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Asymptotic analysis for micropolar fluids

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Abstract

The flow of a micropolar fluid through a wavy constricted channel which depends on a small parameter $\varepsilon \ll 1$ is considered. The asymptotic solution is built and justified thanks to a study of the boundary layers terms. The Stokes and Navier–Stokes problems set in a tube structure were previously considered. The method of partial asymptotic decomposition of domain (MAPPD) is also applied and justified for the micropolar flow problem. This method reduces the initial problem to the problem set in the boundary layers domain. ***To cite this article: D. Dupuy et al., C. R. Mecanique 332 (2004).***

Résumé

Analyse asymptotique des fluides micropolaires. Cet article porte sur l'étude de l'écoulement d'un fluide micropolaire à l'intérieur d'un tube, périodiquement ondulé, de période ε, de largeur d'ordre ε et de longueur d'ordre 1. En utilisant une étude similaire à celle effectuée pour des écoulements de Stokes et de Navier–Stokes dans une structure tubulaire, on considère une analyse asymptotique de ce problème. Une solution asymptotique est construite et les termes de couche limite qui apparaissent au voisinage des extrémités sont étudiés. Après justification de cette approche, la méthode de décomposition asymptotique partielle du domaine est mise en place pour ce problème. ***Pour citer cet article : D. Dupuy et al., C. R. Mecanique 332 (2004).***

Keywords: Computational fluid mechanics; Micropolar fluids; Asymptotic expansion; Partial asymptotic domain decomposition

Mots-clés : Mécanique des fluides numérique ; Fluides micropolaires ; Dévéloppement asymptotique ; Décomposition asymptotique partielle du domaine

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0. Introduction

The classical Navier–Stokes theory is not too appropriate for the description of a class of fluids which exhibit certain microscopic effects arising from the local structure and micro-motions of the fluid elements. A new theory was introduced by Eringen in [1]. A subclass of these fluids is the micropolar fluids. Animal blood, liquid crystals and certain polymeric fluids are a few examples of fluids which may be represented by the mathematical model of micropolar fluids, introduced by Eringen in [2]. From the physical point of view, micropolar fluids are characterized by the following property: fluid points contained in a small volume element, in addition to the usual rigid motion, can rotate about the centroid of the volume element in an average sense, the rotation being described by a skew-symmetric gyration tensor, ω.

Due to their importance in industrial and engineering applications, micropolar fluids were studied in several papers such as: [3–8].

This paper deals with the study of steady incompressible flow of a micropolar fluid (animal blood) through a periodically constricted tube with a period and a thickness of order ε. We consider an asymptotic expansion of the solution and we prove its convergence. Then the method of partial asymptotic decomposition of domain (see [9–11]) is applied for the considered problem. This method was previously used for Stokes and Navier–Stokes flows in several papers such as: [12–15].

In Section 1, we describe the physical problem, we prove the existence and the uniqueness of the solution and we obtain *a priori* estimates.

In Section 2, we introduce an asymptotic expansion of the solution. We study the macroscopic problem for the first approximation $(\mathbf{v}^0, \omega^0, p^1)$. Then, in Section 3, we introduce the boundary layers and in Section 4 we estimate the error between the microscopic solution and the macroscopic one. Some details concerning the approximations of higher order are given in Section 5.

The last section deals with the partial asymptotic decomposition of domain applied to our problem. By introducing several functions, we define an asymptotic solution and we establish some estimates. Then, by writing the initial variational problem on a more regular space, we obtain the solution of the partially decomposed problem and we prove that it represents a good approximation for the solution of the considered problem.

