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Viscoplastic behaviour of steels during phase transformations

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Abstract

That contribution to transformation plasticity of steels arising from the so-called Greenwood–Johnson mechanism is often described using the model developed by Leblond and coworkers. This model made the assumption of purely plastic behaviour. It is extended here to incorporate viscous effects, which are present during some transformations, especially at high temperatures. The predictions of the original and extended models are compared to experimental results for a material for which the second contribution to transformation plasticity, due to the so-called Magee mechanism, is known to be negligible, and it is shown that the incorporation of viscous effects into the model significantly improves its predictions. *To cite this article: Y. Vincent et al., C. R. Mécanique 331 (2003).*

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Résumé

Comportement viscoplastique des aciers en cours de transformation de phase. La contribution à la plasticité de transformation des aciers due au mécanisme dit de Greenwood–Johnson est souvent décrite à l'aide du modèle de Leblond et coll. Ce modèle faisait l'hypothèse d'un comportement purement plastique. On l'étend ici en incorporant les effets visqueux, présents lors de certaines transformations, particulièrement à haute température. Les prédictions des modèles original et étendu sont comparées à des résultats expérimentaux pour un matériau pour lequel la seconde contribution à la plasticité de transformation, due au mécanisme dit de Magee, est connue pour être négligeable, et l'on montre que l'incorporation des effets visqueux dans le modèle en améliore significativement les prédictions. *Pour citer cet article : Y. Vincent et al., C. R. Mécanique 331 (2003).*

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1. Introduction

Numerical simulations of welding of steel structures require adequate modelling of ‘transformation plasticity’. This expression refers to the anomalous plastic flow of metals which can be observed during solid–solid phase transformations, and especially the $\gamma \rightarrow \alpha$ transformation of steels during cooling. It is known to be due to two distinct mechanisms, one proposed by Greenwood and Johnson [1], and the other by Magee [2]. In the Greenwood–Johnson interpretation, transformation plasticity arises from microplasticity in the weaker austenitic phase induced by the difference of specific volume between the phases. In the Magee interpretation (essentially for martensitic transformations), it is due to ‘orientation’ of the newly formed martensite plates by the external load.

The respective importance of these mechanisms depends on the material and the transformation. The Greenwood–Johnson mechanism is important in diffusional transformations such as the ferritic–pearlitic transformation, but also in bainitic and martensitic transformations when the difference of specific volume between the phases is large. It is absent in shape memory alloys in which this volume difference is zero. The Magee mechanism was specifically proposed to explain transformation plasticity during martensitic transformations, but it can also occur during bainitic transformations. It is absent in diffusional transformations.

The Greenwood–Johnson mechanism has been studied in great detail by Leblond and coworkers (Leblond et al. [3–6]). The completely explicit formula obtained for the ‘transformation plastic strain’ has been experimentally assessed for materials and transformations for which the Magee mechanism is known to be negligible; see for instance the works of Desalos [7], Cavallo [8], Taleb [9], Grostabussiat [10], Coret [11] and Vincent [12]. It is often used to account for transformation plasticity in numerical simulations of welding processes.

Although the material law used by Leblond and coworkers to describe the behaviour of individual phases incorporated such refinements as the possibility of strain hardening of isotropic, kinematic or mixed (isotropic + kinematic) type, it did not account for viscous effects. The reason was that it was generally thought up to now that such effects are of little importance in welding simulations because of the relatively short duration of welding processes. However it was recently found by Bru et al. [13] and Leblond et al. [14] that residual *distortions* are sensitive to tiny details of the material behaviour which have little influence on residual *stresses*, such as values of the yield stress at very high temperatures, just below the fusion point. Therefore viscous effects, which are bound to be present during transformations occurring at high temperatures, may significantly influence residual distortions. This makes it desirable to extend Leblond and coworkers’ treatment of transformation plasticity so as to incorporate viscous effects. It is precisely the aim of the present paper to present such an extension.

