Available online at www.sciencedirect.com
SCIENCE DIRECT®
ELSEVIER
C. R. Mecanique 332 (2004) 43–49
COMPTES RENDUS MECANIQUE

On the existence of the low-frequency surface waves in a porous medium

Inna Edelman [1]

Weierstrass Institute for Applied Analysis and Stochastics, Mohrenstr. 39, 10117 Berlin, Germany

Received 26 June 2003; accepted after revision 21 November 2003

Presented by Évariste Sanchez-Palencia

Abstract

The existence and propagation of the surface waves at a vacuum/porous medium interface are investigated in the low frequency range. Two types of surface waves are shown to be possible: the generalized Rayleigh wave, which always exists, and the Stoneley wave, which exists for a limited range of wave numbers. Moreover, within the k-domain of existence the Stoneley wave cannot appear for certain values of elastic parameters of the solid phase. The bifurcation behavior of both the Stoneley wave and the Biot (P2) bulk wave, depending on the wave number, is revealed. The asymptotic formulas for the phase velocities of the surface waves are derived. ***To cite this article: I. Edelman, C. R. Mecanique 332 (2004).***

Résumé

Sur l'existence des ondes de surface basse fréquence en milieux poreux. La nature et la propagation des ondes de surface engendrées par la surface libre d'un milieu poreux sont étudiées en basse fréquence : nous mettons en évidence deux types d'ondes de surface : l'onde de Rayleigh et l'onde de Stoneley. Cette dernière existe pour une gamme limitée de nombres d'onde. Le comportement de bifurcation de l'onde de Stoneley et de l'onde lente de Biot (P2) dépendant du nombre d'onde est mis en évidence. Il est aussi prouvé qu'à l'intérieur du domaine d'existence du nombre d'onde, l'onde de Stoneley ne peut pas apparaître pour certaines valeurs de paramètres élastiques de la phase solide. Les formules asymptotiques des vitesses de phase des ondes de surface sont également présentées. ***Pour citer cet article : I. Edelman, C. R. Mecanique 332 (2004).***

Keywords: Porous media; Waves; Asymptotic analysis; Bifurcation; Interface

Mots-clés : Milieux poreux ; Ondes ; Analyse asymptotique ; Bifurcation ; Interface

E-mail address: inna_edelman@mail.ru (I. Edelman).
[1] Permanent address: Russian Academy of Sciences, United Institute of Physics of the Earth, B.Gruzinskaya 10, Moscow 123242, Russia.

1631-0721/$ – see front matter
doi:10.1016/j.crme.2003.11.004

1. Introduction

Lord Rayleigh discovered that at a free interface of an isotropic elastic half-space there is only one surface wave – the Rayleigh wave [1]. This wave is a nondispersive plane inhomogeneous wave, undamped in its direction of propagation along the surface, and damped normal to the boundary. At a solid-liquid interface two types of surface waves may exist: the true Stoneley surface wave [2,3], propagating parallel to the boundary without attenuation and being exponentially damped in both directions away from the interface; its velocity is lower than all the bulk velocities in the solid and in the liquid, and the generalized Rayleigh wave with a velocity higher than the wave velocity in the fluid; it is attenuated along the propagation direction by 'leaking' energy into the fluid [3]. For many years, the analysis of surface waves has been widely developed. However, very few studies have dealt with a poroelastic solid (details of the current state of art can be found in [4,5]). These works are based on the high frequency limit of the classical Biot model for fluid-saturated porous medium [6]. The model describes three bulk waves in an unbounded fluid-saturated medium: shear (S), fast longitudinal (P1), and slow longitudinal (P2 or the Biot) waves. The shear wave and the longitudinal wave of the first kind (P1) are similar to the waves in an ordinary single phase, isotropic elastic medium. Biot found that in addition to the usual elastic waves there exists longitudinal wave of the second kind (P2) which is propagatory at high frequencies. It is obvious that due to the presence of the P2 wave in a saturated porous medium, the number and properties of surface waves at interfaces of porous solid in contrast to interfaces of elastic solid should be different.