1. The physical problem: existence and uniqueness results

We consider a steady ε-periodic incompressible, two-dimensional flow of a micropolar fluid in a periodically constricted tube G^ε, with $\varepsilon = \frac{1}{n}$ (with $n \in \mathbb{N}^\star$) given by

$$G^\varepsilon = \left\{(x_1, x_2) \in \mathbb{R}^2;\ 0 < x_1 < 1 \text{ and } -h_\varepsilon^-(x_1) < x_2 < h_\varepsilon^+(x_1)\right\}$$

with $h_\varepsilon^\pm : \mathbb{R} \to (0, \varepsilon/2)$ two ε-periodic functions of class $\mathcal{C}^2(\mathbb{R})$ and such that there exists $m > 0$: for all $t \in \mathbb{R}$, $h_\varepsilon^\pm(t) \geqslant \varepsilon m$. For the boundaries, we introduce the notations $\Gamma_\varepsilon^\pm = \{(x_1, \pm h_\varepsilon^\pm(x_1)) \in \mathbb{R}^2;\ 0 < x_1 < 1\}$ and $\Sigma^\varepsilon(i) = \overline{G^\varepsilon} \cap \{x_1 = i\}$ (for $i = 0, 1$).

We introduce a new macroscopic variable: $y = \frac{x}{\varepsilon}$, we define $h^\pm(y_1) = \frac{1}{\varepsilon} h_\varepsilon^\pm(\varepsilon y_1)$ and we denote by $Y = \{y \in \mathbb{R}^2\colon\ 0 < y_1 < 1 \text{ and } -h^-(y_1) < y_2 < h^+(y_1)\}$ the cellular domain.

The steady, incompressible, ε-periodic in x_1, 2-D flow of a micropolar fluid with non homogeneous Dirichlet boundary conditions for the velocity, is described by the following coupled system:

$$\begin{cases} (\mathbf{v}^\varepsilon \cdot \nabla)\mathbf{v}^\varepsilon - (\mu + \chi)\Delta \mathbf{v}^\varepsilon + \nabla p^\varepsilon - \chi\, \mathbf{curl}\, \omega^\varepsilon = \mathbf{f} \quad \text{in } G^\varepsilon \\ j\mathbf{v}^\varepsilon \cdot \nabla \omega^\varepsilon - \gamma \Delta \omega^\varepsilon + 2\chi \omega^\varepsilon - \chi\, \mathrm{curl}\, \mathbf{v}^\varepsilon = g \quad \text{in } G^\varepsilon \\ \mathrm{div}\, \mathbf{v}^\varepsilon = 0 \quad \text{in } G^\varepsilon \\ \mathbf{v}^\varepsilon = 0 \quad \text{on } \Gamma_\varepsilon^\pm, \mathbf{v}^\varepsilon = \varepsilon^2 \boldsymbol{\varphi}_\varepsilon \quad \text{on } \Sigma^\varepsilon(i) \quad \text{for } i = 0, 1 \\ \omega^\varepsilon = 0 \quad \text{on } \partial G^\varepsilon \end{cases} \tag{1}$$

with μ, χ, γ, j positive given constants, $\mathbf{f}$, g and $\boldsymbol{\varphi}_\varepsilon$ given functions and $\mathbf{v}^\varepsilon$, ω^ε, p^ε the unknowns: the velocity, the microrotation (which is a scalar function in the 2-D case) and the pressure of the fluid, respectively.

Let $H^1_{\mathrm{div}}(G^\varepsilon)$ be the space of divergence free vector valued functions from $H^1(G^\varepsilon)^2$ equal zero on the boundaries $\Gamma^\pm_\varepsilon$ and $V(G^\varepsilon)$ the subspace of $H^1_{\mathrm{div}}(G^\varepsilon)$ of functions vanishing on the whole boundary. We denote by $W(Y)$ the subspace of functions in $H^1(G^\varepsilon)^2$ with vanishing divergence, equal zero on the boundaries $\Gamma^\pm = \{(y_1, \pm h^\pm(y_1)) \in \mathbb{R}^2;\ 0 < y_1 < 1\}$ and 1-periodic in y_1.