Just as in previous works of Leblond and coworkers, we shall only consider that contribution to transformation plasticity arising from the sole Greenwood–Johnson mechanism. This does not mean in any way that we claim that the contribution of the Magee mechanism is insignificant. Our reasons for not considering it can be summarized as follows. Greenwood–Johnson’s mechanism is of purely mechanical nature and therefore liable to some theoretical treatment leading to some fully explicit formula for the ‘transformation plastic strain rate’, avoiding the introduction of any ad hoc, adjustable parameters. In contrast, Magee’s mechanism involves complex interactions of metallurgical and mechanical phenomena. As a result, theoretical treatments incorporating this mechanism (see for instance the recent, very refined work of Cherkaoui et al. [15]) unavoidably encounter difficulties which can only be resolved by adopting some more heuristic approach at some stage, and introducing one or more adjustable parameter(s). Now introducing heuristic, adjustable parameters is precisely what we want to avoid, in line with previous works of Leblond and coworkers. This is why we shall focus our attention solely on the Greenwood–Johnson mechanism, even though this unavoidably means that the results obtained will be directly applicable only to materials and transformations for which Magee’s mechanism is negligible.

It should be noted, however, that the theory developed disregarding Magee’s mechanism will be fully compatible with other ones incorporating it, or experiments evidencing its presence; indeed it will always be possible to add to the expression derived here for the contribution of the Greenwood–Johnson mechanism, that derived within some other theoretical framework or directly from experiments, for the contribution of the Magee mechanism. However, in order to avoid the introduction of adjustable parameters, the validity of the theoretical formula derived below

will be experimentally assessed using some material for which several experimental studies have shown that the Magee mechanism is negligible.

2. Theoretical evaluation of the Greenwood–Johnson mechanism including viscous effects

The theoretical treatment is based on homogenization, using some representative volume element (RVE) V sufficiently large to contain both phases, and within which the α phase is developing at the expense of the γ phase. It uses several approximations, most of which were already present in Leblond et al.’s [5,6] treatment disregarding viscous effects. There are only 2 new, related approximations \mathcal{A}_1 , \mathcal{A}_2 aimed at incorporating such effects:

\mathcal{A}_1 : Viscous effects can be accounted for in an approximate way by simply considering austenite as purely plastic with a strain-rate-dependent yield stress.

Taking strain hardening to be of isotropic type for simplicity,¹ we thus assume that the yield stress of the austenitic phase is given, at the microscopic scale, by

$$\sigma_\gamma^Y \equiv k_\gamma(\varepsilon_\gamma^{\text{eq}}, T) + K_\gamma(T)(\varepsilon_\gamma^{\text{eq}})^{M_\gamma(T)} (\dot{\varepsilon}_\gamma^{\text{eq}})^{N_\gamma(T)} \tag{1}$$

where T denotes the temperature, $\dot{\varepsilon}_\gamma^{\text{eq}}$ the microscopic equivalent plastic strain rate, $\varepsilon_\gamma^{\text{eq}}$ the corresponding cumulated plastic strain, $k_\gamma(\varepsilon_\gamma^{\text{eq}}, T)$ the yield stress for a zero strain rate and $K_\gamma(T)$, $M_\gamma(T)$, $N_\gamma(T)$ material parameters. All these parameters can be experimentally determined through tensile tests at various strain rates and creep tests.

\mathcal{A}_2 : The microscopic plastic strain rate $\dot{\varepsilon}_\gamma^p$ in the austenitic phase can be considered as arising only from variations of the macroscopic stress Σ , the temperature T and the proportion z_α of the α phase:²

$$(\dot{\varepsilon}_\gamma^p)_{ij} \equiv \frac{\delta(\varepsilon_\gamma^p)_{ij}}{\delta \Sigma_{kl}} \dot{\Sigma}_{kl} + \frac{\delta(\varepsilon_\gamma^p)_{ij}}{\delta T} \dot{T} + \frac{\delta(\varepsilon_\gamma^p)_{ij}}{\delta z_\alpha} \dot{z}_\alpha \tag{2}$$

Clearly, this property holds rigorously true for a purely plastic behaviour since plastic flow then takes place only if Σ , T and/or z_α vary. It is only an approximation in the presence of viscous effects since creep then occurs even if all 3 parameters are held constant. This approximation is reasonable since both the temperature and the proportions of the phases do vary rather quickly during transformations induced by welding of steels.

