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Homogenization of an elasticity problem in domains with a net of slender bars near surface

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Abstract

We consider an elasticity problem in a domain $\Omega^{(\varepsilon)} = \Omega \setminus F^{(\varepsilon)}$, where Ω is an open bounded domain in $\mathbf{R}^3$, $F^{(\varepsilon)}$ is a connected nonperiodic set in Ω like a net of slender bars, and ε is a parameter characterizing the microstructure of the domain. We consider the case of a surface distribution of the set $F^{(\varepsilon)}$, i.e., for sufficiently small ε, the set $F^{(\varepsilon)}$ is concentrated in arbitrary small neighbourhood of a surface Γ. Under a hypothesis on the asymptotic behaviour of the energy functional, we obtain the macroscopic (homogenized) model. ***To cite this article: M. Goncharenko, L. Pankratov, C. R. Mecanique 331 (2003).***

Résumé

Homogénéisation d'un problème d'élasticité dans des domaines avec un réseau de barres fines proche de la surface. On étudie un problème d'élasticité dans un domaine $\Omega^{(\varepsilon)} = \Omega \setminus F^{(\varepsilon)}$, où Ω est un ouvert borné dans $\mathbf{R}^3$, $F^{(\varepsilon)}$ est un ensemble non-périodique connexe dans Ω de type un réseau de barres fines et ε est un paramétre qui caractérise la microstructure du domaine. On considère le cas d'une distribution surfacique de l'ensemble $F^{(\varepsilon)}$, c'est-á-dire, lorsque $\varepsilon \to 0$, cet ensemble se concentre dans un voisinage d'une surface Γ aussi petit que l'on veut. Sous une hypothèse sur le comportement asymptotique de la fonctionelle d'énergie on obtient le modèle macroscopique (homogénéisé). ***Pour citer cet article : M. Goncharenko, L. Pankratov, C. R. Mecanique 331 (2003).***

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1. Introduction

We study a homogenization problem for the elasticity equation which describes the equilibrium of an elastic medium with rigid inclusions, such as a nonperiodic net of slender bars concentrated in an arbitrary small neigbourhood of a smooth surface. We assume that the inclusions are strongly fastened on the board (see the boundary condition (2) below). Most of the studies on the subject are using periodic homogenization (see, e.g., [1,2,4], and the bibliography herein), except [3], where a nonperiodic case of a bulk distribution of inclusions was considered. In this Note, instead of the classical periodicity assumption on the set of inclusions, we impose a certain condition (see (H1) in Theorem 2.1) on the local energy functional (4). We obtain a global homogenized model giving a general setting for various types of surface distribution of nets formed by slender bars. In Section 3 we show that (H1) is satisfied, in particular, in the case of a periodic net of bars having the diameter exponentially small with respect to the periodicity of the net. In this case the coefficients of the homogenized model are calculated in the explicit form.

Notation. We denote by C a generic constant independent of ε and we adopte the Einstein convention of summation over repeated indices.

2. Problem statement and formulation of the main result

Let Ω be a fixed domain in $\mathbf{R}^3$ with a smooth boundary $\partial\Omega$ and $F^{(\varepsilon)}$ be closed connected set in Ω, such as a net of slender bars. Suppose that $F^{(\varepsilon)}$ depends on a parameter ε such that the distance between the bars and their diameters tends to zero as $\varepsilon \to 0$. We consider the case of a surface distribution of the set $F^{(\varepsilon)}$, i.e., for sufficiently small ε, the inclusions $F^{(\varepsilon)}$ are concentrated in an arbitrary small neighbourhood of a fixed smooth surface $\Gamma \subset \Omega$.

In the domain $\Omega^{(\varepsilon)}$, $\Omega^{(\varepsilon)} = \Omega \setminus F^{(\varepsilon)}$, we consider the boundary value problem:

$$-\mu\Delta\vec{u}^{\,\varepsilon} - (\lambda+\mu)\operatorname{grad}\operatorname{div}\vec{u}^{\,\varepsilon} = \vec{K} \quad \text{in } \Omega^{(\varepsilon)} \tag{1}$$

$$\vec{u}^{\,\varepsilon} = 0 \quad \text{on } \partial\Omega^{(\varepsilon)} \tag{2}$$

where $\vec{K} \in [L^2(\Omega)]^3$ is a given vector-function, and $\lambda,\ \mu$ are Lame's constants.