The focus of this work is on the research of the existence and asymptotic behavior of the surface modes at the free interface of a porous medium in the low frequency range. The asymptotic analysis presented in the paper is based on the mathematical model of saturated poroelastic materials, proposed in [7]. As in the classical Biot model, it describes three bulk waves in an unbounded fluid-saturated medium (see [8] for the comparison of the models). The existence of two surface modes are proven to be possible: the true Stoneley wave and the generalized Rayleigh wave. The behavior of the surface modes depend crucially on the properties of the bulk waves. In the high frequency range there are not peculiarities in propagation of both bulk and surface waves: the velocities of bulk waves are almost constant and the true Stoneley and the generalized Rayleigh waves spread with speeds somewhat less than those of P2 and shear waves, respectively [9,5]. However, in the low frequency range the Biot slow wave is not always propagatory: it is fully attenuated below some critical wave number k_{cr} which depends on the permeability of the media and viscosity of the fluid [10,8]. This critical wave number is the bifurcation point, above which the P2 wave begins to propagate. Because of this complicated behavior of the P2 wave, the properties of low-frequency surface modes are different in comparison with the high frequency range. In the low frequency range, similar to the P2 wave, the Stoneley surface mode possesses a bifurcation in the vicinity of k_{cr}. Moreover, within the k-domain of existence, the Stoneley wave cannot appear for certain values of elastic parameters of the solid phase. Also there exists the generalized Rayleigh wave, which propagates almost without attenuation.

In should be noted that we investigate the propagation of the acoustic bulk waves through an infinite porous medium in the absence of external forces. In this case one must set the wave number k to be real and define the frequency $\omega = \omega(k)$, which can be complex, as a solution of the corresponding dispersion equation, i.e., we solve the initial value problem.

2. The mathematical model

Let an infinite space Ω be occupied by a saturated porous medium. In dimensionless variables the balance equations describing the porous two-phase medium has the following general form ($x \in \Omega,\ t \in [0, T]$) [7,5]:

Mass conservation equations

$$\partial\rho^F/\partial t + \operatorname{div}\big(\rho^F \mathbf{v}^F\big) = 0, \qquad \partial\rho^S/\partial t + \operatorname{div}\big(\rho^S \mathbf{v}^S\big) = 0 \tag{1}$$

Here, ρ is the mass density, $\mathbf{v}$ is the velocity vector and indices F and S indicate fluid or solid phases, respectively. The partial mass densities of the solid and fluid phases ρ^S and ρ^F are connected with the true mass densities ρ^{SR} and ρ^{FR} by the relations: $\rho^S = (1-n)\rho^{SR}$, $\rho^F = n\rho^{FR}$, where n denotes the porosity.

Momentum conservation equations

$$\rho^F\big[\partial/\partial t + \big(v_j^F, \partial/\partial x_j\big)\big]v_i^F - \partial T_{ij}^F/\partial x_j + \pi\big(v_i^F - v_i^S\big) = 0 \tag{2}$$

$$\rho^S\big[\partial/\partial t + \big(v_j^S, \partial/\partial x_j\big)\big]v_i^S - \partial T_{ij}^S/\partial x_j - \pi\big(v_i^F - v_i^S\big) = 0 \tag{3}$$

Here $\mathbf{T}^F$ and $\mathbf{T}^S$ are the partial stress tensors and $\pi = n_0\mu^F/\mathcal{K}$, where n_0 is the initial value of porosity, μ^F is the viscosity of a liquid, $\mathcal{K}$ is the permeability of a porous medium.

Balance equation of porosity

$$\partial n/\partial t + \big(v_i^S, \partial/\partial x_i\big)n + n_0\,\mathrm{div}\big(\mathbf{v}^F - \mathbf{v}^S\big) = -(n - n_0) \tag{4}$$

Constitutive relations for linear poroelastic materials

$$\mathbf{T}^F = -p^F\mathbf{1} - \beta(n-n_0)\mathbf{1}, \qquad p^F = p_0^F + \kappa\big(\rho^F - \rho_0^F\big) \tag{5}$$

$$\mathbf{T}^S = \mathbf{T}_0^S + \lambda^S\,\mathrm{div}\,\mathbf{u}^S\mathbf{1} + 2\mu^S\,\mathrm{sym\,grad}\,\mathbf{u}^S + \beta(n-n_0)\mathbf{1} \tag{6}$$

Here p^F is the pore pressure; p_0^F and ρ_0^F are the initial values of pore pressure and fluid mass density, respectively; κ is the constant material parameter; β is the coupling coefficient of the components; $\mathbf{T}_0^S$ is a constant reference value of the partial stress tensor in the skeleton; λ^S and μ^S are the Lamé constants; $\mathbf{u}^S$ is the displacement vector for the solid phase: $\mathbf{v}^S = \partial\mathbf{u}^S/\partial t$.