Proposition 1.1. *For $\varepsilon > 0$ sufficiently small, for $\mathbf{f} \in H^{-1}(G^\varepsilon)^2$, $g \in H^{-1}(G^\varepsilon)$ such that $\|\mathbf{f}\|_{H^{-1}(G^\varepsilon)^2}$ and $\|g\|_{H^{-1}(G^\varepsilon)}$ are bounded by a constant which does not depend on ε, and $\boldsymbol{\varphi}_\varepsilon(x) = \boldsymbol{\varphi}(\frac{x}{\varepsilon})$ with $\boldsymbol{\varphi} \in W(Y)$, problem* (1) *admits a unique solution $(\mathbf{v}^\varepsilon, p^\varepsilon, \omega^\varepsilon)$ in $H^1_{\mathrm{div}}(G^\varepsilon) \times L^2(G^\varepsilon)/\mathbb{R} \times H^1_0(G^\varepsilon)$.*

2. Asymptotic expansion

In the sequel, we suppose that $\mathbf{f} = (f_1(x_1), 0)^t$ (where f_1 belongs to $L^2(0,1)$) and g is a constant. We seek the formal asymptotic expansion for the solution of problem (1) in the form:

$$\mathbf{v}^\varepsilon(x) = \varepsilon^2 \sum_{l=0}^{+\infty} \varepsilon^l \mathbf{v}^l(x_1, y), \quad p^\varepsilon(x) = \sum_{l=0}^{+\infty} \varepsilon^l p^l(x_1, y) \quad \text{and} \quad \omega^\varepsilon(x) = \varepsilon^2 \sum_{l=0}^{+\infty} \varepsilon^l \omega^l(x_1, y)$$

with $\mathbf{v}^l$, ω^l and p^l 1-periodic functions in y_1.

Introducing these expansions in (1), we obtain $p^0 = p^0(x_1)$ and for the first approximation $(\mathbf{v}^0, p^0, p^1, \omega^0)$, the following problems:

$$\begin{cases} -(\mu+\chi)\Delta_y \mathbf{v}^0 + \nabla_y p^1 = \mathbf{f}(x_1) - \dfrac{\mathrm{d}p^0}{\mathrm{d}x_1}\mathbf{e}_1 & \text{in } (0,1)\times Y \\ \mathrm{div}_y\, \mathbf{v}^0 = 0 \quad \text{in } (0,1)\times Y \\ \mathbf{v}^0 = 0 \quad \text{on } (0,1)\times \Gamma^\pm, \quad \mathbf{v}^0 \text{ and } p^1 \text{ 1-periodic in } y_1 \\ \int_Y \big(\mathbf{v}^0\big)_1 (x_1, y)\,\mathrm{d}y = c_0 \end{cases} \tag{2}$$

and

$$\begin{cases} -\gamma \Delta_y \omega^0 = g \quad \text{in } (0,1)\times Y \\ \omega^0 = 0 \quad \text{on } (0,1)\times \Gamma^\pm, \quad \omega^0 \text{ 1-periodic in } y_1 \end{cases} \tag{3}$$

where $(\cdot)_1$ represents the first component. The condition $(2)_4$ is a consequence of the compatibility condition for the Stokes problem satisfied by the following term $(\mathbf{v}^1, p^2)$ of the asymptotic expansion, hence c_0 will be a constant which does not depend on the variable x_1.

Remark 1. The constant c_0 is not arbitrary but it will be defined later thanks to the condition imposed in order to construct boundary layers which decay exponentially to zero at infinity.

We first prove the following result:

Theorem 2.1. *For any $c_0 \in \mathbb{R}$, there exists $p^0 \in L^2(0,1)/\mathbb{R}$ such that problem* (4) *admits an unique solution $(\mathbf{v}^0, p^1)$ in $L^2(0,1; W(Y)) \times L^2(0,1; L^2(Y)/\mathbb{R})$. Moreover, problem* (3) *has also a unique solution ω^0 in $L^2(0,1; H^1(Y))$.*

Remark 2. In the proof of this theorem, we get $\mathbf{v}^0 = \mathbf{v}^0(y)$, $p^1 = p^1(y)$ and $\omega^0 = \omega^0(y)$.

