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Optimal design of functionally graded plates with thermo-elastic plastic behaviour

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Abstract

The thermo-elastic plastic behaviour of functionally graded plates under extremal thermal loading at different boundary conditions is considered. The plates consist of two phases – ZrO_2 ceramics and Ti_6Al_4V alloy. The layers are distributed exponentially through the thickness. The mechanical and thermal properties of both materials strongly depend on temperature. The stress–strain behaviour is investigated by the FEM. To predict the stable state of the structures of interest, several failure criteria are applied. Two cost functions are introduced to optimize the design of the plate. The main results are discussed and graphically illustrated. ***To cite this article: L. Parashkevova et al., C. R. Mecanique 332 (2004).***

Résumé

Configuration optimale des plaques fonctionnellement graduées à plasticité thermo-élastique. Nous avons étudié le comportement plastique thermo-élastique des plaques fonctionnellement graduées soumises à une charge thermique extrême, avec différentes conditions aux limites. Les plaques étaient contenaient deux parties distinctes : la céramique ZrO_2 et un alliage Ti_6Al_4V, avec une distribution des couches suivant une loi exponentielle. Les propriétés mécaniques et thermiques de ces deux matériaux dépendent fortement de la température. Le comportement tension–élongation est étudié avec la méthode d'éléments finis (MEF). Pour trouver les états stables ou instables de la structure en question, on applique quelques critères d'échec. Deux fonctions sont introduites pour optimiser la configuration de la plaque. Les résultats principaux sont discutés et illustrés par les graphiques. ***Pour citer cet article : L. Parashkevova et al., C. R. Mecanique 332 (2004).***

Keywords: Computational solid mechanics; Functionally graded materials; Thermal loading; Optimal design

Mots-clés : Mécanique des solides numérique ; Matériaux fonctionnellement gradués ; Charge thermique ; Configuration optimale

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1. Introduction

The rapid development of aircraft and space technologies requires that the materials withstand severe environments such as high temperature or large temperature gradients. The functionally graded materials (FGMs) play an irreplaceable role to reduce the temperature through-thickness gradient. Material gradients, induced by the change in material properties, make FGMs different in behaviour from homogeneous and traditional composite materials. FGMs offer good possibilities for optimizing engineering structures to achieve high performance and material efficiency. That is why they have successful applications as electronic devices, optical films, antiwear and anticorrosion coatings and biomaterials.

The majority of the published works on plates with nonuniform through-thickness properties have been directed to layered composites [1–3] and temperature sensitive material properties. The works on application of functionally graded materials with specially varying compositions can be related to the paper [4], where the effect of material composition in functionally graded shells is studied. The analysis based on a first order plate theory of thermo-elastic behavior of functionally graded plates for moderately large deformations [5] shows that the behavior of ceramic-metal FGM plate is not intermediate to that of homogeneous and metallic plates.

Two-phase laminated thin plates are considered in the present paper. The layers are distributed through the plate thickness according to an exponential law. Such a law covers uniform layers distribution, as well as a wide range of nonuniform distributions [2]. The aim of this Note is to obtain an optimal layers distribution, avoiding failure and reducing thermal stresses.

2. Problem formulation

Consider a layered two-phase infinite thin plate. The two phases building the plate are thermal resistant metal alloy and ceramics. Perfect contact between the layers and plain strain condition is assumed (Fig. 1(a)).

The Cartesian coordinate system is introduced; the axes are directed along the base and the thickness, respectively. All material properties (mechanical and thermal) of both constituents are temperature dependent. The initial temperature of the plate is T_0. We assume that the laminated plate is suddenly heated from the lower and upper surfaces by the surrounding media, the temperatures of which are denoted by T_1 and T_2, respectively (Fig. 1(a)). Three kinds of boundary conditions are considered: simply supported plate (Fig. 1(b)), bottom surface of