Although the terms proportional to $\dot{\Sigma}_{kl}$ and \dot{T} can be calculated explicitly, just as in the case of some purely plastic behaviour (see [5,6]), their effect is much less important than that of the term proportional to \dot{z}_α . We shall therefore only present the calculation of the quantity

$$\dot{E}_{ij}^{\text{tp}} \equiv \left\langle \frac{\delta \varepsilon_{ij}^p}{\delta z_\alpha} \right\rangle_V \dot{z}_\alpha \equiv (1 - z_\alpha) \left\langle \frac{\delta(\varepsilon_\gamma^p)_{ij}}{\delta z_\alpha} \right\rangle_{V_\gamma} \dot{z}_\alpha \tag{3}$$

where V_γ denotes the subvolume of the RVE V occupied by the γ phase; \dot{E}^{tp} represents, by definition, the (macroscopic) ‘transformation plastic strain rate’. Use has been made in this equation of the fact that (visco)plastic flow remains essentially localized in the γ phase, since the α phase is much harder.

¹ The theory can also be developed for kinematic or mixed (isotropic + kinematic) hardening, at the expense of greater complexity.

² Increments are denoted here with the unusual symbol δ instead of ∂ to emphasize the fact that the quantities $\delta(\cdot)/\delta(\cdot)$ are *not* true partial derivatives, because ε_γ^p depends on local internal parameters in addition to Σ , T and z_α . Eq. (2) just expresses the hypothesis that the increment of ε_γ^p is a linear function of the increments of Σ , T and z_α .

Extra approximations analogous to those made in the case of a purely plastic behaviour (see [5,6]) are now introduced:

- \mathcal{A}_3 : The local yield stress of austenite σ_γ^Y in the microscopic flow rule may be replaced by some mean value $\overline{\sigma_\gamma^Y}$ (to be defined precisely later);
- \mathcal{A}_4 : Spatial correlations between the quantity $\delta\varepsilon_\gamma^{\text{eq}}/\delta z_\alpha$ and the components $(s_\gamma)_{ij}$ of the microscopic stress deviator in the austenitic phase are negligible;
- \mathcal{A}_5 : The average stress deviator in the austenitic phase $\mathbf{S}_\gamma \equiv \langle s_\gamma \rangle_{V_\gamma}$ may be equated to the overall stress deviator $\mathbf{S} \equiv \langle s \rangle_V$.

Clearly, this last approximation is acceptable only during, say, the first half of the transformation, when the volume V_γ occupied by the γ phase fills most of the RVE V ; but this is unimportant because the Greenwood–Johnson mechanism, which involves microplasticity precisely in this phase, becomes ineffective when its volume decreases.

Equation (3) then yields

$$\begin{aligned} \dot{E}_{ij}^{\text{tp}} &= \frac{3}{2} (1 - z_\alpha) \left\langle \frac{\delta\varepsilon_\gamma^{\text{eq}}}{\delta z_\alpha} \frac{(s_\gamma)_{ij}}{\sigma_\gamma^Y} \right\rangle_{V_\gamma} \dot{z}_\alpha \approx \frac{3}{2} \frac{1 - z_\alpha}{\overline{\sigma_\gamma^Y}} \left\langle \frac{\delta\varepsilon_\gamma^{\text{eq}}}{\delta z_\alpha} (s_\gamma)_{ij} \right\rangle_{V_\gamma} \dot{z}_\alpha \\ &\approx \frac{3}{2} \frac{1 - z_\alpha}{\overline{\sigma_\gamma^Y}} \left\langle \frac{\delta\varepsilon_\gamma^{\text{eq}}}{\delta z_\alpha} \right\rangle_{V_\gamma} \langle (s_\gamma)_{ij} \rangle_{V_\gamma} \dot{z}_\alpha \approx \frac{3}{2} \frac{1 - z_\alpha}{\overline{\sigma_\gamma^Y}} \left\langle \frac{\delta\varepsilon_\gamma^{\text{eq}}}{\delta z_\alpha} \right\rangle_{V_\gamma} S_{ij} \dot{z}_\alpha \end{aligned} \quad (4)$$