To compensate the trace of $\mathbf{v}^0(\frac{x}{\varepsilon})$ and $\omega^0(\frac{x}{\varepsilon})$ on $\Sigma^\varepsilon(i)$ (for $i=0,1$), we introduce boundary layers terms. For this purpose, we define two semi-infinite wavy strips: $G_0=[\bigcup_{l=0}^{+\infty}(\overline{Y}+l\mathbf{e}_1)]'$ and $G_1=[\bigcup_{l=1}^{+\infty}(\overline{Y}-l\mathbf{e}_1)]'$ where for any set A, $\bar{A}$ is the closure and A' is the set of the interior points. Denote by $S_0^\pm=\{(y_1,\pm h^\pm(y_1))\in\mathbb{R}^2;\ y_1\in(0,+\infty)\}$, $S_1^\pm=\{(y_1,\pm h^\pm(y_1))\in\mathbb{R}^2;\ y_1\in(-\infty,0)\}$ and $\Sigma_i=\overline{G}_i\cap\{y_1=0\}$ (for $i=0,1$) the boundaries of these domains.

3. The boundary layers

For $i=0,1$, we consider the exponentially decaying solution of problem:

$$\begin{cases}-(\mu+\chi)\Delta\mathbf{v}_{bli}^0+\nabla p_{bli}^1=0, \quad \operatorname{div}\mathbf{v}_{bli}^0=0 \quad \text{in } G_i\\ \mathbf{v}_{bli}^0=0 \quad \text{on } S_i^\pm, \quad \mathbf{v}_{bli}^0+\mathbf{v}^0=\boldsymbol{\varphi} \quad \text{on } \Sigma_i\end{cases} \tag{4}$$

Suppose that $c_0=\int_{-h^-(0)}^{h^+(0)}(\boldsymbol{\varphi})_1(0,y_2)\,\mathrm{d}y_2$, then problem (4) has a unique solution $(\mathbf{v}_{bli}^0,p_{bli}^1)\in H_{\mathrm{div}}^1(G_i)\times L_{\mathrm{loc}}^2(G_i)/\mathbb{R}$ with $\mathbf{v}_{bli}^0$ decreasing exponentially to zero at infinity, in the sense that: for all $R>0$, the following estimate holds: $\|\mathbf{v}_{bli}^0\|_{H^1(G_i\cap\{|y_1|>R\})^2}\leqslant\sigma\exp(-\lambda R)$ where σ and λ are positive constants (see, e.g., [16]).

For $i=0,1$, let $\omega_{bli}^0\in H^1(G_i)$ be the unique solution of the problem:

$$\begin{cases}-\gamma\Delta\omega_{bli}^0=0 \quad \text{in } G_i\\ \omega_{bli}^0=0 \quad \text{on } S_i^\pm, \quad \omega_{bli}^0+\omega^0=0 \quad \text{on } \Sigma_i\end{cases}$$

This solution satisfies the estimate: for all $R>0$, $\|\omega_{bli}^0\|_{H^1(G_i\cap\{|y_1|>R\})}\leqslant\sigma\exp(-\lambda R)$ where σ and λ are positive constants.

4. Convergence of the asymptotic expansion

We define , for $x\in G^\varepsilon$, the functions:

$$\mathbf{v}^{(0)}(x)=\varepsilon^2\left\{\mathbf{v}^0\left(\frac{x}{\varepsilon}\right)+\mathbf{v}_{bl0}^0\left(\frac{x}{\varepsilon}\right)+\mathbf{v}_{bl1}^0\left(\frac{x}{\varepsilon}-\frac{1}{\varepsilon}\mathbf{e}_1\right)\right\}$$

$$p^{(1)}(x)=p^0(x_1)+\varepsilon\left\{p^1\left(\frac{x}{\varepsilon}\right)+p_{bl0}^1\left(\frac{x}{\varepsilon}\right)+p_{bl1}^1\left(\frac{x}{\varepsilon}-\frac{1}{\varepsilon}\mathbf{e}_1\right)\right\}$$

$$\omega^{(0)}(x)=\varepsilon^2\left\{\omega^0\left(\frac{x}{\varepsilon}\right)+\omega_{bl0}^0\left(\frac{x}{\varepsilon}\right)+\omega_{bl1}^0\left(\frac{x}{\varepsilon}-\frac{1}{\varepsilon}\mathbf{e}_1\right)\right\}$$