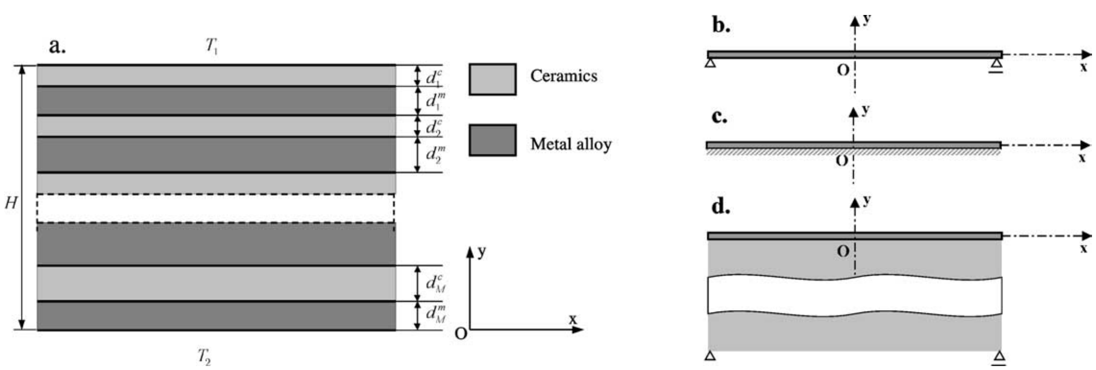

Fig. 1. Cross section and boundary conditions.

Fig. 1. Sections transverses et conditions aux limites.

the plate fixed to a rigid base (Fig. 1(c)) and full contact of the plate with a metal substrate (Fig. 1(d)). The following exponential laws, similar to the ones, given in [6], determine the thickness of the layers of both constituents:

$$d_i^c = \omega_c \frac{k_c H}{M} \exp\bigl[\delta_c (M-i)\bigr], \quad d_i^m = \omega_m \frac{k_m H}{M} \exp\bigl[\delta_m (i-1)\bigr], \quad i = 1, \ldots, M \tag{1}$$

where $\delta_{m(c)}$ and $\omega_{m(c)} > 0$ are dimensionless parameters and their variation permits to condense or to disperse the layers into the range of the thickness $H = \sum_{i=1}^{M}(d_i^c + d_i^m)$; d_i^m and d_i^c denote the thickness of the ith metallic or ceramics layer, respectively. It is assumed that the number of layers M for both materials is the same. The volume concentrations of ceramics and metallic alloys are denoted by k_c and k_m, respectively, where $k_c + k_m = 1$. The following relations of the parameters ω_j and δ_j hold:

$$\omega_j = \frac{M(\exp\delta_j - 1)}{\exp M\delta_j - 1}, \quad j = m, c \tag{2}$$

The coupled thermo-mechanical problem is considered. The materials of both constituents are isotropic. Mises yield condition and the associated flow rule are assumed.

The heat conduction equation has the form:

$$\frac{\partial T}{\partial t} = \frac{\lambda_T}{c_T \rho} T_{,ii} + k\sigma_p \dot{\bar{\varepsilon}}_p + \frac{3\lambda + 2\mu}{c_T \rho} \alpha_T (T - T_0)\dot{\varepsilon}_{kk} \tag{3}$$

where $\lambda_T(T)$ denotes thermal conductivity, $c_T(T)$ is heat capacity, $\rho(T)$ is material density, $\dot{\bar{\varepsilon}}_p$ is the effective plastic strain rate, k is the Taylor–Qwinney coefficient, $\lambda(T)$ and $\mu(T)$ are the Lame coefficients, $\alpha_t(T)$ is the coefficient of linear thermal expansion and $\dot{\varepsilon}_{kk}$ is the elastic volumetric strain rate.

The Cauchy stress tensor σ_{ij} fulfills the equilibrium equations $\sigma_{ij,j} = 0$. The following temperature initial and boundary conditions are fulfilled:

$$T(x, y, 0) = T_0, \quad T(x, 0, t) = T_2, \quad T(x, H, t) = T_1, \quad x, y \in \Omega \tag{4}$$

where Ω is the region occupied by the body under consideration. The criterion $\sigma_{\text{equiv}} = \sigma_{\text{lim}}$ is applied to investigate failure initiation of the multi-layered plate, where the equivalent stress σ_{equiv} and the limit stress σ_{lim} for both phases are explained below.

It is assumed that plastic zones should not be allowed in the metal alloy. Initiation of plastic state is determined by the Mises yield condition:

$$\sigma_{\text{equiv}}^m = \sigma_p, \quad \sigma_{\text{equiv}}^m = \frac{\sqrt{2}}{2}\sqrt{S_2}, \quad S_2 = (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_1 - \sigma_3)^2 \tag{5}$$

denotes the equivalent stress in the metal alloy and $\sigma_1 \geqslant \sigma_2 \geqslant \sigma_3$ are the principal stresses. In that case $\sigma_{\text{equiv}} = \sigma_{\text{equiv}}^m$ and $\sigma_{\text{lim}} = \sigma_p$.