3. Bifurcation of the Biot slow wave at low frequencies

Prior to exploring the existence of the surface modes, let us examine the propagation of the bulk waves through an unbounded fluid-filled porous medium. We focus on the Biot slow wave. To investigate the propagation of the *longitudinal* waves it suffices to study the 1D problem. Consider the propagation of the harmonic waves whose frequency is ω and wave number is k. Following standard procedure one obtains the dispersion equation [10,8]:

$$r\big(\omega^2 - c_f^2k^2\big)\big(\omega^2 - k^2\big) + \mathrm{i}\omega\pi\big((1+r)\omega^2 - k^2\big(1 + rc_f^2\big)\big) = 0 \tag{7}$$

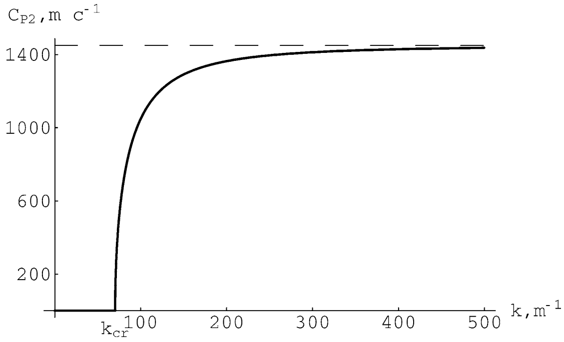
where $r = \rho_0^F/\rho_0^S$ and $c_f = \sqrt{\kappa}/\sqrt{(\lambda^S + 2\mu^S)/\rho_0^S}$. If k is sufficiently large (high frequency range), the root of (7), corresponding to the P2 wave, has the form (note that below $\widetilde{\omega} = \omega/k$ and $\tilde{k} = k/\pi$):

$$\widetilde{\omega}_{P2} = \pm c_f - \frac{\mathrm{i}}{2r}\frac{1}{\tilde{k}} - \frac{1 - c_f^2(1+4r)}{8r^2(1-c_f^2)(\pm c_f)}\frac{1}{\tilde{k}^2} + \mathrm{O}\left(\frac{1}{\tilde{k}^3}\right) \tag{8}$$

It defines the velocity and attenuation of the forward- and backward-directed P2 waves. If k is small enough (low frequency range), one obtains for the forward- and backward-directed P2 waves, respectively:

$$\widetilde{\omega}_{P2}^f = -\mathrm{i}\frac{rc_f^2}{1+rc_f^2}\tilde{k} - \mathrm{i}\frac{r^3c_f^4(1+rc_f^4)}{(1+rc_f^2)^4}\tilde{k}^3 + \mathrm{O}\big(\tilde{k}^4\big), \qquad \widetilde{\omega}_{P2}^b = -\mathrm{i}\frac{1+r}{r}\frac{1}{\tilde{k}} + \mathrm{i}\frac{r(r+c_f)}{(1+r)^2}\tilde{k} + \mathrm{O}\big(\tilde{k}^2\big) \tag{9}$$

The expansions (9) consist of the imaginary terms only, i.e., the phase velocity of the P2 wave equals zero and the wave is fully attenuated. However, the asymptotics (9) are valid only if the wave number k is less then some critical value k_{cr}: k_{cr} is the bifurcation point in a small neighborhood of which the corresponding solution of



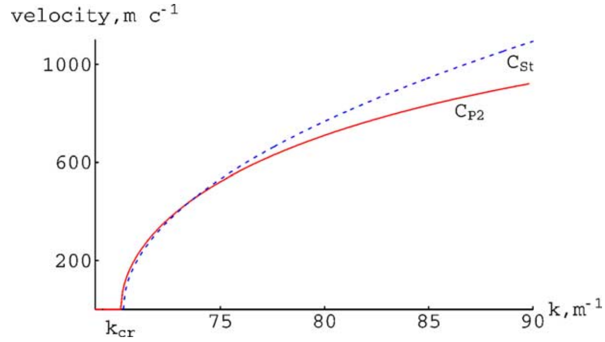


Fig. 1. Velocity of the P2 wave (left-hand side) and velocities of the Stoneley (dashed line) and P2 (solid line) waves (right-hand side), evaluated in physical variables: $k_{cr} \approx 70.22\ m^{-1}$ (water-saturated sandstone [8]).

