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C. R. Mecanique 332 (2004) 619–626

COMPTES RENDUS MECANIQUE

Self-excited stick–slip oscillations of drill bits

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Received 10 January 2004; accepted 30 January 2004

Presented by André Zaoui

Abstract

This Note studies the self-excited stick–slip oscillations of a rotary drilling system with a drag bit, using a discrete model which takes into consideration the axial and torsional vibration modes of the bit. Coupling between these two vibration modes takes place through a bit-rock interaction law which accounts for both frictional contact and cutting processes at the bit-rock interface. The cutting process introduces a delay in the equations of motion which is ultimately responsible for the existence of self-excited vibrations, exhibiting stick–slip oscillations under certain conditions. ***To cite this article: T. Richard et al., C. R. Mecanique 332 (2004).***

Résumé

Oscillations stick–slip auto-entretenues d'un outil de forage. Cette Note traite du problème des oscillations auto-entretenues du type adhérence–glissement (stick–slip), qui se manifestent souvent dans les structures de forage pétrolier avec outils monobloc à taillants. Ce problème est étudié sur la base d'un modèle discret, dont les modes de vibrations axiales et de torsion sont couplés par une loi d'interaction roche-outil qui prend en compte les phénomènes de frottement et de coupe. Le processus de coupe introduit un délai dans les équations du mouvement, qui conduit à l'apparition de vibrations auto-entretenues pouvant dégénérer en oscillations stick–slip dans certaines conditions. ***Pour citer cet article : T. Richard et al., C. R. Mecanique 332 (2004).***

Keywords: Vibrations; Dynamical systems; Stick–slip; Drillstring vibrations; Delay; Self-excited vibrations

Mots-clés : Vibrations ; Systèmes dynamiques ; Stick–slip (adhérence–glissement) ; Vibrations du train de tiges de forage ; Delai ; Vibrations auto-entretenues

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1631-0721/$ – see front matter
doi:10.1016/j.crme.2004.01.016

Version française abrégée

Des vibrations de torsion auto-entretenues du type adhérence–glissement (stick–slip) se manifestent fréquemment dans le train de tiges d'une structure de forage pétrolier. L'utilisation d'outils de forage monoblocs à taillants accentue ce phénomène. L'existence d'un tel régime d'instabilité est souvent interpretée comme étant la manifestation d'une loi d'interface outil-roche, caracterisée par une diminution du couple résistant à l'outil avec la vitesse de rotation [2,1,3]. Cependant, on reconnaît également que la nature particulière de l'interaction outil-roche est une source de vibrations axiales de la structure de forage [5].

Dans cette Note, on étudie le phénomène d'oscillations stick–slip sur la base d'un système équivalent à deux degrés de liberté qui peut vibrer en torsion et axialement, et d'une loi d'interaction outil-roche qui est indépendante de la vitesse angulaire, voir Fig. 1(a). La loi d'interface impose un couplage entre les deux modes de vibrations. Le modèle discret est constitué d'un ressort de torsion de rigidité C (correspondant à la section supérieure du train de tige) et en un point de masse M et d'inertie I (représentant les masses-tiges au-dessus de l'outil), cf. (1). Les conditions en surface sont interpretées comme correspondant à une vitesse angulaire constante $\Omega_\circ$ et une force verticale constante $H_\circ$ (dirigée vers le haut) appliquées au ressort. La réponse de l'outil est décrite par deux jeux de variables conjuguées : le poids sur l'outil W et la vitesse d'avancement axiale V d'une part, le couple T et la vitesse angulaire Ω d'autre part. Les positions axiale U et angulaire Φ de l'outil sont obtenues par intégration de V et Ω. Le système d'équations qui combine les conditions en surface (2), les équations de mouvement (3), et la loi d'interaction roche-outil (9) accepte une réponse triviale $\mathcal{R}_\circ$, qui n'évolue pas dans le temps t (stationnaire). Les variables correspondantes $W_\circ,\ V_\circ, T_\circ,\ \Omega_\circ$, dépendent alors uniquement des conditions imposées en surface et de la loi d'interaction roche-outil, cf. (2) et (9).

