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Combined action of anticonvection and thermocapillarity in two- and three-layer systems

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Abstract

Anticonvection generated by the joint action of the external heating and heat sources (sinks) on the interface in the layers with finite thicknesses is studied. Anticonvective structures in fluid systems subject to the anticonvective instability only in the presence of heat sources (sinks) on the interface have been obtained. The nonlinear regimes of anticonvection in the system of three immiscible viscous fluids heated from above are investigated. The specific phenomena caused by direct and indirect interaction of anticonvective and thermocapillary mechanisms of instability are considered. In particular, different oscillatory configurations where anticonvection arises mainly near the upper interface and thermocapillary convection appears mainly near the lower interface, have been studied. ***To cite this article: I.B. Simanovskii, C. R. Mecanique 332 (2004).***

Résumé

Action simultanée d'anticonvection et de thermocapillarité dans des systèmes multi-couches. On étudie l'anticonvection engendrée par l'action simultanée d'un apport thermique venant de l'extérieur et à partir des sources (puits) de chaleur se trouvant aux interfaces des couches d'épaisseur finie. On obtient les structures anticonvectives présentant une instabilité anticonvective uniquement pour les systèmes fluides en présence de sources (puits) de chaleur à l'interface. On étudie également les régimes d'anticonvection non-linéaire dans un système de trois fluides visqueux immiscibles, chauffés par dessus. Les phénomènes spécifiques provoqués par une interaction directe et indirecte entre les mécanismes d'instabilité anticonvectifs et thermocapillaires sont considérés. En particulier, on démontre l'existence des configurations oscillatoires diverses, dans lesquelles l'anticonvection se produit principalement à proximité d'interface supérieur, et la convection thermocapillaire apparaît près d'interface inférieur. ***Pour citer cet article : I.B. Simanovskii, C. R. Mecanique 332 (2004).***

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1. Introduction

It is known that there are two basic physical phenomena that produce convective instability in fluid systems with an interface: buoyancy and thermocapillarity. When heating is from below, the buoyancy instability generates the Rayleigh–Benard convection, while the thermocapillary effect is the origin of the Marangoni–Benard convection.

However, the Rayleigh instability is not the only possible type of the buoyancy-driven instability. Although it can be found in many textbooks (see, for example, [1]) that for fluids with a positive heat expansion coefficient the buoyancy convective instability of the mechanical equilibrium appears only by heating from below, this opinion is not actually correct in the presence of the interface between two fluids. It was shown by Welander [2] that when the temperature gradient is directed vertically upwards, the two-layer system, consisting of two immiscible viscous fluids of semi-infinite thicknesses, may become unstable with respect to the monotonic disturbances. The specific non-Rayleigh mechanism of instability (anticonvection) takes place in the system. This mechanism of instability may appear only under defined conditions: the fluids with considerably different physical properties must be considered. In particular, the heat expansion coefficient of the upper layer must be much smaller than that of the lower layer, and the thermal diffusivity of the lower layer must be much higher than that of the upper one (or vice versa). The linear stability boundaries (for the layers of finite thicknesses) were determined in [3] and the finite-amplitude regimes of anticonvection were obtained in [4] (see also [5]).

For a long time it was a common opinion that the phenomenon of anticonvection was rather exotic and, as the matter of fact, only one physical system (water–mercury), satisfying the conditions for the existence of anticonvection was found [3]. It turns out, however, that the appearance of anticonvection can be simplified when the interface serves as a source or sink of heat. Specific examples of an active influence of the interface on the heat transfer are absorption of light, evaporation, mass transfer through the interface, heterogeneous chemical reactions, and so on. In the presence of the uniformly distributed heat source (sink) at the interface, the anticonvective and buoyancy instability mechanisms can act simultaneously. The interaction between both instability mechanisms for the water–mercury system was studied first by Nepomnyashchy and Simanovskii [6]. In [7,8] it was found, that in the presence of an interfacial heat source (sink) the anticonvection could be generated in any system of two semi-infinite layers. The consideration of the anticonvection in semi-infinite layers reveals the existence of the instability, but it is not sufficient for the calculation of critical values of the temperature gradient and the critical wavenumber. For this aim, the investigation of layers with finite thicknesses is necessary. The general analysis of conditions for the appearance of anticonvection in the system of layers with finite thicknesses was made in [9]. It was shown that the anticonvection appears in the situation where the temperature gradient in one fluid is much smaller than that in another fluid. The difference between the temperature gradients sufficient for the appearance of the anticonvection, can be caused by the natural difference of thermophysical parameters of fluids, as in the case of the water–mercury system, or it can be produced artificially by heating or cooling the interface.