equation (7) splits into several branches. The critical wave number k_{cr} and the pertinent critical frequency are defined asymptotically:

$$k_{cr} \approx c_f\bigl(1 + 1/\bigl(2rc_f^2\bigr)\bigr)\pi, \qquad \omega_{cr} = -i\pi\Omega_{cr}, \qquad \Omega_{cr} \approx 1/(2r) + 2c_f^2\bigl(1 + 3rc_f^2 - 2c_f^2\bigr) \tag{10}$$

If $k > k_{cr}$, then the P2 mode becomes propagatory. For any small parameter ϵ and wave number $k = k_{cr}(1+\epsilon^2 k_2) + \pi O(\epsilon^3)$ the asymptotic expansion for its frequency has the form:

$$\omega_{P2} = \omega_{cr} + \varepsilon\omega_1 + \pi O\bigl(\epsilon^2\bigr) \qquad \omega_1 = 2k_{cr}\sqrt{k_2/\mathcal{A}} \tag{11}$$

$$\mathcal{A} = \frac{1+c_f^2}{c_f^2} + \frac{1-c_f^2}{c_f^2 g(\Omega_{cr})\sqrt{g(\Omega_{cr})}}\bigl(-r^3\bigl(1-c_f^2\bigr)^3\Omega_{cr}^3 + 3r^2\bigl(1-c_f^2\bigr)^2\bigl(1-rc_f^2\bigr)\Omega_{cr}^2$$
$$- 3r\bigl(1-c_f^2\bigr)\bigl(1+r^2c_f^4\bigr)\Omega_{cr} + \bigl(1-rc_f^2\bigr)\bigl(1+rc_f^2\bigr)^2\bigr) > 0 \tag{12}$$

$$g(\Omega) = \Omega^2 r^2\bigl(1-c_f^2\bigr)^2 - 2r\Omega\bigl(1-c_f^2\bigr)\bigl(1-rc_f^2\bigr) + \bigl(1+rc_f^2\bigr)^2. \tag{13}$$

Thus, the velocity of the forward-directed P2 wave in the low frequency range (more precisely, in a small neighborhood of the bifurcation point) equals $\mathrm{Re}(\omega_{P2})/k$. The bifurcation point describes the transition from the low to high frequency regimes of the propagation of the P2 mode. The asymptotic formula $(10)_1$ demonstrates that k_{cr} is directly proportional to the parameter π and, consequently, the Biot wave behavior is dominated by the permeability of a medium. In Fig. 1 (left-hand side) the phase velocity of the P2 wave is evaluated in physical variables for water-saturated sandstone (see example in [8]).

4. Surface waves

Consider two semi-infinite spaces, Ω^- and Ω^+, having a common interface Γ. Let the region Ω^- be occupied by a saturated porous medium and the region Ω^+ be occupied by the vacuum. System (1)–(4) describes the porous medium ($x \in \Omega^-,\ t \in [0, T]$). Further, we linearize (1)–(4) about some equilibrium state $\rho^F = \rho_0^F,\ \rho^S = \rho_0^S,\ \mathbf{v}^F = \mathbf{0},\ \mathbf{v}^S = \mathbf{0},\ n = n_0$ and introduce the displacement vector for the fluid phase $\mathbf{u}^F$. We investigate 2D problem (xy plane) of the propagation of the surface waves along an interface of a porous medium, which occupies the semi-infinite space $y > 0$ (region Ω^-) and is bounded by the vacuum, which fills the semi-infinite space $y < 0$ (region Ω^+). On the interface $y = 0$, separating the porous medium and the vacuum, the following linearized boundary conditions have to be satisfied: the relative normal velocity must be equal to zero and the total stress vector must vanish:

$$\partial\big(u_2^F - u_2^S\big)/\partial t|_{y=0} = 0, \qquad \big(\partial u_1^S/\partial y + \partial u_2^S/\partial x\big)|_{y=0} = 0 \tag{14}$$

$$\big(\lambda^S \operatorname{div}\mathbf{u}^S + 2\mu^S \partial u_2^S/\partial y - \kappa\big(\rho^F - \rho_0^F\big)\big)|_{y=0} = 0 \tag{15}$$

Our goal is to prove that the boundary value problem (1)–(4), (15) and (16) has solutions in the form of surface waves, i.e., solutions which decrease sufficiently fast as $|y| \to \infty$. For this purpose we investigate the propagation of a harmonic wave whose wave number is $k \in \mathbb{R}^1$, frequency is $\omega = \omega(k)$, and whose amplitude depends on y. Thus, $\mathrm{Re}(\omega/k)$ defines the phase velocity of waves, while $\mathrm{Im}(\omega)$ defines the attenuation.