Pour l'étude de la réponse non-triviale, qui se manifeste par des vibrations de l'outil, on considère un outil de forage de rayon a, constitué de n lames radiales espacées par un angle de $2\pi/n$, voir Fig. 1(b). Chaque lame possède un méplat d'épaisseur ℓ_n, en contact frottant avec la roche. En l'absence de vibrations latérales, la profondeur de coupe d_n est constante et identique pour chaque lame, cf. (4). Elle est fonction du délai $t_n(t)$ qui est le temps nécessaire à l'outil pour tourner d'un angle $2\pi/n$ et atteindre sa position actuelle au temps t, cf. (5). En conditions normales de forage ($V > 0,\ \Omega > 0$), les forces de coupe agissant sur l'outil sont proportionnelles à la profondeur de passe et à l'énergie spécifique ε, tandis que les forces de frottement mobilisées le long des méplats correspondent à une contrainte de contact σ admise constante [7] et à un coefficient de frottement μ, cf. (6) et (7) [6]. Dans le cas où l'outil se déplace vers le haut ($V < 0$), on admet une perte complète du contact méplat-roche. Durant une phase d'adhérence, l'outil demeure immobile ; dû à la rotation continue en surface, le couple appliqué sur l'outil augmente jusqu'à ce que la condition de glissement soit satisfaite, cf. (8). On suppose que l'amplitude de la composante de frottement du couple est suffisante pour empêcher toute rotation rétrograde.

On définit de nouvelles variables adimensionelles fonctions d'un temps normalisé τ, par le biais d'un temps caractéristique t_* et d'une longueur caractéristique L_* : $u,\ v,\ \varphi, \omega,\ \mathcal{W},\ \mathcal{T}$, cf. (11). La réponse du système dépend de deux groupes de paramètres adimensionels : (i) deux paramètres opératoires, le poids sur l'outil $\mathcal{W}_\circ$ et la vitesse angulaire de surface $\omega_\circ$, (ii) trois nombres décrivant l'outil, la roche et la structure de forage, β, λ et ψ, outre le nombre de lames n. Les perturbations $u(\tau)$ et $\varphi(\tau)$ de la solution triviale, cf. (14), sont régies par les équations de mouvement, les lois d'interaction roche-outil, l'équation de delai et les conditions de stick–slip sont exprimées sous forme adimensionelle, cf. (15). Une analyse de stabilité linéaire permet de conclure que la solution triviale est instable.

Les conditions initiales sont données par la solution triviale. Une perturbation est introduite au temps $t = 0$, sous la forme d'une variation soudaine de l'énergie spécifique ε. La convergence vers le cycle limite est illustrée à la Fig. 2 pour $\beta = 0{,}3$ et $\beta = 1{,}3$. Dans le premier cas, l'amplitude des oscillations croit jusqu'à ce qu'un régime d'oscillations périodiques auto-entretenues avec stick–slip soit atteint ; dans le second cas, les oscillations auto-entretenues restent de faibles amplitudes. La variation de $v,\ \omega,\ \mathcal{W},\ \mathcal{T}$ une fois que le système a atteint un cycle limite est illustrée à la Fig. 3 pour $\beta = 0{,}3$.

1. Introduction

Rotary drilling systems equipped with drag bits, which consist of fixed blades or cutters mounted on the surface of a bit body, are used to drill deep boreholes. Downhole measurements [1] indicate that these systems always experience torsional vibrations, which can often degenerate into stick–slip oscillations, characterized by sticking phases with the bit stopping completely and slipping phases with the angular velocity of the tool Ω increasing up to two times the imposed angular velocity.

Modelling of the stick–slip oscillations of drag bits is typically carried out by considering only the torsional vibration of the drill string and by reducing the bit-rock interface to an equivalent frictional contact with a velocity weakening friction coefficient [2,3]. Stick–slip vibrations have also been shown to take place with a constant friction coefficient, provided that axial vibrations are introduced in the model [4]; however, the torsional oscillations should more appropriately be considered as forced (by the axial vibrations) rather than self-excited. Finally, it has also been recognized that the particular nature of the bit rock interaction is a source of axial instabilities [5]. In this Note, we describe a discrete model of the drilling system which excludes any rate-dependence in the description of the bit-rock interface. This model takes into consideration the axial and torsional vibration modes and the coupling between these two modes through bit-rock interaction laws which account for both frictional contact and cutting processes at the bit-rock interface. We show that this model experiences self-excited vibrations, which can degenerate into stick–slip oscillations under certain conditions.