The first equality here is a consequence of the Prandtl–Reuss flow rule, the second one of approximation \mathcal{A}_3 , the third one of approximation \mathcal{A}_4 and the fourth one of approximation \mathcal{A}_5 .

The quantity $\langle \delta\varepsilon_\gamma^{\text{eq}}/\delta z_\alpha \rangle_{V_\gamma}$ must now be evaluated. This is done by considering a simple model geometry, namely an austenitic sphere in which a spherical core of α phase is growing. The calculation is performed at the lowest order with respect to the overall stress tensor Σ , that is for $\Sigma = \mathbf{0}$; this means that no external load is applied, so that spherical symmetry is preserved. In the initial configuration, at time $t = 0$, the sphere, of radius b , is entirely austenitic. At time t , a spherical core of radius $a(t)$ in the reference configuration has been transformed into α phase, so that the volume fraction of this phase is $z_\alpha = a^3(t)/b^3$. Because the $\gamma \rightarrow \alpha$ transformation induces an increase of specific volume, the actual volume of this spherical core at time t is in fact (slightly) larger than its value of $\frac{4}{3}\pi a^3(t)$ in the reference configuration; this generates plastic deformations in the external austenitic crust, which must be evaluated.

The calculation is made much easier by neglecting elasticity (although this is by no means compulsory). Since plasticity is incompressible, the only source of volume change is then the $\gamma \rightarrow \alpha$ transformation. Thus the actual volume of the spherical core of α phase at time t is $\frac{4}{3}\pi a^3(t)(1 + \Delta V/V)$ where $\Delta V/V$ denotes the change of specific volume induced by the transformation (which is a function of temperature). This implies that the radial displacement $u(a(t), t)$ at $r = a(t)$ and time t is $(\Delta V/V)a(t)/3$. Because of incompressibility in the external austenitic crust, the radial displacement $u(r, t)$ in this crust at the same instant is $(\Delta V/V)a^3(t)/3r^2$, so that the variation of this displacement between instants t and $t + \delta t$, corresponding to positions $a(t)$ and $a(t + \delta t) \equiv a(t) + \delta a$ of the transformation front in the reference configuration, is $(\Delta V/V)a^2(t)\delta a/r^2$. The corresponding increment of equivalent plastic strain is $2(\Delta V/V)a^2(t)\delta a/r^3$, and the average value of this increment in the austenitic crust, divided by the increment $\delta z_\alpha \equiv 3a^2(t)\delta a/b^3$ of the proportion of α phase, is given by

$$\left\langle \frac{\delta\varepsilon_\gamma^{\text{eq}}}{\delta z_\alpha} \right\rangle_{V_\gamma} = \frac{1}{\frac{4}{3}\pi(b^3 - a^3(t))} \int_{a(t)}^b \frac{2(\Delta V/V)a^2(t)\delta a/r^3}{3a^2(t)\delta a/b^3} 4\pi r^2 dr = -\frac{2}{3} \frac{\Delta V}{V} \frac{\ln z_\alpha}{1 - z_\alpha} \quad (5)$$

Combination of Eqs. (4) and (5) yields the final expression of the transformation plastic strain rate:

$$\dot{\varepsilon}_{ij}^{\text{tp}} \approx -\frac{\Delta V/V}{\overline{\sigma}_\gamma} S_{ij}(\ln z_\alpha) \dot{z}_\alpha \quad (6)$$