The function $\mathbf{v}^{(0)}$ belongs to $H_{\mathrm{div}}^1(G^\varepsilon)$ but does not verify the second condition of $(1)_4$ and $\omega^{(0)}$ does not vanish on $\Sigma^\varepsilon(i)$ for $i=0,1$. However, they can be modified by adding a function $\boldsymbol{\mathcal{D}}^\varepsilon\in H_{\mathrm{div}}^1(G^\varepsilon)^2$ such that $\mathbf{v}^{(0)}-\boldsymbol{\mathcal{D}}^\varepsilon=\varepsilon^2\boldsymbol{\varphi}_\varepsilon$ on ∂G^ε and a function $\mathcal{C}^\varepsilon\in H^1(G^\varepsilon)$ such that $\omega^{(0)}-\mathcal{C}^\varepsilon=0$ on ∂G^ε. Moreover, these functions satisfy the estimates $\|\boldsymbol{\mathcal{D}}^\varepsilon\|_{H^1(G^\varepsilon)^2}\leqslant\varepsilon^2\sigma\exp(-\lambda/\varepsilon)$ and $\|\mathcal{C}^\varepsilon\|_{H^1(G^\varepsilon)}\leqslant\varepsilon^2\sigma\exp(-\lambda/\varepsilon)$ where σ and λ are two positive constants which do not depend on ε.

Denote by $\mathbf{v}_a^\varepsilon=\mathbf{v}^{(0)}-\boldsymbol{\mathcal{D}}^\varepsilon$ and $\omega_a^\varepsilon=\omega^{(0)}-\mathcal{C}^\varepsilon$.

Theorem 4.1. *For $f_1\in L^2(0,1)$, the following estimates hold*:

$$\begin{cases}\|\mathbf{v}^\varepsilon-\mathbf{v}_a^\varepsilon\|_{H^1(G^\varepsilon)^2}\leqslant C\varepsilon^{5/2}\\ \|\omega^\varepsilon-\omega_a^\varepsilon\|_{H^1(G^\varepsilon)}\leqslant C\varepsilon^{5/2}\end{cases}$$

where C is a positive constant which does not depend on ε.

5. The higher order approximations

The higher order approximations can be written in the following general form: for $k \geqslant 2$, we seek the solution $(\mathbf{v}^{k-1}, p^k, \omega^{k-1})$ of problems

$$\begin{cases} -(\mu+\chi)\Delta_y \mathbf{v}^{k-1} + \nabla_y p^k = \mathbf{f}\big(\mathbf{v}^{i-1}, p^i, \omega^{i-1}\big)\ (i<k), \quad \operatorname{div}_y \mathbf{v}^{k-1} = -\dfrac{\partial}{\partial x_1}\big(\mathbf{v}^{k-2}\big)_1 \quad \text{in } (0,1)\times Y \\ \mathbf{v}^{k-1} = 0 \quad \text{on } (0,1)\times \Gamma^{\pm}, \quad \mathbf{v}^{k-1} \text{ and } p^k \text{ 1-periodic in } y_1, \quad \displaystyle\int_Y \big(\mathbf{v}^{k-1}\big)_1(x_1,y)\,\mathrm{d}y = c_{k-1} \end{cases}$$

and

$$\begin{cases} -\gamma\Delta_y \omega^{k-1} = g\big(\mathbf{v}^{i-1}, \omega^{i-1}\big)\ i<k \quad \text{in } Y\times(0,1) \\ \omega^{k-1} = 0 \quad \text{on } \Gamma^{\pm}\times(0,1), \quad \omega^{k-1} \text{ 1-periodic in } y_1 \end{cases}$$

where $\mathbf{f}$ and g are some functions.

Remark 3. It can be proved recursively that $p^k = h_1^k(y) + h_2^k(x_1)$ for all $k \geqslant 2$. Then we obtain $\operatorname{div}_y \mathbf{v}^{k-1} = 0$ and the last condition for the velocity is satisfied.