2. Model of a drilling system

A rotary drilling structure consists essentially of a rig, a drill string, and a bit. The principal components of the drill string are the bottom hole assembly (BHA) composed mainly of heavy steel tubes to provide a large downward force on the bit, and a set of drill pipes made of thinner tubes. We assume that a constant upward force H_o and a constant angular velocity Ω_o are applied by the rig on the drill string, that the borehole is vertical, and that there are no spurious lateral motions of the bit.

The bit motion and the forces acting on the bit can be calculated knowing the prescribed surface boundary conditions, the mechanical properties of the drill string and the bit-rock interaction law. We consider a discrete model of the drill string stripped to its essential elements, i.e., a point mass M and moment of inertia I to represent the BHA and a linear spring of torsional stiffness C to model the drillpipes, see Fig. 1(a). Expressions for M, I, and C are readily obtained by assuming that both the drillpipes and the BHA correspond to continuous shafts with constant cross-sections and density ρ

$$M = \rho\pi\left[\left(r_b^o\right)^2 - \left(r_b^i\right)^2\right]L_b, \quad I = \rho J_b L_b, \quad C = \frac{GJ_p}{L_p}, \quad J = \frac{\pi}{2}\left[\left(r^o\right)^4 - \left(r^i\right)^4\right] \tag{1}$$

with r_p^i (r_b^i), r_p^o (r_b^o), L_p (L_b), J_p (J_b) denoting respectively the inner radius, outer radius, length, and polar moment of inertia of the drillpipe (BHA).

The bit response, which is generally a function of time t, consists of two sets of conjugated quantities: the weight-on-bit W and the vertical bit velocity V on the one hand, and the torque-on-bit T and the angular bit velocity Ω on the other hand. (Refer to Fig. 1(a) for the sign convention.) It is also convenient to introduce the vertical position U and the angular position Φ of the bit.

There is a steady-state (trivial) response of the bit, characterized by constant quantities W_o, V_o, T_o, Ω_o. Although complete determination of this trivial response requires the knowledge of the bit-rock interface laws, it can already be stated that

$$W_o = W_s - H_o, \qquad \Omega = \Omega_o, \qquad \Phi_o = \Omega_o t - T_o/C \tag{2}$$

where W_s is the submerged weight of the drillstring. The non-trivial response can be determined in principle from the surface conditions and bit-rock interface laws, together with the following angular and axial equations of motion

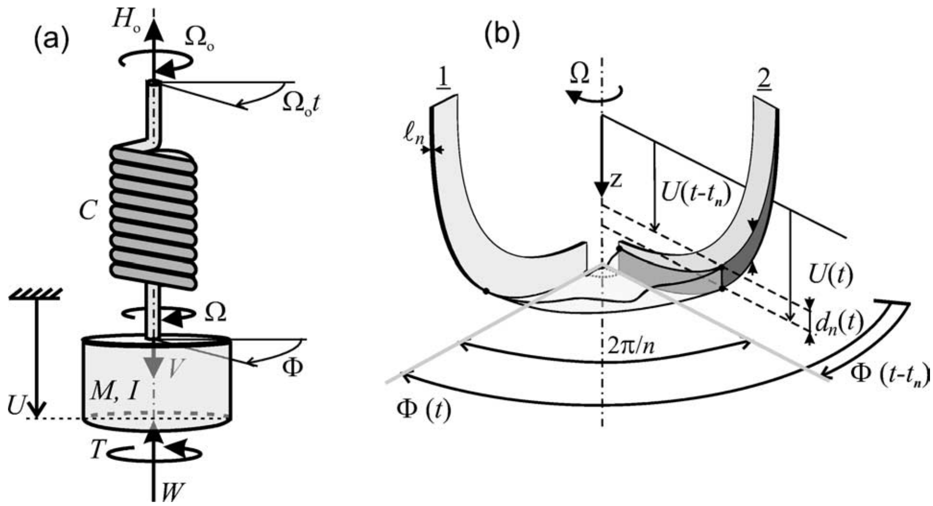

Fig. 1. (a) Simplified model of a drilling system; (b) section of the bottom-hole profile located between two successive blades of a drill bit.