Another interfacial physical effect that may cause a convective instability is the thermocapillary effect which can generate stationary [10] and oscillatory motions [11–13]. In a real situation, different instability mechanisms may act simultaneously. The combined action of anticonvective and thermocapillary mechanisms of instability in a two-layer system was considered in [14,15], where the steady convective motions with different spatial structures were obtained in the system.

Three-layer systems can differ considerably from systems with a single interface. The essentially new effect is the possibility of the interaction between two interfaces. Evidently, the indirect interaction of anticonvective and thermocapillary mechanisms of instability (when both mechanisms act on different interfaces) cannot be realized in a two-layer system. The investigation of multi-layer systems was started in [16–18] where the Marangoni–Benard and Rayleigh–Benard convections were considered. The first experimental results on the Marangoni–Benard instability under microgravity conditions in a symmetric three-layer system confirm the existence of the oscillatory instability in theoretically predicted interval of parameters [19].

An anticonvective mechanism of instability in a three-layer system was investigated in [20]. The combined action of anticonvective and thermocapillary mechanisms of instability in the case of multilayer fluid system was discussed in [21].

2. Formulation of the problem

Let the space between two parallel rigid plates $z = a_1$ and $z = -a_2$ be filled by two immiscible viscous fluids with different physical properties. The plates are kept at constant but different temperatures (the total temperature drop is θ). A constant heat release of the rate Q_0 (Q_0 may be positive or negative) is set on the interface. All variables referring to the upper layer are marked by index 1, and the variables referring to the lower layer are marked by index 2. Let us use the following notations:

$$\rho = \rho_1/\rho_2, \quad \nu = \nu_1/\nu_2, \quad \eta = \eta_1/\eta_2, \quad \kappa = \kappa_1/\kappa_2$$
$$\chi = \chi_1/\chi_2, \quad \beta = \beta_1/\beta_2, \quad a = a_2/a_1, \quad L = l/a_1$$

Here ρ_m, ν_m, η_m, κ_m, χ_m, β_m and a_m are, respectively, density, kinematic and dynamic viscosities, heat conductivity, thermal diffusivity, heat expansion coefficient and the thickness of the mth layer ($m = 1, 2$). As the units of length, time, velocity, pressure and temperature we use a_1, a_1^2/ν_1, ν_1/a_1, $\rho_1\nu_1^2/a_1^2$ and θ, respectively. The complete nonlinear equations of convection in the framework of the Boussinesq approximation (see [23]) for both fluids have the following form:

$$\begin{aligned}
&\frac{\partial \vec{v}_1}{\partial t} + (\vec{v}_1 \cdot \nabla)\vec{v}_1 = -\nabla p_1 + \triangle \vec{v}_1 + G T_1 \vec{\gamma} \\
&\frac{\partial T_1}{\partial t} + \vec{v}_1 \cdot \nabla T_1 = \frac{1}{P}\triangle T_1 \\
&\nabla \vec{v}_1 = 0
\end{aligned} \tag{1}$$

$$\begin{aligned}
&\frac{\partial \vec{v}_2}{\partial t} + (\vec{v}_2 \cdot \nabla)\vec{v}_2 = -\rho\nabla p_2 + \frac{1}{\nu}\triangle \vec{v}_2 + \frac{G}{\beta} T_2 \vec{\gamma} \\
&\frac{\partial T_2}{\partial t} + \vec{v}_2 \cdot \nabla T_2 = \frac{1}{\chi P}\triangle T_2 \\
&\nabla \vec{v}_2 = 0
\end{aligned} \tag{2}$$

where $G = g\beta_1\theta a_1^3/\nu_1^2$ is the Grashof number, characterizing the intensity of the external heating (g is the gravity acceleration), $P = \nu_1/\chi_1$ is the Prandtl number for the liquid in layer 1, $\vec{\gamma}$ is the unit vector directed vertically upward. The conditions on the isothermic rigid horizontal boundaries are:

$$z = 1: \quad \vec{v}_1 = 0; \quad T_1 = 0 \tag{3}$$

$$z = -a: \quad \vec{v}_2 = 0; \quad T_2 = s \tag{4}$$

$s = -1$ for heating from above; $s = +1$ for heating from below.