A solution in the region Ω^- is sought in the following form:

$$\mathbf{u}^F = \operatorname{grad}\varphi^F + \operatorname{rot}\boldsymbol{\Psi}^F, \quad \mathbf{u}^S = \operatorname{grad}\varphi^S + \operatorname{rot}\boldsymbol{\Psi}^S \tag{16}$$

where $\boldsymbol{\Psi}^F = (0, 0, \psi^F)$, $\boldsymbol{\Psi}^S = (0, 0, \psi^S)$ and

$$\varphi^F = A^F(y)\exp\big(\mathrm{i}(kx - \omega t)\big), \qquad \varphi^S = A^S(y)\exp\big(\mathrm{i}(kx - \omega t)\big) \tag{17}$$

$$\psi^F = B^F(y)\exp\big(\mathrm{i}(kx - \omega t)\big), \qquad \psi^S = B^S(y)\exp\big(\mathrm{i}(kx - \omega t)\big) \tag{18}$$

$$\rho^F - \rho_0^F = A_\rho^F(y)\exp\big(\mathrm{i}(kx - \omega t)\big), \qquad \rho^S - \rho_0^S = A_\rho^S(y)\exp\big(\mathrm{i}(kx - \omega t)\big) \tag{19}$$

$$n - n_0 = A_\Delta \exp\big(\mathrm{i}(kx - \omega t)\big) \tag{20}$$

For the unknown amplitudes A^F, A^S, and B^S one gets (for the others one obtains algebraic relations):

$$\begin{pmatrix} A^F \\ A^S \end{pmatrix} = C_1 \begin{pmatrix} R_1^F \\ R_1^S \end{pmatrix} \exp(-\gamma_1 y) + C_2 \begin{pmatrix} R_2^F \\ R_2^S \end{pmatrix} \exp(-\gamma_2 y) \tag{21}$$

$$B^S = C_s \exp(-\gamma_s y), \qquad \gamma_s = \sqrt{k^2 - \frac{\omega^2}{c_s^2} + \frac{\mathrm{i}\pi\omega^2 r}{c_s^2(\omega r + \mathrm{i}\pi)}}, \qquad c_s = \sqrt{\frac{\mu^S}{\lambda^S + 2\mu^S}} \tag{22}$$

It should be emphasized that we are interested in the solutions in the form of surface waves, i.e., in the solutions which attenuate with y. Thus, the solution (21), (22) bounded in y requires:

$$\mathrm{Re}\,\gamma_s > 0, \qquad \mathrm{Re}\,\gamma_j > 0, \quad j = 1, 2 \tag{23}$$

Radicals γ_j and vectors $(R_j^F, R_j^S)^T$, $j = 1, 2$, are defined from the construction of the asymptotic solution and have different structure in the high and low frequency ranges. Substituting (21), (22) into the boundary conditions (14), (15) one gets the system of equations for $C_1,\ C_2,\ C_s$. Requiring that the determinant of this system must vanish yields the dispersion equation. Next consider two limit cases:

(1) $|k| \gg 1$, i.e., short waves or high frequency range. In this case [9,5]

$$\tilde\gamma_1 = \gamma_1/|k| = \sqrt{1 - \tilde\omega^2/c_f^2}, \quad \tilde\gamma_2 = \gamma_2/|k| = \sqrt{1 - \tilde\omega^2}, \quad \tilde\gamma_s = \gamma_s/|k| = \sqrt{1 - \tilde\omega^2/c_s^2}, \quad \tilde\omega = \omega/k$$

Also, $(R_1^F, R_1^S) = (1, 0)$, $(R_2^F, R_2^S) = (0, 1)$. The corresponding dispersion equation has two roots. One of them defines the true Stoneley surface wave, which propagates almost without attenuation. Its speed is less that the velocities of all bulk waves in a porous medium and has the order $\mathrm{O}(c_f)$. Another one corresponds to the generalized (leaky) Rayleigh wave. Its velocity is a bit bigger than that one of the classical Rayleigh wave and it attenuates while propagating. A part of its energy is absorbed by the P2 bulk wave.