6. The method of asymptotic partial decomposition of domain

This method for variational problems was introduced in [11]. In this section, we shall apply the ideas of this paper for the variational problem obtained from (1):

$$\begin{cases} \text{Find } (\mathbf{u}^\varepsilon, \omega^\varepsilon) \in V\big(G^\varepsilon\big)\times H_0^1\big(G^\varepsilon\big) \text{ such that} \\ (\mu+\chi)\displaystyle\int_{G^\varepsilon} \nabla\mathbf{u}^\varepsilon\cdot\nabla\mathbf{z} + \int_{G^\varepsilon} (\mathbf{u}^\varepsilon\cdot\nabla)\mathbf{u}^\varepsilon\mathbf{z} + \varepsilon^2\Big\{\int_{G^\varepsilon} (\boldsymbol{\varphi}_\varepsilon\cdot\nabla)\mathbf{u}^\varepsilon\mathbf{z} + \int_{G^\varepsilon} (\mathbf{u}^\varepsilon\cdot\nabla)\boldsymbol{\varphi}_\varepsilon\mathbf{z}\Big\} - \chi\int_{G^\varepsilon} \mathbf{curl}\,\omega^\varepsilon\cdot\mathbf{z} \\ \quad = \displaystyle\int_{G^\varepsilon} \mathbf{f}\cdot\mathbf{z} - \varepsilon^4\int_{G^\varepsilon} (\boldsymbol{\varphi}_\varepsilon\cdot\nabla)\boldsymbol{\varphi}_\varepsilon\mathbf{z} + \varepsilon^2(\mu+\chi)\int_{G^\varepsilon} \nabla\boldsymbol{\varphi}_\varepsilon\cdot\nabla\mathbf{z} \\ \gamma\displaystyle\int_{G^\varepsilon} \nabla\omega^\varepsilon\cdot\nabla\rho + j\int_{G^\varepsilon} \mathbf{u}^\varepsilon\cdot\nabla\omega^\varepsilon\rho + j\varepsilon^2\int_{G^\varepsilon} \boldsymbol{\varphi}_\varepsilon\cdot\nabla\omega^\varepsilon\rho + 2\chi\int_{G^\varepsilon}\omega^\varepsilon\rho - \chi\int_{G^\varepsilon} \operatorname{curl}\mathbf{u}^\varepsilon\rho \\ \quad = \displaystyle\int_{G^\varepsilon} g\rho + \varepsilon^2\chi\int_{G^\varepsilon} \operatorname{curl}\boldsymbol{\varphi}_\varepsilon\rho \\ \quad \text{for all } \mathbf{z}\in V\big(G^\varepsilon\big) \text{ and for all } \rho\in H_0^1\big(G^\varepsilon\big) \end{cases} \tag{5}$$

We introduce another parameter $\delta = K\varepsilon[|\ln\varepsilon|]$, with some finite $K \in \mathbb{N}^\star$.

Let $V_{\mathrm{dec}}^{\delta,\varepsilon}(G^\varepsilon)$ and $H_{\mathrm{dec}}^{\delta,\varepsilon}(G^\varepsilon)$ be the spaces

$$V_{\mathrm{dec}}^{\delta,\varepsilon}\big(G^\varepsilon\big) = \Big\{\mathbf{u}\in V\big(G^\varepsilon\big)\colon \mathbf{u}(x) = \varepsilon^2 c\Big[\mathbf{v}^0\Big(\frac{x}{\varepsilon}\Big) - \boldsymbol{\varphi}_\varepsilon(x)\Big],\ c\in\mathbb{R},\ \text{if } x_1\in(\delta,1-\delta)\Big\}$$

$$H_{\mathrm{dec}}^{\delta,\varepsilon}\big(G^\varepsilon\big) = \Big\{\zeta\in H_0^1\big(G^\varepsilon\big)\colon \zeta(x) = \varepsilon^2 c\,\omega^0\Big(\frac{x}{\varepsilon}\Big),\ c\in\mathbb{R},\ \text{if } x_1\in(\delta,1-\delta)\Big\}$$

