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Contribution to 3D impact problems: collisions between two slender steel bars

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Abstract

This Note deals with the three-dimensional phenomenon of collision between two slender steel bars. The problem posed is whether the restitution concept developed in rigid-body theory is relevant in the case of such slender contactors. Some elements of an answer are provided through the use of two complementary approaches of collision, a theoretical one based on coefficients of restitution and series of experiments. Our main conclusion is that the alleged Newton coefficient of restitution varies according to the impact location on the rods. ***To cite this article: C. Le Saux et al., C. R. Mecanique 332 (2004).***

Résumé

Une contribution aux problèmes d'impact en trois dimensions : collisions entre deux barres élancées. Cette Note présente une étude du phénomène de collision entre deux barres d'acier élancées. La question est posée de savoir si la notion de restitution développée dans le contexte de corps rigides est pertinente pour de tels contacteurs. Par le biais de deux approches complémentaires de ce phénomène, l'une théorique à coefficients de restitution et l'autre expérimentale, nous apportons des éléments de réponse à la question posée. Principalement on démontre la variation du prétendu coefficient de restitution de Newton en fonction de la position du point d'impact sur les barres. ***Pour citer cet article : C. Le Saux et al., C. R. Mecanique 332 (2004).***

Keywords: Dynamics of rigid or flexible systems; Collisions; Slender bars; Coefficient of restitution; Flexural vibrations; Multiple micro-collisions

Mots-clés : Dynamique des systèmes rigides ou flexibles ; Collisions ; Barres élancées ; Coefficient de restitution ; Vibrations de flexion ; Multiple micro-collisions

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Les phénomènes de chocs jouent un rôle important dans de nombreux systèmes mécaniques intéressant divers domaines de recherche tels que le génie civil (milieux granulaires, éboulements, séismes) la robotique (manipulateurs, marcheurs) et les mécanismes (disjoncteurs électriques, vibro-impacteurs). Dans le contexte de

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corps supposés rigides, les lois de chocs élaborées reposent pour la plupart sur la définition d'un coefficient de restitution normale et sur le frottement de Coulomb pour la description du comportement tangentiel. Elles ont été établies pour certaines formes de solides ; si par exemple dans le cas de corps sphériques elles conduisent à des résultats acceptables [2], de récents travaux [3], relatifs à la collision d'une barre élancée sur un bâti rigide, démontrent leur insuffisance pour reproduire correctement le phénomène physique.

Dans le cadre du développement d'une technique numérique, de type event-driven, pour calculer l'évolution dans l'espace de systèmes constructifs pliables/dépliables à barres et câbles [4], on s'interroge sur l'aptitude d'un modèle de choc basé sur la notion de restitution à décrire convenablement le phénomène de collision entre barres élancées. Une étude de ce phénomène est initiée par deux approches complémentaires, l'une numérique, l'autre expérimentale.

C'est le modèle numérique de collision de Moreau [5] qui est mis en œuvre ; il compte parmi les rares modélisations en 3D capables de traiter les collisions en présence de contacts multiples, phénomènes qui concernent le pliage des systèmes étudiés. Il s'agit d'un modèle à trois paramètres empiriques μ, e_{N}, e_{T} qui s'identifient respectivement à un coefficient de frottement, un coefficient de restitution normale et un coefficient de restitution tangentielle. Le dispositif expérimental est constitué (Fig. 1) de deux barres cylindriques. La première, S_1, reliée au bâti par l'intermédiaire d'un seul câble, se comporte comme un pendule. La seconde, S_2, est suspendue horizontalement au bâti par l'intermédiaire de 2 câbles verticaux de même longueur, formant comme un trapèze. Chacune des barres est munie d'un accéléromètre piézoélectrique associé à un amplificateur et relié à un analyseur de signaux (Fig. 1(c)). Le protocole d'un essai consiste en un lâcher du pendule, à vitesse initiale nulle, dans un plan $P(d)$ (Fig. 1(a)) perpendiculaire à la position initiale de S_2 ; l'angle de lâcher est de 45° (Fig. 1(b)). Ce dispositif permet différentes configurations d'impact d'excentricité d, dans lesquelles les deux barres sont orthogonales et produit dans chaque cas un choc (quasi) direct. Un choc entre deux solides est dit « direct » lorsque la valeur, $\mathbf{v}_{\mathrm{T}}^{-}$, avant choc de la vitesse relative tangentielle de ces solides est nulle.