Several criteria are applied to evaluate failure of the brittle elastic ceramics material. For all criteria $\sigma_{\text{equiv}}^c = \sigma_{\text{Bt}}$, i.e., $\sigma_{\text{equiv}} = \sigma_{\text{equiv}}^c$ and $\sigma_{\text{lim}} = \sigma_{\text{Bt}}$, where σ_{Bt} is the tensile strength.

Mohr criterion [7]:

$$\sigma_{\text{equiv}}^c = \sigma_1 - \frac{\sigma_{\text{Bt}}}{\sigma_{\text{Bc}}}\sigma_3 \tag{6}$$

where σ_{Bc} is the compressive strength.

Deformation ctriterion [8]: $\sigma_{\text{equiv}}^c = \sigma_1 - \nu(\sigma_2 + \sigma_3)$.

Maximal stress criterion [8]: $\sigma_{\text{equiv}}^c = \sigma_1$.

Balandin criterion [9]: $\sigma_{\text{equiv}}^c = (S_2 + 2(\sigma_{\text{Bc}} - \sigma_{\text{Bt}})S_1)/(2\sigma_{\text{Bc}})$, $S_1 = \sigma_1 + \sigma_2 + \sigma_3$.

The last criterion as stated is a special case of Tsai-Wu failure criterion and Hoffman criterion [8] if both are transformed for isotropic materials.

We introduce the function

$$G(y) = 1 - \frac{\sigma_{\text{equiv}}}{\sigma_{\text{lim}}} \tag{7}$$

In order to avoid failure, the condition $G(y) > 0$ should be fulfilled.

Optimal design analysis of the multi-layer plate is provided. The functions Ψ_1 and Ψ_2 are defined as cost functions in the safe zone:

$$\Psi_1(\omega_m, \omega_c, k_m) = \sum_{j=1}^{\widetilde{M}} \frac{\int_{y_{1j}}^{y_{2j}} G(y)\,\mathrm{d}y}{y_{2j} - y_{1j}} \Rightarrow \min, \qquad \Psi_2(\omega_m, \omega_c, k_m) = \frac{\sum_{j=1}^{\widetilde{M}} \int_{y_{1j}}^{y_{2j}} G(y)\,\mathrm{d}y}{H} \Rightarrow \min \tag{8}$$

where y_{1j} and y_{2j} are the y-coordinates of the upper and lower surfaces of a strip in the jth layer (metal or ceramics). Inside the strip the function G is positive. $\widetilde{M} = 2M - M_f$, where M_f is the number of totally damaged layers. The thickness of the strip $y_{2j} - y_{1j}$ may be less than the thickness of the jth layer if in part of that layer $G \leqslant 0$. The thickness of the strip is equal to the thickness of the layer, in case $G > 0$ at each through-thickness point of the layer. If $G \leqslant 0$ in the whole thickness of a particular layer, this layer does not contribute to the functions Ψ_1 and Ψ_2. Criterion (8), concerning Ψ_1 takes into account existence of large deviations of the equivalent stress from the limit one in the single layers. Criterion (8), concerning Ψ_2 considers the average in the whole cross section deviation of the equivalent stress from the limit one. Criteria (8) lead to a plate structure with maximal using of the material carrying capacity.

The conditions $\Psi_1(\omega_m, \omega_c, k_m) \Rightarrow \max$ or $\Psi_2(\omega_m, \omega_c, k_m) \Rightarrow \max$ lead to a plate structure with maximally reduced stresses in the plate.

3. Numerical examples

A functionally graded plate consisting of two phases, titanium alloy Ti_6Al_4V and ceramics ZrO_2, is considered. The plate is subjected to temperature loading $T_1 = 1100$ K and $T_2 = T_0 = 300$ K. The mechanical characteristics of both phases are taken from [10] and [6]. There are three laminates of each phase $(M = 3)$. The total thickness of the plate is $H = 0.84428$ mm. The plate is taken in Ox direction long enough, so that the edge effect is negligible. The laminates are distributed exponentially, according to Eqs. (1). We consider the cases $d_1^c \geqslant d_2^c \geqslant d_3^c$ and $d_1^m \leqslant d_2^m \leqslant d_3^m$, for which $\delta_{m(c)} \geqslant 0$ and which are of practical interest. This leads to the intervals $\omega_{m(c)} \in (0, 1]$. At fixed metal alloy concentration $k_m = 0.5528$ the values of ω_c and ω_m are varied.

The FEM is applied for solving the coupled thermo-mechanical problem. The finite element package MARC is used. 4-node quadrilateral isoparametric finite elements are applied.