Fig. 1. (a) Modèle simplifié d'une structure de forage ; (b) section du profile du fond de trou située entre deux lames voisines d'un outil.

$$I\frac{\mathrm{d}^2\Phi}{\mathrm{d}t^2}+C(\Phi-\Phi_\mathrm{o})=T_\mathrm{o}-T,\qquad M\frac{\mathrm{d}^2U}{\mathrm{d}t^2}=W_\mathrm{o}-W \tag{3}$$

noting that both T and W are, through the bit-rock interaction laws, functionals of the history of Φ (denoted by ${}_0^t\Phi$) and U (denoted by ${}_0^tU$), i.e., $T=T({}_0^t\Phi,{}_0^tU)$ and $W=T({}_0^t\Phi,{}_0^tU)$.

3. Bit-rock interaction

We restrict consideration to an idealized drag bit of radius a, consisting of n identical radial blades regularly spaced by an angle equal to $2\pi/n$, see Fig. 1(b). Each blade is characterized by a sub-vertical cutting surface and a wearflat of width ℓ_n orthogonal to the bit axis. In the absence of any lateral motion, the depth of cut per blade d_n (i.e. the thickness of the rock ridge in front of the blade) is constant along the blade and identical for each blade

$$d_n(t)=U(t)-U(t-t_n) \tag{4}$$

The time $t_n(t)$ required for the bit to rotate by $2\pi/n$ to its current position at time t is a $t_n(t)$ is a solution of

$$\Phi(t)-\Phi(t-t_n)=2\pi/n \tag{5}$$

The combined depth of cut and equivalent wear flat for the bit are $d=nd_n$ and $\ell=n\ell_n$.

Consider first the 'normal' case when $V>0$ and $\Omega>0$. The drilling action of a drag bit consists of a pure cutting process in front of each blade and a frictional process along the wear flats. Both the weight-on-bit W and the torque-on-bit T can thus be expressed as

$$W=W_\mathrm{c}+W_\mathrm{f},\qquad T=T_\mathrm{c}+T_\mathrm{f} \tag{6}$$

where the subscript c denotes the cutting component, and f the frictional component. The forces associated with the cutting process are taken to be proportional to the depth of cut d while the frictional forces mobilized along the wearflats depend on a rate-independent friction coefficient μ. The cutting and frictional components of W and T are explicitly given by [6]

$$W_\mathrm{c}=a\zeta\varepsilon d,\quad W_\mathrm{f}=a\ell\sigma,\quad T_\mathrm{c}=\frac{a^2}{2}\varepsilon d,\quad T_\mathrm{f}=\frac{a^2}{2}\gamma\mu\ell\sigma \tag{7}$$

where ε is the intrinsic specific energy (the amount of energy required to cut a unit volume of rock), ζ characterizes the inclination of the cutting force on the cutting face (typically, $0.6\leqslant\zeta\leqslant0.8$), and σ is the magnitude of the normal stress acting across the wear flat interface. The number γ globally characterizes the spatial orientation and distribution of the *chamfers/*wearflats; for a flat-bottom bit, $1 \leqslant \gamma \leqslant 4/3$. Single cutter experiments on rock show that σ is virtually constant (and approximately equal to ε) once the wearflat is in conforming contact with the rock [7]. Here, we assume that σ is a constant when $V > 0$ and $\Omega > 0$ and the bit is cutting rock ($d > 0$). Thus drilling occurs in the normal case, only if $W > W_{\mathrm{f}}$ and $T > T_{\mathrm{f}}$.

Consider next the case $V < 0$ and $\Omega > 0$ when the bit is moving upwards. We assume a complete loss of contact between the wearflat and the rock, so that the frictional components W_{f} and T_{f} vanish. Note that $d < 0$ corresponds to cases of bit bouncing, when the bit losses momentarily contact with the rock.

Finally the case $V = 0$ and $\Omega = 0$ corresponds to the stick phase when the bit remains immobile. During this phase, $U = U(t_k)$ and $\Phi = \Phi(t_k)$, with t_k denoting the time at which the bit sticks. Since rotation of the drill pipes continues at the surface, the torque applied by the drillstring on to the BHA builds up until its magnitude is sufficient to overcome the reacting torque; the bit slips at time t_p when

$$C\left[\Omega_{\circ} t_p - \Phi(t_k)\right] = \frac{a^2}{2}\left[\varepsilon d(t_k) + \mu\gamma l\sigma\right] \tag{8}$$

The magnitude of the frictional torque is assumed to be sufficient to restrain the bit from rotating backward (anti-clockwise).