Les essais numériques mettent en évidence, indépendamment de l'excentricité de la configuration d'impact, que seul le coefficient de restitution normale e_{N} a une influence notable sur le bilan du choc : impulsion de contact, variation d'énergie cinétique. Une étude de sensibilité de e_{N} à l'excentricité d des configurations d'impact est menée : les signaux temporels, délivrés par les accéléromètres au cours de différentes campagnes d'essais à d variable, fournissent des informations quantitatives sur le comportement du système qui sont comparées à celles déduites du modèle numérique pour différentes valeurs de e_{N} ; la valeur de e_{T} est choisie arbitrairement et celle du coefficient de frottement (ici acier/acier) est fixée à la valeur usuelle 0,15. Les résultats obtenus (Figs. 2 et 3) font apparaître une dépendance du coefficient de restitution normale à l'égard de la position du point d'impact sur les barres. Il est aussi mis en évidence que la dissipation d'énergie induite par une collision n'est pas localisée au voisinage de l'impact mais se produit à l'échelle globale du système par l'intermédiaire de vibrations de flexion des barres ; la dépendance de e_{N} à l'égard de l'excentricité d de la configuration d'impact pourrait s'expliquer par un comportement vibratoire des barres différent lorsque d varie (différents modes de flexion excités). De plus, les essais expérimentaux ont montré que les vibrations des barres en flexion induites par un choc sont à l'origine d'un phénomène de multiples micro-collisions. Les effets du comportement vibratoire des barres, sur la cinétique du système, ne sont pas pris en compte dans la théorie des chocs de corps rigides. Celle-ci doit donc être utilisée avec précaution et dans des situations spécifiques. Les résultats présentés dans cette Note apportent une contribution aux problèmes d'impact en trois dimensions ; dans le cas de contacteurs élancés, ils mettent en doute la pertinence de la notion de coefficient de restitution.

1. Introduction

In rigid-body mechanics, a collision is represented as an instantaneous phenomenon. At the instant t_{c} of a collision, the function of time $\mathbf{u}$ which represents the system velocity is expected to be discontinuous (but with bounded variation) [1] while the function $\mathbf{q}$ which represents the system configuration may be assumed continuous in time. Among the computational techniques proposed in literature to calculate the evolution of a set of three-dimensional rigid bodies submitted to unilateral contact constraints, the methods of the event-driven sort take into account the nonsmoothness in time of such a problem. These methods break down the time interval into a succession of regular evolution phases and identify the transitions between them: breaking of contacts, collisions and so on. The collision process is described by means of an impulsion balance connecting the jump $(\mathbf{u}^+(t_c) - \mathbf{u}^-(t_c))$ to contact percussions $\mathbf{S}_\alpha$, α ranging over the set J of contacts effective at instant t_c. However, these relations do not determine uniquely the post-collision velocities $\mathbf{u}^+(t_c)$, used as initial velocities for further evolution. Information about the percussions $\mathbf{S}_\alpha$ is needed. In this context, it is convenient to state such relations in the form of macroscopic laws called collision laws. The most popular laws are based on the definition of a normal coefficient of restitution and a Coulomb friction hypothesis. They were elaborated for bodies of a specific shape; while in the case of spherical bodies they lead to convincing results [2], recent works such as [3] demonstrate that they do not correctly predict, for instance, the impact of a bar freely dropped onto a massive external body.

In the course of developing an event-driven technique applied to the folding and unfolding of reticulated systems made of bars and cables [4], we questioned the capability of restitution-based models to describe correctly collisions between slender bars. Some elements of an answer are provided through the use of two complementary approaches: numerical and experimental.