The distribution of the normal stress σ_x through the plate thickness was investigated for the different types of boundary conditions mentioned above. At the contact surface between two layers, discontinuity of the stress distribution takes place due to the change of material properties. Calculations showed that in the case of simply supported plate deformations due to temperature and stress jumps are higher than in the rest cases. Therefore we are dealing later only with this kind of boundary conditions. Applying the optimization criteria mentioned above, we try to reduce the stress jumps.

The modifications of the function G for different failure criteria were studied. Computations indicated that for all types of boundary conditions the *Deformation criterion* is too restrictive and leads to failure of ceramics in the prevailing part of the cross section. In contrary, the *Balandin* (*Tsai-Wu*) *criterion* is too weak and does not show failure anywhere. That is why we consider further the *Mohr criterion* (6) which leads to results close to the results obtained by the *Maximal stress criterion*; additionally, it takes into account the difference between tensile and compressive strength, a property important for ceramic materials.

The safe zone (a) and unsafe zones (b) and (c) at fixed concentration $k_m = 0.5528$ are seen in Fig. 2. In zone (b) the limit stress for Ti_6Al_4V is reached. In zone (c) the limit stress of ZrO_2 according to *Mohr criterion* (6) is also

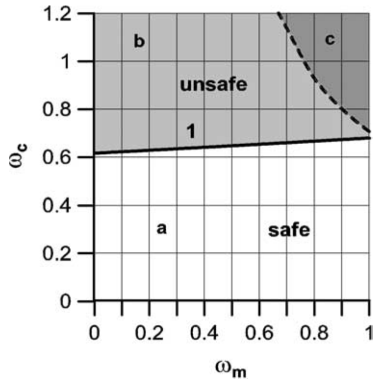

Fig. 2. (a) Safe zone; (b) zone in which $\sigma_{\lim}$ of Ti_6Al_4V is reached; (c) zone in which $\sigma_{\lim}$ of both phases is reached; 1 – neutral curve.

Fig. 2. (a) Zone de sécurité ; (b) zone dans laquelle $\sigma_{\lim}$ de Ti_6Al_4V est atteint ; (c) zone dans laquelle $\sigma_{\lim}$ des deux phases est atteint ; 1 = courbe neutre.

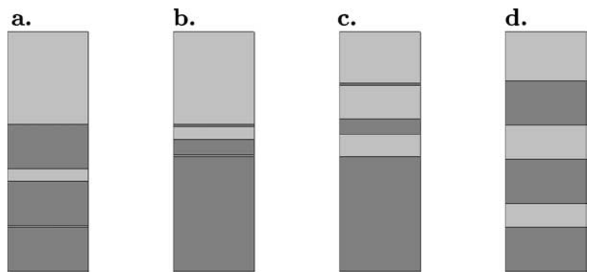

Fig. 3. Layers distribution at: (a) max Ψ_1; (b) max Ψ_2; (c) min Ψ_1; (d) min Ψ_2.

Fig. 3. La distribution des couches correspondant à : (a) max Ψ_1 ; (b) max Ψ_2 ; (c) min Ψ_1 ; (d) min Ψ_2.

reached. Taking into account that $d_1^c > d_2^c > d_3^c$ and relation (1) note that the thickness of the first ceramics layer $d_1^c \sim (0.6\text{–}0.7)\frac{k_c H}{M}$ is of particular importance for preventing titanium failure (thermal coating). At $\omega_c > (0.6\text{–}0.7)$ no distribution of the titanium layers can ensure safety. The minimum value of Ψ_1 is reached at $\omega_m = 0.0476$ and $\omega_c = 0.6150$, while the minimum value of Ψ_2 is at $\omega_m = 1$ and $\omega_c = 0.667$ along the neutral curve 1.

The through-thickness layers distribution is seen in Fig. 3 for extremum values of the cost functions Ψ_1 and Ψ_2.

4. Conclusions

Using the FEM, the influence of several failure criteria, boundary conditions, layers distribution on thermo-mechanical stress behaviour is provided. The optimal design of material composition for a two-phase exponentially graded plate is analysed. It is shown that the chosen cost function is effective to reduce the jumps in thermal stress through-thickness distribution. The numerical results show that the thickness of the outer ceramic layer is the most important for preventing titanium failure. This fact leads to the existance of a minimum necessary ceramic volume concentration for given plate thickness. The investigation provided can help in designing optimal layers distribution. This has an application in technologies of thermal barrier coatings.

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