(2) $|k| \ll 1$, i.e., long waves or low frequency range. In this case

$$\tilde\gamma_1 = \sqrt{1 - \mathrm{i}\frac{\tilde\omega}{\tilde k}\left(1 + \frac{1}{rc_f^2}\right) - \frac{\tilde\omega^2}{c_f^2}\frac{1 + rc_f^4}{1 + rc_f^2}} + \mathrm{O}\big(\sqrt{\tilde k}\big), \qquad \begin{pmatrix} R_1^F \\ R_1^S \end{pmatrix} = \begin{pmatrix} 1 \\ -rc_f^2 \end{pmatrix}\big(1 + \tilde k + \mathrm{O}\big(\tilde k^2\big)\big) \tag{24}$$

$$\tilde\gamma_2 = \sqrt{1 - \tilde\omega^2\frac{1 + r}{1 + rc_f^2}} + \mathrm{O}\big(\sqrt{\tilde k}\big), \qquad \begin{pmatrix} R_2^F \\ R_2^S \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}\big(1 + \tilde k + \mathrm{O}\big(\tilde k^2\big)\big) \tag{25}$$

Remark 1. The expansions (25) are not valid in a neighborhood of the bifurcation point k_{cr}.

In the low frequency range the dispersion equation has two approximations depending on the region of wave numbers. First consider the region $k \leqslant k_{\mathrm{cr}}$, where the Biot bulk wave does not propagate. One can prove, that in this region the dispersion equation has a unique root, corresponding to the generalized Rayleigh surface wave. Its asymptotic expansion has the form:

$$\widetilde{\omega}_{R'} = c_R + \epsilon \Omega_1 + \mathrm{O}(\epsilon^2) \tag{26}$$

where c_R is the speed of the classical Rayleigh wave in an elastic half-space, $\epsilon \equiv r$, and Ω_1 is defined by

$$\Omega_1 \frac{\mathcal{P}_R(\widetilde{\omega})}{d\widetilde{\omega}}\bigg|_{\widetilde{\omega}=c_R} = \left(1 - \frac{c_R^2}{2c_s^2}\right)(3c_R^2 - 2c_s^2 c_f^2) + \sqrt{1-c_R^2}\sqrt{1-c_R^2/c_s^2}\left(1 - \frac{c_R^2}{c_s^2 - c_R^2} - \frac{c_R^2 - c_f^2}{1 - c_R^2}\right)$$

$$\mathcal{P}_R(\widetilde{\omega}) = \left(2 - \frac{\widetilde{\omega}^2}{c_s^2}\right)^2 - 4\sqrt{1-\widetilde{\omega}^2}\sqrt{1-\widetilde{\omega}^2/c_s^2}$$

As is easily verified, the provisos (23) are fulfilled for (26). Thereby, the solution (21), (22) indeed has the structure of a surface wave. It follows from (26) that the generalized Rayleigh wave propagates almost without attenuation in the region $k \leqslant k_{\mathrm{cr}}$.

Next consider the surface modes which can appear in a small neighborhood of the bifurcation point k_{cr}, where $k > k_{\mathrm{cr}}$ and the P2 bulk wave is propagatory. One can prove that in this region the dispersion equation has one more root, which corresponds to the Stoneley surface mode. The bifurcation behavior of the Biot slow wave dictates that the Stoneley wave must also possess a bifurcation. Indeed, this surface mode exists for a limited range of wave numbers. For the wave number $k = \frac{\pi}{2rc_f}(1 + \epsilon^2 k_2 + \cdots)$ the asymptotic expansion of the root, which defines the Stoneley surface mode, has the form:

$$\widetilde{\omega}_{\mathrm{St}} = -\mathrm{i}\epsilon + \sqrt{2(k_2 - 2r)}\,\epsilon^2 + \mathrm{O}(\epsilon^3) \tag{27}$$

where $\epsilon \equiv c_f$. Obviously, $\mathrm{Re}\,\widetilde{\omega}_{\mathrm{St}} \neq 0$ if the expression under the square root in (27) is positive. Thus, similar to the P2 wave, the Stoneley mode has a bifurcation behavior in a neighborhood of the point

$$k_{\mathrm{cr}} \approx \frac{\pi}{2rc_f}\left(1 + c_f^2 k_2\right), \quad \text{where } k_2 = 2r \tag{28}$$