2. Moreau's numerical model of collision

Moreau's model, which involves restitution coefficients, is one of the few 3D approaches able to handle collisions occurring in multi-contact configurations, which is the case in the folding process under study. We proceed now to describe Moreau's model in some detail. For every contact indexed by $\alpha \in J$, Moreau [5] relates the contact percussion $\mathbf{S}_\alpha$ and the so-called average velocity $\mathbf{v}_\alpha^{\mathrm{m}}$ through Signorini's condition and Coulomb's friction law:

$$\alpha \in J \quad \begin{cases} S_{\mathrm{N}\alpha} \geqslant 0, \quad v_{\mathrm{N}\alpha}^{\mathrm{m}} \geqslant 0, \quad S_{\mathrm{N}\alpha} v_{\mathrm{N}\alpha}^{\mathrm{m}} = 0 \\ |\mathbf{S}_{\mathrm{T}\alpha}| \leqslant \mu_\alpha S_{\mathrm{N}\alpha}, \quad \mathbf{v}_{\mathrm{T}\alpha}^{\mathrm{m}} \neq \mathbf{0} \quad \Rightarrow \quad \mathbf{S}_{\mathrm{T}\alpha} = -\mu_\alpha S_{\mathrm{N}\alpha} \mathbf{v}_{\mathrm{T}\alpha}^{\mathrm{m}} / \left|\mathbf{v}_{\mathrm{T}\alpha}^{\mathrm{m}}\right| \end{cases} \tag{1}$$

Subscripts N and T refer to the normal and tangential components, relative to a local frame at M_α, of the concerned vectors. The value $\mathbf{v}_\alpha^{\mathrm{m}}$ is defined as a weighted means of the (known) pre-collision and (unknown) post-collision relative velocities of the colliding bodies at contact point M_α, say $\mathbf{v}_\alpha^-$ and $\mathbf{v}_\alpha^+$.

$$v_{\mathrm{N}\alpha}^{\mathrm{m}} = \frac{e_{\mathrm{N}\alpha}}{1+e_{\mathrm{N}\alpha}} v_{\mathrm{N}\alpha}^- + \frac{1}{1+e_{\mathrm{N}\alpha}} v_{\mathrm{N}\alpha}^+, \quad \mathbf{v}_{\mathrm{T}\alpha}^{\mathrm{m}} = \frac{e_{\mathrm{T}\alpha}}{1+e_{\mathrm{T}\alpha}} \mathbf{v}_{\mathrm{T}\alpha}^- + \frac{1}{1+e_{\mathrm{T}\alpha}} \mathbf{v}_{\mathrm{T}\alpha}^+, \quad e_{\mathrm{N}\alpha}, e_{\mathrm{T}\alpha} \in [0,1] \tag{2}$$

Velocities $\mathbf{v}_\alpha^-$ and $\mathbf{v}_\alpha^+$ can be expressed as affine functions of $\mathbf{u}$ in the form $\mathbf{v}_\alpha^\pm = G_\alpha \mathbf{u}^\pm + W_\alpha$; here G_α and W_α are continuous functions of time. The empirical parameters $e_{\mathrm{N}\alpha}$ and $e_{\mathrm{T}\alpha}$ are respectively the normal and tangential coefficients of restitution, a terminology which will be justified below. The contact law stated is of the prospective type, in the sense of Moreau; this implies that $\mathbf{S}_\alpha$ can be non zero only if $v_{\mathrm{N}\alpha}^{\mathrm{m}} = 0$, i.e., $v_{\mathrm{N}\alpha}^+ = -e_{\mathrm{N}\alpha} v_{\mathrm{N}\alpha}^-$ (see Eq. (2)), which is formally Newton's restitution law. But such a formulation is richer than a restitution law separately applied at each contact α; it also permits $\mathbf{S}_\alpha = \mathbf{0}$, in which case only the inequality $v_{\mathrm{N}\alpha}^{\mathrm{m}} \geqslant 0$ is asserted. The global calculation discussed below involves all the contacts together, through the equations of dynamics, and decides between the two branches of the alternative. Similarly the global calculation may end in the zero sliding case of Coulomb's law at contact α. Then Eq. (2) gives $\mathbf{v}_{\mathrm{T}\alpha}^+ = -e_{\mathrm{T}} \mathbf{v}_{\mathrm{T}\alpha}^-$, which is a law of tangential restitution. The non-linear problem of collisions is formulated in (3), where H denotes the inertia matrix.