We study the asymptotic behaviour of $\vec{u}^{\,\varepsilon}(x)$ solution of problem (1), (2) as $\varepsilon \to 0$.

Let us introduce quantitative characteristics of set $F^{(\varepsilon)}$. Let S be an arbitrary part of surface Γ and $T(S,\delta) = \{y + \alpha n_y,\ y \in S,\ |\alpha| \leqslant \delta\}$, where n_y is the normal vector to S at the point y. Thus $T(S,\delta)$ is a layer of thikness 2δ with the central surface S. Denote by S_δ^+ and S_δ^- the bases of the layer and by $T^{(\varepsilon)}(S,\delta) = T(S,\delta) \setminus F^{(\varepsilon)}$.

Consider the functional

$$Q_{\vec{a}}(\varepsilon, S, \delta) = \inf_{v^\varepsilon} \int\limits_{T^{(\varepsilon)}(S,\delta)} W(\vec{v}^{\,\varepsilon})\,\mathrm{d}x \tag{3}$$

Here the infimum is taken in the class of vector functions $\vec{v}^{\,\varepsilon} \in [H^1(T^{(\varepsilon)}(S,\delta))]^3$ equal zero on $F^{(\varepsilon)}$ and $\vec{a}$ on $S_\delta^\pm$, where $\vec{a}$ is an arbitrary vector in $\mathbf{R}^3$; $W(\vec{u}) = W(\vec{u},\vec{u})$ is given by

$$W(\vec{u},\vec{v}) = C_{iklm}\mathcal{E}_{ik}(\vec{u})\mathcal{E}_{lm}(\vec{v}), \quad \mathcal{E}_{ik}(\vec{u}) = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_k} + \frac{\partial u_k}{\partial x_i}\right)$$

where C_{iklm} are the constants appearing in Hook's law.

The functional $Q_{\vec{a}}(\varepsilon,S,\delta)$ can be represented in the form

$$Q_{\vec{a}}(\varepsilon,S,\delta) = q^{ik}(\varepsilon,S,\delta)a_i a_k$$

with

$$q^{ik}(\varepsilon,S,\delta)=\int\limits_{T^{(\varepsilon)}(S,\delta)} W(\vec v^{\,k,\varepsilon},\vec v^{\,i,\varepsilon})\,\mathrm{d}x \tag{4}$$

where $\vec v^{\,k,\varepsilon}$ is the vector-function which minimizes (3) when $\vec a=\vec e^{\,k}$ ($\vec e^{\,k}$ is the unit vector of the axis x_k).

The main result of the Note is the following:

Theorem 2.1. *Assume that for any $S\subset\Gamma$ the following condition is fulfilled*:

$$\lim_{\delta\to 0}\varliminf_{\varepsilon\to 0} q^{ik}(\varepsilon,S,\delta)=\lim_{\delta\to 0}\varlimsup_{\varepsilon\to 0} q^{ik}(\varepsilon,S,\delta)=\int\limits_S q^{ik}(x)\,\mathrm{d}S \tag{H1}$$

where $q^{ik}(x)$ is a continuous function on S. Then $\vec u^{\varepsilon}$ solution of problem (1), (2), *extended by zero on $F^{(\varepsilon)}$, converges in $[L^2(\Omega)]^3$ to $\vec u$ solution of*

$$-\Delta\vec u-(\lambda+\mu)\,\mathrm{grad}\,\mathrm{div}\,\vec u=\vec K\quad \text{in } \Omega\setminus\Gamma \tag{5}$$

$$(\vec u)^+=(\vec u)^-\quad \text{on } \Gamma \tag{6}$$

$$\big(\vec t(\vec u)\big)^+-\big(\vec t(\vec u)\big)^-=Q\vec u\quad \text{on } \Gamma \tag{7}$$

where $Q=Q(x)$ is a matrix with the elements $q^{ik}=q^{ik}(x)$ and

$$\vec t(\vec u)=C_{iklm}\mathcal{E}_{lm}(\vec u)\cos(\vec n,\vec e^{\,i})\vec e^{\,k}$$

is the stress vector. Here the signs $\pm$ indicate the values of the functions $\vec u$ and $t(\vec u)$ on the different sides of the surface Γ.