It remains to give a precise definition for the ‘mean yield stress’ of austenite $\overline{\sigma}_\gamma$. This ‘mean yield stress’ is taken as that value provided by Eq. (1), the local equivalent strain rate $\dot{\varepsilon}_\gamma^{\text{eq}}$ and cumulated strain $\varepsilon_\gamma^{\text{eq}}$ being replaced by their macroscopic counterparts $\dot{E}_\gamma^{\text{eq}}$, E_γ^{eq} :

$$\overline{\sigma}_\gamma \equiv k_\gamma(E_\gamma^{\text{eq}}, T) + K_\gamma(T)(E_\gamma^{\text{eq}})^{M_\gamma(T)}(\dot{E}_\gamma^{\text{eq}})^{N_\gamma(T)} \quad (7)$$

$$\dot{E}_\gamma^{\text{eq}} \equiv \langle \dot{\varepsilon}_\gamma^{\text{eq}} \rangle_{V_\gamma} \equiv \left\langle \frac{\delta \varepsilon_\gamma^{\text{eq}}}{\delta z_\alpha} \right\rangle_{V_\gamma} \dot{z}_\alpha, \quad E_\gamma^{\text{eq}} \equiv \int_0^t \dot{E}_\gamma^{\text{eq}}(\tau) d\tau \quad (8)$$

where $\langle \delta \varepsilon_\gamma^{\text{eq}} / \delta z_\alpha \rangle_{V_\gamma}$ is recalled to be given by Eq. (5).

Eqs. (5)–(8) fully define the model developed.

3. Comparison with experimental results for a bar with clamped ends undergoing thermal cycles

The validity of the theory developed will now be assessed by comparing its predictions to experimental results (stress versus temperature) for a bar with clamped ends undergoing thermal cycles. Such an experiment is called a ‘Satoh test’ with reference to the pioneering work of this author (Satoh [16]), who invented it in order to develop qualitative insight into the development of residual stresses during welding. The material used will be 16MND5 steel (American standard A508 cl. 3), for which several earlier studies (Desalos [7], Cavallo [8], Taleb [9], Grostabusiat [10], Coret [11] and Vincent [12]) have shown that the Magee mechanism is negligible, even during the martensitic transformation. Such a conclusion was reached notably through direct inspection of the microstructure of the newly formed phase, and also through observation that repeated thermal cycles result in perfectly cumulative ‘transformation plastic strains’.³

Prior to performing such experiments, free dilatometry tests were performed in order to determine the transformation kinetics and the change of specific volume induced by the transformations; tensile tests at various strain rates and creep tests were also conducted to determine the material parameters $k_\gamma(\varepsilon_\gamma^{\text{eq}}, T)$, $K_\gamma(T)$, $M_\gamma(T)$, $N_\gamma(T)$. Knowledge of all these parameters is indeed necessary for the sake of comparison of theory and experiment.

Each Satoh test involved 3 thermal cycles with decreasing maximum temperatures (1100 °C, 900 °C, 650 °C); transformations occurred during both heating and cooling during the first 2 cycles but were absent during the 3rd, the maximum temperature being too low. The heating rate was quite fast, about 80 °C·s⁻¹, and 2 cooling rates of -0.3 °C·s⁻¹ and -12 °C·s⁻¹, corresponding to ‘Satoh Tests 1 and 2’, were considered. The $\gamma \rightarrow \alpha$ transformation was bainitic during Test 1 and martensitic during Test 2.

Fig. 1 displays the axial stress recorded during the 1st cycle of Tests 1 and 2 versus temperature, together with theoretical results calculated using plastic and viscoplastic models. Prior to commenting upon the influence of transformation plasticity upon these curves, it is useful to qualitatively explain their overall shape. During heating (lower part of each diagram), compressive stresses develop due to thermal expansion. The stress first decreases linearly, due to elasticity, but linearity is soon lost because the yield limit is reached. Strain hardening becomes more and more balanced in time by thermal softening so that the stress goes back to zero at very high temperatures.