$$\begin{aligned} & H\left(\mathbf{u}^+ - \mathbf{u}^-\right) = \sum_{\alpha \in I_n} {}^T G_\alpha \mathbf{S}_\alpha \\ & \forall \alpha \in J \quad \begin{cases} S_{\mathrm{N}\alpha} \geqslant 0, \quad v_{\mathrm{N}\alpha}^{\mathrm{m}} \geqslant 0, \quad S_{\mathrm{N}\alpha} v_{\mathrm{N}\alpha}^{\mathrm{m}} = 0 \\ |\mathbf{S}_{\mathrm{T}\alpha}| \leqslant \mu_\alpha S_{\mathrm{N}\alpha}, \ \mathbf{v}_{\mathrm{T}\alpha}^{\mathrm{m}} \neq \mathbf{0} \quad \Rightarrow \quad \mathbf{S}_{\mathrm{T}\alpha} = -\mu_\alpha S_{\mathrm{N}\alpha} \mathbf{v}_{\mathrm{T}\alpha}^{\mathrm{m}} / \left|\mathbf{v}_{\mathrm{T}\alpha}^{\mathrm{m}}\right| \end{cases} \end{aligned} \tag{3}$$

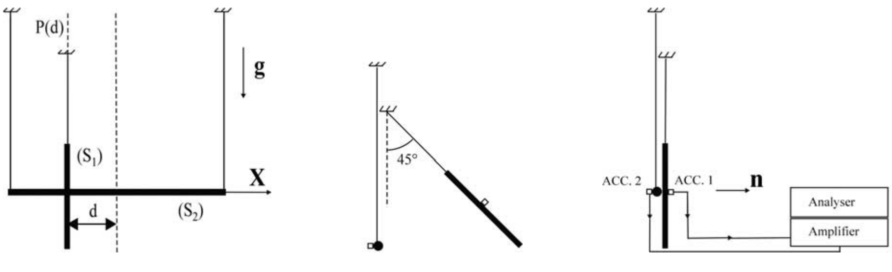


Fig. 1. Experimental set-up: release configuration, (a) front view, (b) lateral view; (c) impact configuration.

Fig. 1. Dispositif expérimental : configuration de lâcher, (a) vue de face, (b) vue latérale ; (c) configuration d'impact.

Problem (3) may be solved through various methods [5,6]; we use an iteration technique à la Gauss–Seidel [5] which amounts to treating a succession of single-contact problems. Since the contact law (1) allows one to convincingly reproduce known phenomena such as the exotic behaviour of a "superball" or the rocking of a slender block on a table, one may ask whether it is able to correctly describe collisions between two slender bars.

3. Experimental set-up

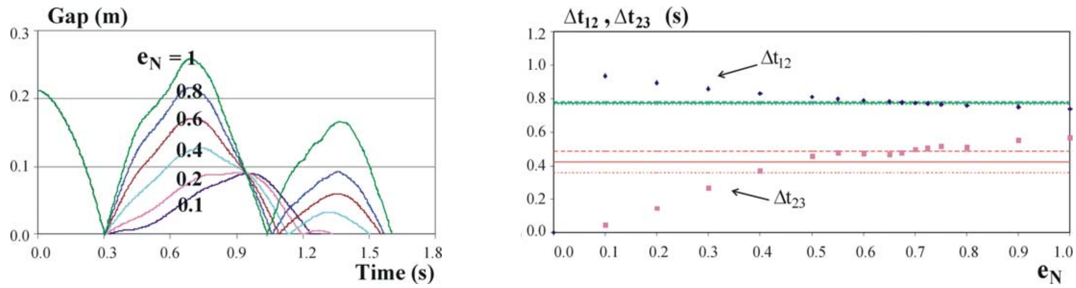
Our study is currently limited to some simple situations. The experimental set-up (Fig. 1) allows for different impact configurations in which the bars are orthogonal, and produces a quasi-direct impact in each case ("direct" means that $\mathbf{v}_T^- = \mathbf{0}$). The first bar, S_1, of length $L_{S_1} = 0.305$ m, is suspended like a pendulum to the framework through a cable of length $L_{C_1} = 0.150$ m. The second bar, S_2, of length $L_{S_2} = 0.610$ m, hangs horizontally from the framework by two identical cables of length $L_{C_2} = 0.594$ m; the whole looks like a trapeze. Cables made up of 7 twisted wires are 1 mm in diameter and have Young's modulus evaluated to 10^{11} MPa; S_1 and S_2 are full cylindrical steel bars 12 mm in diameter.