Let us discuss the boundary conditions at the interface between the two fluids. One should be very careful on taking into account the deformation of the interface when using the Boussinesq approximation, because it is known that the interfacial deformation is a non-Boussinesq effect [22]. Indeed, the Boussinesq approximation is based on the assumption $\varepsilon_\beta = \beta_1\theta \ll 1$, $G = \mathrm{O}(1)$, and therefore the Galileo number $Ga = G/\varepsilon_\beta = ga_1^3/\nu_1^2 \gg 1$. However, the balance of normal stresses on the interface shows that the interface deformation is proportional to $1/Ga\delta$, where $\delta = \rho^{-1} - 1$ (see [23]). Because $1/Ga\delta = \varepsilon_\beta/G\delta$ is small unless $\delta \ll 1$, we come to the conclusion that in the framework of the Boussinesq approximation the interfacial deformation has to be neglected, if the densities of the fluids are not close to each other. The case of close densities is not considered in the present paper. Note that if the deformation is included in the consideration, while the corrections of the same order $O(\varepsilon_\beta)$ in the continuity equation are neglected, this may lead to erroneous results (see [23]).

The boundary conditions on the interface include conditions for the tangential stresses:

$$z=0: \quad \eta\frac{\partial v_{1x}}{\partial z}-\frac{\partial v_{2x}}{\partial z}-\frac{\eta M}{P}\frac{\partial T_1}{\partial x}=0 \tag{5}$$

$$\eta\frac{\partial v_{1y}}{\partial z}-\frac{\partial v_{2y}}{\partial z}-\frac{\eta M}{P}\frac{\partial T_1}{\partial y}=0 \tag{6}$$

the continuity of the velocity field:

$$v_1=v_2 \tag{7}$$

the continuity of the temperature field:

$$T_1=T_2 \tag{8}$$

and the continuity of the heat flux normal components:

$$\kappa\frac{\partial T_1}{\partial z}-\frac{\partial T_2}{\partial z}=-\kappa\frac{G_Q}{G} \tag{9}$$

Here $M=\alpha\theta a_1/\eta_1\chi_1$ is the Marangoni number, $G_Q=g\beta_1 Q_0 a_1^4/\nu_1^2\kappa_1$ is the modified Grashof number determined by the intensity of the interfacial heat release. The boundary-value problem (1)–(9) for any choice of parameters has the solution:

$$\vec{v}_m=0, \quad T_m=T_m^0(z), \quad p_m=p_m^0(z), \quad m=1,2 \tag{10}$$

corresponding to the mechanical equilibrium state. The temperature gradients in the equilibrium state are:

$$A_1=\frac{\mathrm{d}T_1^0}{\mathrm{d}z}=-\frac{sG+a\kappa G_Q}{G(1+a\kappa)}, \qquad A_2=\frac{\mathrm{d}T_2^0}{\mathrm{d}z}=-\frac{\kappa(sG-G_Q)}{G(1+a\kappa)} \tag{11}$$

In the following sections we shall investigate the case $s=-1$ (heating from above).

The origin of anticonvective mechanism of instability is explained in [3,15].

3. Anticonvection with heat release on the interface

In this section we consider the example when, in the absence of heat release on the interface, there is no anticonvection in the system. It will be shown that the presence of the interfacial heat release leads to the appearance of the anticonvective motion. We present the results of the direct numerical solution of the eigenvalue problem for the real two-liquid system silicone oil – 10*cs* – ethylene glycol with the following set of parameters: $\eta=0.549$, $\nu=0.6493$, $\kappa=0.6194$, $\chi=1.096$, $\beta=1.4516$; the Prandtl number $P=94$. The calculations were made with $a=1$. One can expect the appearance of anticonvection for two cases: intensive heat sources ($G_Q>0$) and intensive heat sinks ($G_Q<0$). The anticonvection is the only possible instability mechanism if the temperature gradients are positive in both layers. Such a temperature profile is obtained in two cases: (a) $sG<-a\kappa G_Q$, $G_Q>0$; (b) $sG<G_Q<0$. The neutral curve corresponding to the case of heat sources ($G_Q=6000$), is shown in Fig. 1 (line 1). The minimum of this neutral curve $sG\approx-3729$ is less than $-a\kappa G_Q\approx-3718$, thus both gradients A_1 and A_2 are positive, and the anticonvection is the only instability mechanism. Note, that at the minimum, the ratio $A_1/A_2\approx 2.4\times10^{-3}$, i.e., there is a strong heating from above in the bottom layer, and nearly neutral stratification in the top layer. Line 2 in Fig. 1 ($G_Q=-8835$) corresponds to the opposite case $G_Q<0$, $sG\approx G_Q$