Steady-state drilling conditions ($W = W_{\mathrm{o}},\ \Omega = \Omega_{\mathrm{o}}$) represent particular instances of the normal case ($V > 0$, $\Omega > 0$). The delay t_n is constant and given by $t_n = 2\pi/n\Omega_{\mathrm{o}}$ and thus $d = 2\pi V_{\mathrm{o}}/\Omega_{\mathrm{o}}$. The penetration rate V_{o} and torque-on-bit T_{o} are then readily deduced from the bit-rock interaction laws (6) and (7)

$$V_{\mathrm{o}} = \frac{(W_{\mathrm{o}} - W_{\mathrm{f}})\Omega_{\mathrm{o}}}{2\pi a\zeta\varepsilon}, \quad T_{\mathrm{o}} = T_{\mathrm{f}} + \frac{a(W_{\mathrm{o}} - W_{\mathrm{f}})}{2\zeta} \quad \text{if } W_{\mathrm{o}} > W_{\mathrm{f}} \tag{9}$$

The bit is not drilling if $W_{\mathrm{o}} \leqslant W_{\mathrm{f}}$, i.e. $V_{\mathrm{o}} = 0$ and $T_{\mathrm{o}} = a\gamma\mu W_{\mathrm{o}}/2$.

4. Dimensionless formulation

It is convenient to formulate the model in dimensionless form. First, we introduce a characteristic time $t_* = \sqrt{I/C}$ and a characteristic length $L_* = 2C/\varepsilon a^2$ (typically, $t_* \sim 1$ s and $L_* \sim 1$ mm). The scaled kinematic and dynamic quantities at the bit, $u,\ v,\ \varphi,\ \omega,\ \mathcal{W},\ \mathcal{T}$ are then defined as

$$u = \frac{U - U_{\mathrm{o}}}{L_*}, \quad v = \frac{Vt_*}{L_*}, \quad \varphi = \Phi - \Phi_{\mathrm{o}}, \quad \omega = \Omega t_*, \quad \mathcal{W} = \frac{W}{\zeta\varepsilon a L_*}, \quad \mathcal{T} = \frac{T}{C} \tag{10}$$

where $U_{\mathrm{o}} = V_{\mathrm{o}} t$ is the vertical bit position according to the trivial response. All these quantities are function of the dimensionless time $\tau = t/t_*$. The scaled bit response $\mathcal{B}(\tau) = \{\mathcal{W},\ v,\ \mathcal{T},\ \omega\}$ depends on two sets of parameters:

- the control parameters $\mathcal{W}_{\mathrm{o}} = W_{\mathrm{o}}/\zeta\varepsilon a L_*$ and $\omega_{\mathrm{o}} = \Omega_{\mathrm{o}} t_*$;
- the problem parameters characterizing the geometry and wear state of the bit, the rock, and the drilling structure, i.e., the number of blades n, and the lumped parameters $\beta = \mu\gamma\zeta,\ \lambda = \sigma\ell/\zeta\varepsilon L_*$, and $\psi = \zeta\varepsilon a I/MC$.

The typical range of variation of both control parameters is [1, 10]. The bluntness number λ is of order 1, while the bit-rock interaction number β is typically within the interval [0.1, 1]. The system number ψ is the only large number of this problem as it is of order 10^2. The largeness of ψ dictates that the inertia of the BHA in the axial direction cannot be neglected.

In the absence of vibrations, the response $\mathcal{B}$ is the trivial solution $\{\mathcal{W}_{\mathrm{o}}, v_{\mathrm{o}},\ \mathcal{T}_{\mathrm{o}},\ \omega_{\mathrm{o}}\}$, with v_{o} and $\mathcal{T}_{\mathrm{o}}$ given by

$$v_{\mathrm{o}} = \frac{\omega_{\mathrm{o}}}{2\pi}(\mathcal{W}_{\mathrm{o}} - \lambda), \qquad \mathcal{T}_{\mathrm{o}} = \mathcal{W}_{\mathrm{o}} + \lambda(\beta - 1) \tag{11}$$

if cutting is taking place ($\mathcal{W}_o > \lambda$). The non-trivial response $\mathcal{B}(\tau)$ is governed by the conditions at the surface and at the bit-rock interface, and by the momentum balance equations (3), which can be rewritten as

$$\ddot{u} = \psi(\mathcal{W}_o - \mathcal{W}), \qquad \ddot{\varphi} + \varphi = \mathcal{T}_o - \mathcal{T} \tag{12}$$