Tests are performed by releasing S_1, with zero initial velocity, in a plane P orthogonal to body S_2 at rest; the release angle is 45° (Fig. 1 (a) and (b)). Solid S_1, under gravity and cable tension, moves in plane P towards S_2. The impact takes place at S_1 half-length (Fig. 1(c)), whatever is the geometrical variable "d" of the tests; the latter characterizes both the release plane P and the eccentricity of the impact configuration. Each bar is fitted with a piezoelectric accelerometer (4393V Brüel and Kjaer) associated with an amplifier and connected to a signal analyser. The time signals, delivered by the accelerometers during different trial campaigns (d varying), provide qualitative information about the behaviour of the system. This information will be compared to numerical results, obtained with different pairs (e_N, e_T); the steel/steel friction coefficient is given the usual value of 0.15. This procedure aims at establishing either the possibility of identifying a value for the pair (e_N, e_T) or its dependence on the impact eccentricity.

4. Numerical and experimental results

Whatever is d in $[0, L_{S_2}/2]$, numerical simulations show that $|\mathbf{v}_{T\alpha}^-| = 5.7 \times 10^{-2}$ m·s^{-1} is small compared with $|v_{N\alpha}^-| = 1.27$ m·s^{-1}, so that impact may be considered as direct. The value assigned to e_T has little effect on the shock balance: contact impulsion, kinetic energy loss. This has been checked with different values of e_N, in the various collision configurations with eccentricity $d_0 = 0,\ d_1 = 7.5,\ d_2 = 15,\ d_3 = 26.5$ cm: the relative effect on the shock balance is less than 4%. The sensitivity of e_N to the impact eccentricity is studied by considering cases d_0 to d_3. On the accelerograms collected during tests there appear peaks which reveal successive inelastic collisions. Consequently in each case e_N is nonzero.

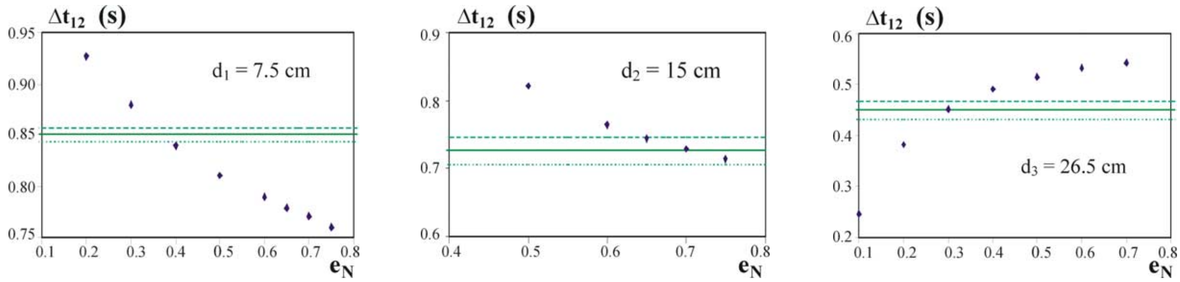
For a release in the $P(d_0)$ plane, numerical tests show that the first two collisions occurring at S_1 and S_2 half-length are (quasi) directs; among the two coefficients of restitution e_T and e_N, only the second one has a substantial



(a) (b)

Fig. 2. Impact of eccentricity d_0: (a) gap, (b) time-intervals between successive collisions.

Fig. 2. Impact d'excentricité d_0 : (a) écart normal, (b) intervalles de temps entre collisions successives.



(a) (b) (c)

Fig. 3. Time-intervals between successive collisions: impact of eccentricity (a) d_1, (b) d_2, (c) d_3.

Fig. 3. Intervalles de temps entre collisions successives : impact d'excentricité (a) d_1, (b) d_2, (c) d_3.

influence on the system motion computed from t_0 to t_{shock3}. Computation results, e_N varying and arbitrarily fixed e_T, are presented on Fig. 2(a) by the time-evolution of the gap between S_1 and S_2. It clearly appears that the lengths Δt_{12} and Δt_{23}, of the time intervals $[t_{\text{shock1}},\ t_{\text{shock2}}]$ and $[t_{\text{shock2}},\ t_{\text{shock3}}]$, vary according to e_N. Moreover, a set of 12 experimental releases was carried out: the measurements of the lengths Δt_{12} and Δt_{23}, presented in terms of average and experimental uncertainty, are compared with the numerical study (Fig. 2(b)). One can thus identify a value of e_N of about 0.65. Concerning the releases in the $P(d_1)$, $P(d_2)$, and $P(d_3)$ planes, simulations point out that the second collision and the following ones are not direct. So, only the Δt_{12} numerical results are compared (see Fig. 3) with the sets of experimental measurements. Figs. 2 and 3 clearly show the dependence of e_N on the eccentricity of impact configurations.