³ In the Greenwood–Johnson interpretation, transformation plasticity is cumulative since it arises microplasticity in the austenitic phase; in the Magee interpretation, the transformation plastic strain cannot exceed a maximum value corresponding to identical orientations of all martensite plates.

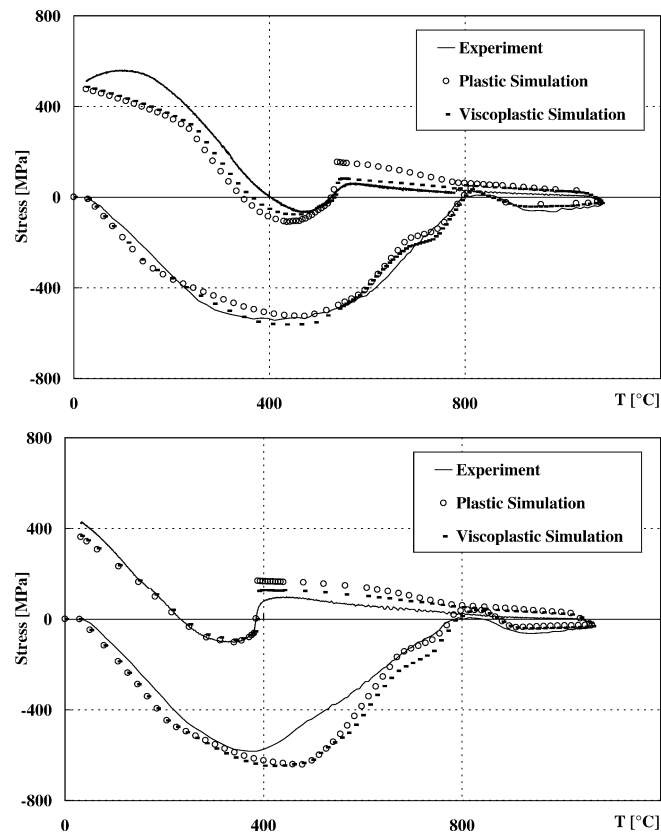


Fig. 1. Stress versus temperature during cycle 1 of Satoh tests 1 (top) and 2 (bottom).

The austenitic transformation is visible in the slight ‘bump’ which can be observed around a temperature of 800 °C and arises from the contraction due to the transformation. During cooling (upper part of each diagram), the stress becomes tensile due to thermal contraction, but remains small at first because the yield stress of austenite is quite low. The $\gamma \rightarrow \alpha$ transformation induces a notable drop of the stress because of the large accompanying expansion. When the transformation is complete, the stress starts increasing again because of thermal contraction.

The effect of transformation plasticity during the $\gamma \rightarrow \alpha$ transformation is to partially counterbalance the accompanying stress drop through (visco)plastic flow. Of course, it is impossible to suppress transformation plasticity in actual experiments, so that this effect cannot be illustrated on the experimental curves; but it is easily done in numerical simulations, and the stress drop during the $\gamma \rightarrow \alpha$ transformation found numerically without transformation plasticity (not shown here because the figures would become illegible) is much larger than that actually observed.