3. Sketch of the proof of Theorem 2.1

The solution $\vec u^{\,\varepsilon}$ of problem (1), (2) minimizes the functional

$$J^{\varepsilon}(\vec u^{\,\varepsilon})=\int\limits_{\Omega^{(\varepsilon)}}\big\{W(\vec u^{\,\varepsilon})-2(\vec K,\vec u^{\,\varepsilon})\big\}\,\mathrm{d}x \tag{8}$$

in the class of vector-functions $\vec u^{\,\varepsilon}\in[H_0^1(\Omega^{(\varepsilon)})]^3$. We extend $\vec u^{\,\varepsilon}$ by zero on $F^{(\varepsilon)}$ keeping for these functions the same notation. Then Korn's and Friedrichs inequalities imply

$$\|\vec u^{\,\varepsilon}\|_{[H^1(\Omega)]^3}\leqslant C\|\vec K\|_{[L^2(\Omega)]^3} \tag{9}$$

Therefore the sequence of functions $\{\vec u^{\,\varepsilon}\}$ is a weakly compact set in $[H^1(\Omega)]^3$ and one can extract a subsequence $\{\vec u^{\,\varepsilon},\ \varepsilon=\varepsilon_k\to 0\}$ weakly convergent to a function $\vec u\in[H^1(\Omega)]^3$.

Let us show that $\vec u$ is the solution of problem (5)–(7).

Let $\Gamma=\bigcup_i S_i$ with $S_i\cap S_j\neq\emptyset$ (the boundary l_i of S_i is assumed to be piecewise smooth). We assume that the diameters of S_i are of order h, where $\varepsilon\ll h\ll 1$. Let δ' be a small parameter such that $\delta'<\frac{\delta}{2}$. Let $T(S_i,\delta')$ be a layer of thickness $2\delta'$ with center surface S_i. Consider the function $\varphi_{\delta'}\in W_2^1(\bigcup_i T(S_i,\delta'))$ such that $\varphi_{\delta'}\equiv 0$ in the $2\delta'$-neighbourhood of curve $l=\bigcup_i l_i$ and $\varphi_{\delta'}\equiv 1$ outside of $\delta/2$-neighbourhood of curve l, $0\leqslant\varphi_{\delta'}\leqslant 1$ everywhere and

$$\lim_{\delta'\to 0}\int\limits_{\Omega}|\nabla\varphi_{\delta'}|^2\,\mathrm{d}x\to 0$$

Let $\vec{w}$ be a vector-function from the space $[C_0^2(\Omega)]^3$. Consider a function

$$\vec{w}^{\,\varepsilon}(x)=\begin{cases}\vec{w}(x) & \text{in } \Omega\setminus\bigcup_i T(S_i,\delta)\\ \vec{w}(x)\varphi_{\delta'}(x) & \text{in } \big(\bigcup_i(T(S_i,\delta))\big)\setminus\big(\bigcup_i(T(S_i,\delta'))\big)\\ w_k\vec{v}^{\,k,i,\varepsilon}\varphi_{\delta'} & \text{in } \bigcup_i T^{(\varepsilon)}(S_i,\delta')\end{cases}\tag{10}$$

where $\vec{v}^{\,k,i,\varepsilon}$ is a vector-function minimizing (3) when $\vec{a}=\vec{e}^{\,k}$ in $T(S_i,\delta')$.

Since $\vec{u}^{\,\varepsilon}$ is a function minimazing (8), we have

$$J^\varepsilon(\vec{u}^{\,\varepsilon})\leqslant J^\varepsilon(\vec{w}^{\,\varepsilon})\tag{11}$$

Let us estimate the right-hand side of (11).