Therefore, if $k_2 \leqslant 2r$, i.e., $k \leqslant k_{\mathrm{cr}}$, then the Stoneley wave does not propagate: it is fully attenuated. Otherwise, if $k_2 > 2r$, i.e., $k > k_{\mathrm{cr}}$, it begins to emerge with a velocity very close to the speed of the P2 wave. Fig. 1 (right-hand side) shows the velocities of the P2 and of the Stoneley waves in water-saturated sandstone as functions of the wave number. One sees that in the vicinity of k_{cr} the speed of the Stoneley wave $c_{\mathrm{St}} = \mathrm{Re}\,\widetilde{\omega}_{\mathrm{St}}$ is very close to, but somewhat less than, the speed c_{P2} of the P2 wave. Further deviation of c_{St} from c_{P2} results from the fact that only one term of the asymptotic expansion to c_{St} was taken into account (see (27)). Unlike what occurs in the high frequency limit [9,5], the Stoneley surface mode is strongly attenuated at low frequencies (leaky mode).

Let us verify whether the conditions (23) hold true for (27), i.e., whether the solution (21), (22) indeed has the form of a surface wave. It is easily seen that $\mathrm{Re}\,\gamma_s > 0$ and $\mathrm{Re}\,\gamma_2 > 0$. Solution for $\tilde{\gamma}_1$ in a vicinity of k_{cr} has the form $\tilde{\gamma}_1 = \Gamma_1\epsilon^2 + \Gamma_2\epsilon^3 + \cdots$, where

$$\Gamma_1 = \frac{r(4c_s^2 - 1)(2(1+2r) - 1/(2c_s^2)(1-2r))}{(1+2r)(1-c_s^2)} \tag{29}$$

For example, if $r = 0.1$ (water-saturated sandstone [8]), then $\Gamma_1 < 0$ and, consequently, $\mathrm{Re}\,\gamma_1 < 0$, if $0.41 < c_s < 0.5$. Thereby, the solution (21), (22) is not a surface wave and at low frequencies the Stoneley mode cannot appear at the free interface of the porous medium if the elastic moduli of its solid skeleton are such that the ratio of the speeds of the shear and longitudinal waves $c_s \in (0.41, 0.5)$.

Acknowledgement

Support by the Alexander von Humboldt Foundation is gratefully acknowledged.

References

[1] Lord Rayleigh, On waves propagated along the plane surface of an elastic solid, Proc. London Math. Soc. 17 (1885) 4–11.
[2] R. Stoneley, Elastic waves at the surface of separation of two solids, Proc. Roy. Soc. London Ser. A 106 (1924) 416–428.
[3] L.M. Brekhovskikh, Waves in Layered Media, Academic Press, New York, 1960.
[4] P.B. Nagy, Acoustics and ultrasonics, in: Experimental Methods in the Physical Sciences, Academic Press, 1999, pp. 161–221.
[5] I. Edelman, K. Wilmanski, Asymptotic analysis of surface waves at vacuum/porous medium and liquid/porous medium interfaces, Continuum Mech. Thermodyn. 14 (1) (2002) 25–44.
[6] M.A. Biot, Mechanics of deformation and acoustic propagation in porous media, J. Appl. Phys. 33 (4) (1962) 1482–1498.
[7] K. Wilmanski, Lagrangean model of two-phase porous material, J. Non-Equilib. Thermodyn. 20 (1995) 50–77.
[8] I. Edelman, Bifurcation of the Biot slow wave in a porous medium, J. Acoust. Soc. Am. 114 (1) (2002) 90–97.
[9] I. Edelman, Waves on boundaries of porous media, Phys. Dokl. 46 (2001) 517–521.
[10] I. Edelman, On the bifurcation of the Biot slow wave in a porous medium, WIAS, Preprint 738 (2002).[2]

[2] WIAS-Preprints are easily available from the webpage http://www.wias-berlin.de/publications/preprints as pdf- or ps-files.