Considering S_2 as a non pre-stressed beam with uniform characteristics and free support conditions, one deduces their flexural vibration modes in plane ($\mathbf{X}$,$\mathbf{n}$) (see Fig. 1) from the classic theory of vibration. Fourier's analysis of the signals delivered by the S_2 accelerometer gave results in keeping with theory: induced by the first collision, acceleration peaks observed (see Fig. 4) at certain eigenfrequencies reveal flexural vibrations of S_2 and thus show that some elastic energy is stored in the solid. It is then dissipated due to the internal damping of the bar. By means of deformation gauges, the quantization of this elastic energy should allow one to examine whether the vibrations of the bars play a crucial role in the energy-loss phenomenon induced by a collision. If so, as one may think according to results presented in [3] and [7], the dependence of e_N on the eccentricity of impact configurations could be related to d-dependent vibrational motions of the bars, in which different flexural modes are excited.

In order to measure the duration of collisions, we realised other series of experiments in which the contact between the bars is used to close an electrical circuit. Experiments (d varying) reveal that the first impact which to

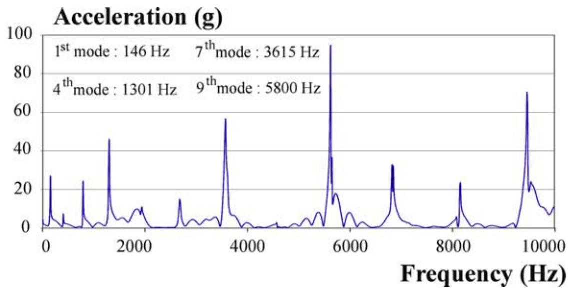


Fig. 4. Vibrational motion of S_2: impact of eccentricity d_0 (accelerometer in $P(d_1)$ plane).

Fig. 4. Vibrations de S_2 : impact d'excentricité d_0 (accéléromètre positionné dans le plan $P(d_1)$).

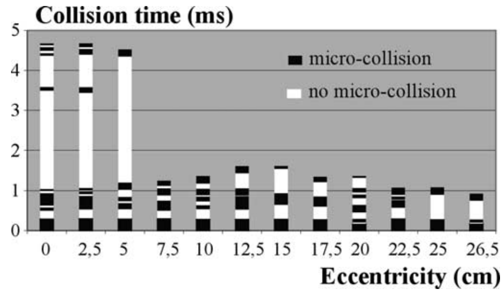


Fig. 5. Collision time and number of micro-collisions versus eccentricity of impact.

Fig. 5. Durée de choc et nombre de micro-collisions selon l'excentricité d'impact.

the naked eye seems to be single, consists in reality of several collisions in quick succession; this phenomenon is related to the flexibility of the bars.

Fig. 5 shows that the collision is realised as groups of micro-impacts, and that the number of such groups varies according to the impact eccentricity. Rigid-body collision theory does not account for the influence of the vibrational motions of the bars on the dynamics of the system, so this theory must be applied carefully and in specific situations.

5. Conclusion

This note deals with the three-dimensional phenomenon of collision between two slender steel bars. We first exhibit the variation of the apparent Newton coefficient of restitution with the impact location on the rods. There appears experimentally that the energy dissipation induced by a shock is not located at the impact but takes place in the whole rods via flexural vibrations; these vibrations are responsible for multiple micro-collisions. These observations induce one to question the relevance of the restitution concept in the case of such slender colliders. An alternative approach accounting for deformations should be implemented. For instance, finite element models or lumped spring-dashpot based models, could be considered, but according to recent works [8] and [9], the definition of a new concept of restitution that incorporates the vibrational loss of energy could also be envisaged; a continuous medium approach might be considered for this purpose. The presented work is a part of a general study on the folding and unfolding of space reticulated systems (LMGC, University of Montpellier II).

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