It follows from the conditions of Theorem 2.1 and the definitions of $\vec{w}^{\,\varepsilon}$ and $\varphi_{\delta'}$ that

$$\overline{\lim_{\varepsilon\to0}}\,J^\varepsilon(\vec{u}^{\,\varepsilon})\leqslant J_{\text{hom}}(\vec{w})\overset{\text{def}}{=}\int\limits_\Omega\big\{W(\vec{w})-2(\vec{K},\vec{w})\big\}\,\mathrm{d}x+\int\limits_\Gamma q^{lk}w_lw_k\,\mathrm{d}\Gamma\tag{12}$$

Let us show now that if $\vec{u}^{\,\varepsilon}$ converges weakly in $[H^1(\Omega)]^3$ to $\vec{u}\in[H^1(\Omega)]^3$ then

$$J_{\text{hom}}(\vec{u})\leqslant\lim_{\varepsilon=\varepsilon_k\to0}J^\varepsilon(\vec{u}^{\,\varepsilon})\tag{13}$$

Let $\vec{u}^{\,\sigma}\in C_0^2$ be a vector-function such that

$$\|\vec{u}-\vec{u}^{\,\sigma}\|_{[H^1(\Omega)]^3}<\sigma$$

We construct the function $\vec{w}^{\,\sigma,\varepsilon}$ by (10) with $\vec{w}^{\,\sigma}=\vec{u}-\vec{u}^{\,\sigma}$ instead of $\vec{w}$. It is easy to see that, for sufficiently small ε,

$$\|\vec{w}^{\,\sigma,\varepsilon}\|_{[H^1(\Omega)]^3}\leqslant C\|\vec{w}^{\,\sigma}\|_{[H^1(\Omega)]^3}$$

and $\vec{w}^{\sigma,\varepsilon}$ converges weakly in $[H^1(\Omega)]^3$ to $\vec{w}^\sigma$.

Let $\vec{u}^{\,\sigma,\varepsilon}=\vec{u}^{\,\varepsilon}-\vec{w}^{\,\sigma,\varepsilon}$. Then it is evident that

$$\|\vec{u}^{\,\sigma,\varepsilon}-\vec{u}^{\,\varepsilon}\|_{[H^1(\Omega)]^3}\leqslant\|\vec{u}-\vec{u}^{\,\sigma}\|_{[H^1(\Omega)]^3}$$

Let $\Gamma=\bigcup_i S_i$ with $S_i\cap S_j=\emptyset$ $(i\neq j)$ and let $T(S_i,\delta)$ be the layer of thikness 2δ with the center surface S_i. Define $\Omega_{\sigma,A}=\{x\colon x\in\Omega,|u_i^\sigma|>A\}$, where $A>0$. In the set $\Omega_{\sigma,A}$ we consider the vector-function $\vec{V}^{\,i}$ with the components

$$V_j^i(x)=\frac{u^{\sigma,\varepsilon}(x)}{u_j^\sigma(x)}u_j^\sigma(x^i)+\chi_\delta(x)\big(u_j^\varepsilon(x)-u_j(x)\big)\frac{u_j^\sigma(x^i)}{u_j^\sigma(x)},\quad j=1,2,3\tag{14}$$

where $x_i\in S_i$ and $\chi_\delta\in C_0^\infty(\Omega)$ such that $\chi_\delta(x)=0$ for $x\in\bigcup_i T(S_i,\delta')$ and $\chi_\delta(x)=1$ for $x\in\Omega\setminus\bigcup_i T(S_i,\delta)$.

The function $\vec{V}^{\,i}$ equals $\vec{u}^{\,\sigma}(x^i)$ for $x\in\Omega\setminus\bigcup_i T(S_i,\delta)$ and zero for $x\in F^{(\varepsilon)}$. Therefore,

$$\int\limits_{T(S_i,\delta)}W(\vec{V}^{\,i})\,\mathrm{d}x\geqslant Q_{\vec{u}^\sigma(x^i)}(s,S_i,\delta)\tag{15}$$

It follows now from (15) that

$$\int\limits_{T(S_i,\delta)}W(\vec{u}^{\,\sigma,\varepsilon})\,\mathrm{d}x\geqslant\int\limits_{T(S_i,\delta)}W(\vec{u}^{\,\sigma})\,\mathrm{d}x+Q_{\vec{u}^\sigma(x^i)}(s,S_i,\delta')+E_i(s,S_i,\delta,u^\sigma)\tag{16}$$

where

$$\lim_{\varepsilon=\varepsilon_k\to0}E_i(s,S_i,\delta,\vec{u}^\sigma)=\mathrm{O}(h\delta^{1/2}\sigma)\tag{17}$$

