taken into account in the homogenized model. On the other hand, it is clear that due to these effects $\lambda^{\max}$ decreases to zero as ϕ decreases to zero; in this case, inequality (11) is strict.

It has been shown in this Note how the procedure for determining yield strength properties of periodic composite media from the local strength properties of their components [3,4,8] can be extended to the homogenization of thin periodic plates. The proposed homogenization procedure, which leads asymptotically to an upper-bound estimate for the ultimate failure of the heterogeneous plate, amounts to solve a yield design boundary value problem on the unit cell. Here, the boundary conditions on the unit cell are those introduced in [5] within the framework of linear elasticity. The procedure has been applied to the determination of the anisotropic out of plane strength criterion for masonry material and a simple example has been provided.

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