The partially decomposed solution $(\mathbf{v}_d^\varepsilon, \omega_d^\varepsilon)$ is defined by $\mathbf{v}_d^\varepsilon = \mathbf{u}_d^\varepsilon + \varepsilon^2\boldsymbol{\varphi}_\varepsilon$, where $(\mathbf{u}_d^\varepsilon, \omega_d^\varepsilon)$ is the unique solution of the following variational problem:

$$
\left\{
\begin{array}{l}
\text{Find } (\mathbf{u}_d^\varepsilon, \omega_d^\varepsilon) \in V_{\text{dec}}^{\delta,\varepsilon}(G^\varepsilon) \times H_{\text{dec}}^{\delta,\varepsilon}(G^\varepsilon) \text{ such that} \\
(\mu+\chi)\displaystyle\int_{G^\varepsilon} \nabla \mathbf{u}_d^\varepsilon \cdot \nabla \mathbf{z} + \int_{G^\varepsilon} (\mathbf{u}_d^\varepsilon \cdot \nabla)\mathbf{u}_d^\varepsilon \mathbf{z} + \varepsilon^2 \left\{ \int_{G^\varepsilon} (\boldsymbol{\varphi}_\varepsilon \cdot \nabla)\mathbf{u}_d^\varepsilon \mathbf{z} + \int_{G^\varepsilon} (\mathbf{u}_d^\varepsilon \cdot \nabla)\boldsymbol{\varphi}_\varepsilon \mathbf{z} \right\} - \chi \int_{G^\varepsilon} \mathbf{curl}\, \omega_d^\varepsilon \cdot \mathbf{z} \\
\quad = \displaystyle\int_{G^\varepsilon} \mathbf{f} \cdot \mathbf{z} - \varepsilon^4 \int_{G^\varepsilon} (\boldsymbol{\varphi}_\varepsilon \cdot \nabla)\boldsymbol{\varphi}_\varepsilon \mathbf{z} + \varepsilon^2(\mu+\chi) \int_{G^\varepsilon} \nabla \boldsymbol{\varphi}_\varepsilon \cdot \mathbf{z} \\
\gamma \displaystyle\int_{G^\varepsilon} \nabla \omega_d^\varepsilon \cdot \nabla \rho + j \int_{G^\varepsilon} \mathbf{u}_d^\varepsilon \cdot \nabla \omega_d^\varepsilon \rho + j\varepsilon^2 \int_{G^\varepsilon} \boldsymbol{\varphi}_\varepsilon \cdot \nabla \omega_d^\varepsilon \rho + 2\chi \int_{G^\varepsilon} \omega_d^\varepsilon \rho - \chi \int_{G^\varepsilon} \operatorname{curl} \mathbf{u}_d^\varepsilon \rho \\
\quad = \displaystyle\int_{G^\varepsilon} g\rho + \varepsilon^2 \chi \int_{G^\varepsilon} \operatorname{curl} \boldsymbol{\varphi}_\varepsilon \rho \\
\quad \text{for all } \mathbf{z} \in V_{\text{dec}}^{\delta,\varepsilon}(G^\varepsilon) \text{ and for all } \rho \in H_{\text{dec}}^{\delta,\varepsilon}(G^\varepsilon)
\end{array}
\right.
\tag{6}
$$

Theorem 6.1. *If* $(\mathbf{u}^\varepsilon, \omega^\varepsilon)$ *is the solution of problem* (5) *and* $(\mathbf{v}_d^\varepsilon, \omega_d^\varepsilon)$ *the partially decomposed solution defined by problem* (6)*, then, for sufficiently small* ε:

$$
\left\{
\begin{array}{l}
\|\mathbf{v}^\varepsilon - \mathbf{v}_d^\varepsilon\|_{H^1(G^\varepsilon)^2} \leqslant C\varepsilon^{5/2} \\
\|\omega^\varepsilon - \omega_d^\varepsilon\|_{H^1(G^\varepsilon)} \leqslant C\varepsilon^{5/2}
\end{array}
\right.
$$

where $\mathbf{v}^\varepsilon = \mathbf{u}^\varepsilon + \varepsilon^2 \boldsymbol{\varphi}_\varepsilon$ *and* C *is a positive constant independent of* ε.

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