Therefore, (15)–(17) imply that

$$J^{\varepsilon}(\vec{u}^{\,\sigma,\varepsilon}) \geqslant \int\limits_{\Omega_{\sigma,A}} W(\vec{u}^{\,\sigma})\,\mathrm{d}x + \sum_i q^{jk}(s, S_i, \delta')u_j^{\sigma}(x^i)u_k^{\sigma}(x^i)$$
$$- 2\int\limits_{\Omega} (\vec{K}, \vec{u}^{\,\sigma,\varepsilon})\,\mathrm{d}x + \mathrm{O}(\delta'^2) + \mathrm{O}(\delta^{1/2}h^{-1}\varepsilon^{1/2})$$

Passing to the limit in this inequality as $\varepsilon \to 0$ and $\delta \to 0$ and then as $A \to 0$ and $\sigma \to 0$ we get (13).

Finally, it follows from (11), (13) that

$$J_{\mathrm{hom}}(\vec{u}) \leqslant J_{\mathrm{hom}}(\vec{w})$$

where $\vec{u}$ is the weak limit of the solutions $\vec{u}^{\,\varepsilon}$ of problem (1), (2) and $\vec{w}$ is a vector-function from the space $[C_0^2(\Omega)]^3$. Then it remains true for any $\vec{w} \in [H_0^1(\Omega)]^3$. Since the solution of problem (5)–(7) is unique, all the sequence of solutions of problem (1), (2) extended by zero on $F^{(\varepsilon)}$ converges to $\vec{u}$ solution of (5)–(7).

Theorem 2.1 is proved. □

4. Periodic example

In a general case, the elements of the matrix Q in (7) are given by the complicated characteristic (3) of the set $F^{(\varepsilon)}$. However, in some special cases they can be calculated explicitely.

Suppose that the set $F^{(\varepsilon)}$ consists of a large number of thin cylinders whose axes belong to the surface $\Gamma = \{x = (x_1, x_2, x_3),\ x_3 = 0\}$. We denote by $d^{\varepsilon} = \mathrm{e}^{-a/\varepsilon}$ the diameters of the cylinders. The axes of the cylinders are parallel to the axes x_1 and x_2. The distance between the cylinder axes we denote by $l^{\varepsilon} = l\varepsilon$.

We represent Γ as a union of squares K_i, $K_i = K(x^i, h)$, centered at x^i and the side length $h > 0$. We suppose that the sides of the squares K_i are parallel to the axes of the cylinders. We denote by $T(K_i, \delta)$ the layer with the center surface K_i.

Let us calculate q^{11}. To this end we introduce the functions

$$\vec{V}^{\,1} = \begin{cases} \vec{e}^{\,1}, & r < d^{\varepsilon} \\ \left(\ln \dfrac{r}{r_1^{\varepsilon}}\right)\left(\ln \dfrac{d^{\varepsilon}}{r_1^{\varepsilon}}\right)^{-1} \psi^{\varepsilon}\left(\dfrac{r}{r_1^{\varepsilon}}\right)\vec{e}^{\,1}, & d^{\varepsilon} < r < r_1^{\varepsilon} \\ (0,0,0), & r > r_1^{\varepsilon} \end{cases} \tag{18}$$

$$\vec{V}^{\,2} = \begin{cases} \vec{e}^{\,1}, & r < d^{\varepsilon} \\ (\xi(r,\varphi), 0, \eta(r,\varphi))\psi^{\varepsilon}\left(\dfrac{r}{r_1^{\varepsilon}}\right), & d^{\varepsilon} < r < r_1^{\varepsilon},\ 0 \leqslant \varphi \leqslant 2\pi \\ (0,0,0), & r > r_1^{\varepsilon} \end{cases} \tag{19}$$