The bit-rock interaction laws are functions of the combined depth of cut $\delta = d/L_*$ for the bit. We choose to express δ as a perturbation from the depth of cut per revolution for the trivial motion, $\delta_o = 2\pi v_o/\omega_o$; i.e., $\delta = \delta_o + \hat{\delta}$ with

$$\hat{\delta} = n v_o \hat{\tau}_n + n(u - \tilde{u}) \tag{13}$$

where $\tilde{u}(\tau) = u(\tau - \tau_n)$ is the delayed motion, and $\hat{\tau}_n = \tau_n - \tau_{no}$ with $\tau_n(\tau)$ the actual delay and $\tau_{no} = 2\pi/n\omega_o$ the constant delay for the trivial response. The algebraic equation governing the delay perturbation $\hat{\tau}_n(\tau)$ is deduced from (5) and the above definitions to be

$$\omega_o \hat{\tau}_n + \varphi(\tau) - \tilde{\varphi}(\tau) = 0 \tag{14}$$

with $\tilde{\varphi}(\tau) = \varphi(\tau - \tau_n)$.

The bit-rock interaction laws can be summarized as follows. If $\dot{\varphi} > -\omega_o$,

$$\mathcal{W} - \mathcal{W}_o = \hat{\delta} + \mathcal{W}_f - \lambda, \qquad \mathcal{T} - \mathcal{T}_o = \hat{\delta} + \beta(\mathcal{W}_f - \lambda) \tag{15}$$

where $\mathcal{W}_f = \lambda$ if $\dot{u} > -v_o$ and $\mathcal{W}_f = 0$ if $\dot{u} < -v_o$. The bit sticks at time τ_k when $\dot{\varphi}(\tau_k) = -\omega_o$ and then slips at time τ_p given by

$$\tau_p = \tau_k + \left[\varphi(\tau_k) + \mathcal{T}(\tau_k) - \mathcal{T}_o\right]/\omega_o \tag{16}$$

The prescribed rotation speed ω_o of the drillstring enters therefore the governing equations via the delay function $\tau_n(\tau)$, as a consequence of the nature of the bit-rock interface.

5. Self-excited oscillations

A linear stability analysis indicates that the trivial motion is unstable. We consider a small perturbation (u, φ) of the trivial motion starting at $\tau = 0$ and analyze the growth of (u, φ) at a time τ when the motion at the delayed time $\tau - \tau_n(\tau)$ was already perturbed, i.e., $\tilde{u} \neq 0$ and $\tilde{\varphi} \neq 0$. We assume, however, that the perturbation has remained small enough until τ that $\dot{u} > -v_o$ from the beginning; hence, $\mathcal{W}_f = \lambda$. Under these conditions, the equations governing (u, φ) can be written as

$$\ddot{u} + \psi\hat{\delta} = 0, \quad \ddot{\varphi} + \varphi + \hat{\delta} = 0, \quad \hat{\delta} = n v_o \hat{\tau}_n + n(u - \tilde{u}), \quad \omega_o \hat{\tau}_n + \varphi - \tilde{\varphi} = 0 \tag{17}$$

Expressions for the perturbation in the delay $\hat{\tau}_n$ and in the depth of cut $\hat{\delta}$ in terms of u and φ and their time derivatives at τ can be obtained by expanding $\tilde{u}$ and $\tilde{\varphi}$ in Taylor series to the second order, and by substituting the truncated series in $(17)_c$ and $(17)_d$ after approximating τ_n as τ_{no}. It follows that

$$\hat{\tau}_n = -\frac{\tau_{no}}{\omega_o}\left(\dot{\varphi} - \frac{\tau_{no}}{2}\ddot{\varphi}\right), \qquad \hat{\delta} = n\hat{\tau}_n v_o + n\tau_{no}\left(\dot{u} - \frac{\tau_{no}}{2}\ddot{u}\right) \tag{18}$$

Introducing the above expressions in the momentum balance equations $(17)_a$ and $(17)_b$ gives, after some manipulations, two identical third order differential equations for φ and u

$$a_3 \dddot{f} + a_2 \ddot{f} + a_1 \dot{f} + a_0 f = 0 \tag{19}$$

where f represents either u or φ, and $a_3 = n\omega_o^3 - 2\pi^2(\psi\omega_o - v_o)$, $a_2 = 2\pi n\omega_o(\psi\omega_o - v_o)$, $a_1 = \omega_o(n\omega_o^2 - 2\pi^2\psi)$, and $a_0 = 2\pi n\omega_o^2\psi$. An exhaustive analysis of the roots of the characteristic polynomial for a realistic range of values of n, ψ, v_o and ω_o indicates that there is at least one root with a positive real part. We can conclude that the trivial solution is unstable and that the vibrations of the system are of a self-excited nature. These conclusions are independent of β, which does not appear in the equations governing the evolution of the perturbation.