For Test 1 (top diagram in Fig. 1), the difference between results calculated with plastic and viscoplastic models is quite large at the beginning of cooling, the viscoplastic simulation being in better agreement with experiment than the plastic simulation; the predicted stresses differ by a factor of 2 just before the beginning of the $\gamma \rightarrow \alpha$ transformation. The reason is that the cooling rate being quite low ($-0.3 \text{ } ^\circ\text{C}\cdot\text{s}^{-1}$), the strain rate is also small, resulting in a low yield stress when viscous effects are accounted for. The predicted stress evolutions during the $\gamma \rightarrow \alpha$ transformation are also largely different, since the final stresses found are quite close in spite of the marked initial difference. One may be tempted to conclude from this final coincidence that the incorporation of viscous effects is of little importance in numerical simulations of welding. This conclusion is wrong for 2 reasons: firstly,

the Satoh test is only a crude 1D experiment, and the influence of viscous effects on final stresses may be much larger in more complex 3D situations; secondly, and more importantly, it has been shown by Bru et al. [13] and Leblond et al. [14] that *residual distortions* are quite sensitive to the *whole sequence of successive stresses* and not only to their final values; therefore accurate prediction of residual distortions requires accurate simulation of the whole sequence of stresses, and it is clear from the top curve in Fig. 1 that this can be achieved only by incorporating viscous effects.

For Test 2 (bottom diagram in Fig. 1), the difference between results calculated during cooling with plastic and viscoplastic models is less important. This is because the cooling rate is much greater ($-12\text{ }^{\circ}\text{C}\cdot\text{s}^{-1}$) than for Test 1, so that the strain rate is also much larger, resulting in a less important effect of this strain rate upon the yield stress.

Results for cycle 2 of both tests are not shown because they yield similar conclusions. However results for cycle 3 are displayed in Fig. 2 because they show an interesting effect, although this effect has nothing to do with phase transformations (which do not occur since the maximum temperature reached is too low). Indeed, for both tests, both simulations without and with viscous effects yield similar stresses during heating, but predicted stresses during cooling are markedly different, that calculated with viscous effects being larger and much closer to that actually observed. The reason is that during that period when the temperature is close to its maximum, stress relaxation occurs in the viscoplastic simulation, resulting in an increase of the stress since it is negative then; the gap between the 2 predicted stresses is preserved afterwards since their evolution is then purely (thermo)elastic. This feature again emphasizes the need for incorporation of viscous effects in welding simulations, in order to correctly capture the evolution of the stresses in time, which governs final residual distortions.

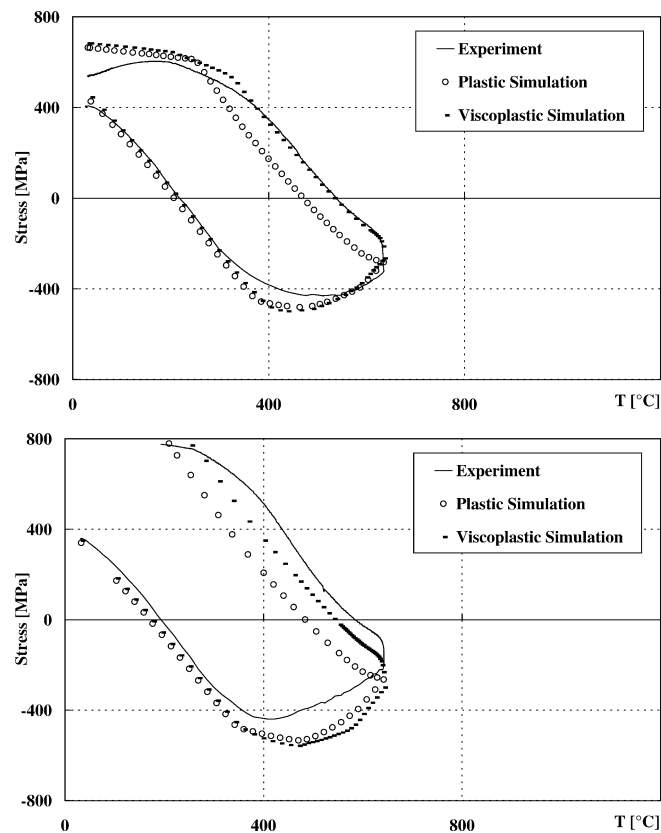


Fig. 2. Stress versus temperature during cycle 3 of Satoh tests 1 (top) and 2 (bottom).

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