Here $\psi^{\varepsilon} \in C_0^2(\Omega)$ such that $\psi^{\varepsilon}(t) = 1$ for $0 \leqslant t \leqslant 1/2$ and $\psi^{\varepsilon}(t) = 0$ for $t \geqslant 3/4$; $r_1^{\varepsilon} = \varepsilon^{\beta}$ $(\beta > 3/2)$; the functions $\xi(r,\varphi)$ and $\eta(r,\varphi)$ are the components of the vector-function $\vec{g} = (\xi, \eta)$ which is the solution of the following auxiliary problem:

$$A^0 g = -\mu\Delta\vec{g} - (\lambda + \mu)\,\mathrm{grad}\,\mathrm{div}\,\vec{g} = 0, \quad d^{\varepsilon} < r < r_1^{\varepsilon}, \quad 0 \leqslant \varphi \leqslant 2\pi$$
$$\vec{g} = \vec{e}^{\,1}, \quad r = d^{\varepsilon}; \qquad \vec{g} = 0, \quad r = r_1^{\varepsilon}$$

Consider the functions

$$u^{\varepsilon} = \vec{e}^{\,1} - \sum_{(T^{(\varepsilon)}(K,\delta))_1} \vec{V}^{\,1}, \qquad \hat{u}^{\varepsilon} = \vec{e}^{\,1} - \sum_{(T^{(\varepsilon)}(K,\delta))_2} \vec{V}^{\,2} \tag{20}$$

Here the indices $_1$ and $_2$ mean that the summation is taken over the cylinders that parallel to the axes x_1 and x_2, respectively.

We introduce the function $\vec{w}^{\,\varepsilon}$ by setting

$$\vec{w}^{\,\varepsilon}(x) = \left(\hat{u}_1^\varepsilon u_1^\varepsilon, 0, u_1^\varepsilon \hat{u}_3^\varepsilon\right) \tag{21}$$

It is clear that $\vec{w}^{\,\varepsilon}$ belongs to the same class that the function minimizing (4), therefore, for sufficiently small ε,

$$q^{11}\left(\varepsilon, T^{(\varepsilon)}(K,\delta), \delta\right) \leqslant \int\limits_{T^{(\varepsilon)}(K,\delta)} W(\vec{w}^{\,\varepsilon})\,\mathrm{d}x = 2\pi\mu \frac{h^2}{l^\varepsilon|\ln d^\varepsilon|}\left(1 + 2\frac{\lambda+2\mu}{\lambda+3\mu}\right) + \mathrm{o}(\delta) \tag{22}$$

as $\delta \to 0$.

Using now the explicit form of the function $\vec{w}^\varepsilon$ one can show that $q^{11}(\varepsilon, T^{(\varepsilon)}(K,\delta), \delta)$ can be also estimated as follows

$$2\pi\mu \frac{h^2}{l^\varepsilon|\ln d^\varepsilon|}\left(1 + 2\frac{\lambda+2\mu}{\lambda+3\mu}\right) + \mathrm{o}(\delta) \leqslant q^{11}\left(\varepsilon, T^{(\varepsilon)}(K,\delta), \delta\right) \tag{23}$$

as $\delta \to 0$. Then from (22) and (23) we finally get

$$q^{11} = 2\pi\mu \frac{1}{la}\left(1 + 2\frac{\lambda+2\mu}{\lambda+3\mu}\right)$$

In a similar way we obtain that

$$\begin{cases} q^{22} = q^{11} = 2\pi\mu \dfrac{1}{la}\left(1 + 2\dfrac{\lambda+2\mu}{\lambda+3\mu}\right) \\ q^{33} = 4\pi\mu \dfrac{1}{la}\dfrac{\lambda+2\mu}{\lambda+3\mu} \\ q^{ik} = 0 \quad (i \neq k) \end{cases}$$

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References

[1] N.S. Bakhvalov, G.P. Panasenko, Homogenization: Averaging Processes in Periodic Media, Kluwer, Dordrecht–Boston–London, 1989.
[2] D. Cioranescu, J. Saint Jean Paulin, Homogenization of Reticulated Structures, in: Applied Mathematical Sciences, Vol. 136, Springer-Verlag, New York–Berlin–Heidelberg, 1999.
[3] E. Khruslov, V. Marchenko, Boundary Value Problems in Domains with Fine-Grained Boundaries, Naukova Dumka, 1974 (in Russian).
[4] É. Sanchez–Palencia, Nonhomogeneous Media and Vibration Theory, in: Lecture Notes in Phys., Vol. 127, Springer-Verlag, New York, 1980.