6. Numerical results and limit cycle

Numerical solution of the system of Eqs. (12)–(15) confirms that any perturbations to the trivial motion cause the system to evolve towards one regime of self-excited axial and torsional vibrations. Several such regimes exist, with some characterized by stick–slip oscillations or bit bouncing. It appears that most of these regimes of solutions are either periodic characterized by a limit cycle or quasi-periodic, i.e. they evolve extremely slowly compared to the characteristic time of the system. The evolution of the bit angular velocity ω towards a limit cycle is illustrated in Fig. 2 for two cases corresponding to $\beta = 0.3$ and $\beta = 1.3$. For $\beta = 0.3$, the amplitude of the torsional oscillations grows until a stationary regime of stick–slip oscillations is established; for $\beta = 1.3$, the regime of oscillations remains at a low amplitude, even though the starting perturbation is initially damped. In both cases, the dominant vibration frequency is slightly less than the torsional resonance frequency of the discrete model. The variation with time of the bit response over four periods, once the system has reached a limit cycle, is shown in Fig. 3 for $\beta = 0.3$. In contrast to the quasi-monochromatic character of the angular velocity ω, all the other quantities $\mathcal{W}$, $\mathcal{T}$, υ experience fluctuations over a wide range of frequencies.

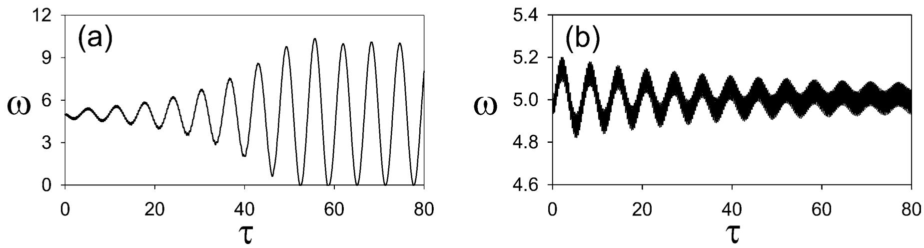

Fig. 2. Influence of the bit parameter β on the evolution of ω, following a perturbation at $\tau = 0$: (a) $\beta = 0.3$; (b) $\beta = 1.3$ ($\mathcal{W}_o = 7,\ \omega_o = 5,\ \lambda = 5,\ \psi = 50,\ n = 6$).

Fig. 2. Influence du paramètre β sur l'évolution de ω, suite à une perturbation à $\tau = 0$: (a) $\beta = 0,3$; (b) $\beta = 1,3$ ($\mathcal{W}_o = 7,\ \omega_o = 5,\ \lambda = 5,\ \psi = 50,\ n = 6$).

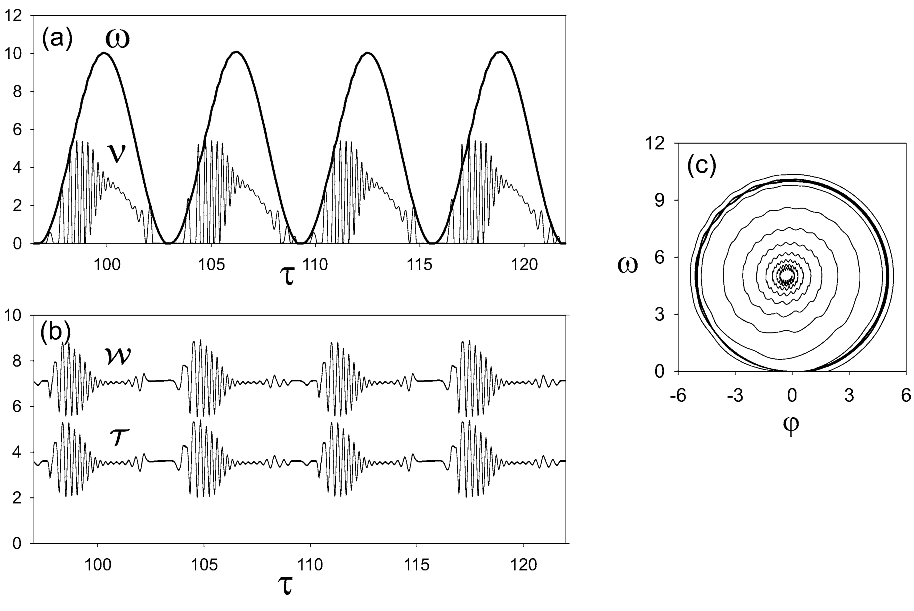

Fig. 3. Evolution of the bit response during a limit cycle: (a) $\omega(\tau),\ \upsilon(\tau)$; (b) $\mathcal{W}(\tau),\ \mathcal{T}(\tau)$; (c) phase plane for angular position ($\mathcal{W}_o = 7,\ \omega_o = 5,\ \beta = 0.3,\ \lambda = 5,\ \psi = 50,\ n = 6$).

Fig. 3. Evolution de la réponse de l'outil pendant un cycle limite : (a) $\omega(\tau),\ \upsilon(\tau)$; (b) $\mathcal{W}(\tau),\ \mathcal{T}(\tau)$; (c) diagramme de phase pour la position angulaire ($\mathcal{W}_o = 7,\ \omega_o = 5,\ \beta = 0,3,\ \lambda = 5,\ \psi = 50,\ n = 6$).

The root cause of the self-excitation is the cutting process which introduces, via the delayed axial position of the bit, a feedback into the equations of motion. Growth of the axial oscillations is hindered, however, by an intermittent 'high-frequency' loss of contact at the wearflat/rock interface, which is limiting the transfer of energy from the torsional to the axial motion. The coefficient β, which encapsulates the asymmetry between the axial and torsional interface laws, affects the ratio between the energy going into the cutting process and the energy dissipated in frictional contact. In fact, stick–slip oscillations take place only if $\beta < 1$. Interestingly, β directly controls the variation of $\mathcal{T}$ with the drilling efficiency $\eta = \mathcal{T}_c/\mathcal{T}$ under constant $\mathcal{W}$; $\mathcal{T}$ increases with η if $\beta < 1$ and decreases with η if $\beta < 1$.

The results of an extensive series of numerical experiments can be summarized into the following general observations. In the absence of bit bouncing, the system reaches strictly a limit cycle whenever $\beta > 1$, or if stick–slip develops when $\beta < 1$, see Fig. 2. If there is no stick–slip and bit bouncing in the case $\beta < 1$, the response reaches 'rapidly' a quasi-periodic regime, with the amplitude of the torsional oscillations increasing extremely slowly, presumably towards an limit cycle with stick–slip oscillations. The susceptibility to stick–slip motion increases with a reduction of the angular velocity ω_o and with an increase of $\mathcal{W}_o$, in accordance with field observations.

7. Concluding remarks

In this paper we have described a novel model for investigating the axial and torsional vibrations of a fixed cutter bit, that are often observed to take place when drilling deep boreholes. The proposed model differs in significant ways from the standard approach used to analyze stick–slip torsional vibrations. First, we consider both axial and torsional vibrations of the bit, as well as the coupling between the two vibration modes through the bit-rock interaction laws. Second, the interface laws account both for cutting of the rock and for frictional contact between the cutter wearflats and the rock. The cutting forces are formulated in terms of the depth of cut, a variable which brings into the equations the position of the bit at a previous a priori unknown time. Also, the nature of the contact at the wearflat/rock interface is such that this contact is lost as soon as the bit instantaneous axial velocity is directed upwards. Finally, all the parameters characterizing the interface laws are rate-independent, and can in principle be determined from single cutter experiments. Within the framework of the discrete model considered here in this paper, the evolution of the system is governed by two coupled delay differential equations, with the delay being part of the solution, and by discontinuous contact conditions. This type of delay differential equations appears to be quite unusual, and work is currently under way to understand their full implications.

Acknowledgements

The authors would like to thank Martyn Fear, Robert Delwiche, and Oliver Matthews for the benefit of many useful discussions. The research was supported by grants from BP-Amoco, Security-DBS, the University of Minnesota, and Diamant Drilling Services.

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