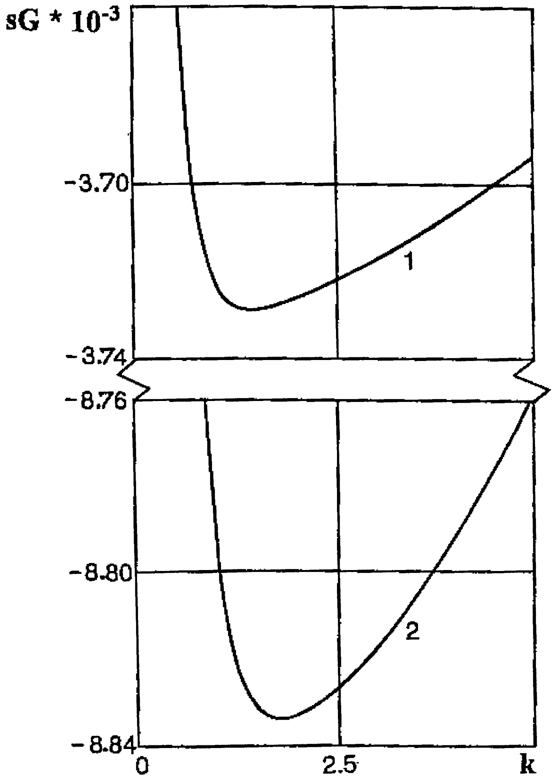
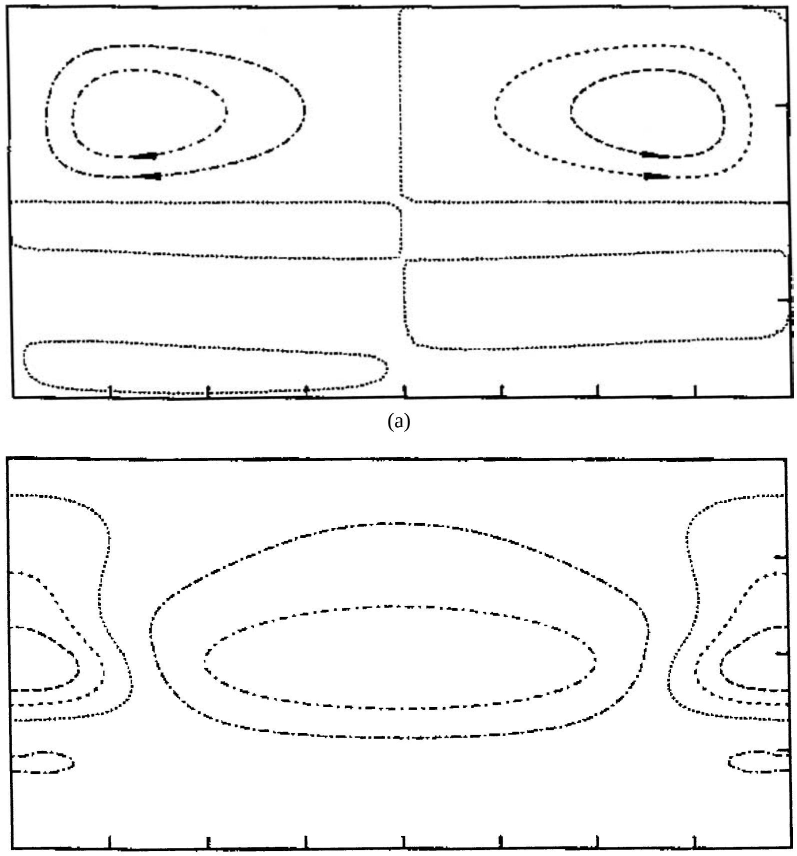


Fig. 1. Neutral curves for $G_Q > 0$ (line 1) and $G_Q < 0$ (line 2).

characterized by the nearly vanishing temperature gradient in the bottom layer, and the strong positive gradient in the top layer.

Now let us present the results of the nonlinear simulation of the two-dimensional anticonvection in the above mentioned system. The calculations were made with $L = 4$, $a = 1$. Recall that in the absence of heat release the anticonvection in this system is impossible. The nonlinear simulation confirms the existence of anticonvection. A typical anticonvective flow structure in the case of heat sources ($G = 3717$, $G_Q = 6000$) is shown in Fig. 2. The streamline patterns are shown in Fig. 2(a), while Fig. 2(b) presents the fields of the temperature deviations $T_m(x,z) - T_m^0(z)$, where $T_m^0(z)$ is the equilibrium temperature field. In the middle of the cavity, there exists a negative deviation of the temperature (Fig. 2(b)) that produces an extensive descending motion in the top layer (with nearly vanishing temperature gradient), and a relatively weak viscosity induced ascending motion in the bottom layer (with the strong positive gradient) near the interface (Fig. 2(a)). However, because of the difference between the temperature gradients in both fluids, heat transfer to the boundary caused by both motions is balanced, so that a stationary anticonvective flow takes place. Near the lateral boundaries, a positive deviation of the temperature generates an ascending motion in the top layer and a descending motion near the interface in the bottom layer, in a similar way. Note that thermal and viscous coupling between two layers generates a multi-store flow structure in the bottom layer. Let us note that at the same values of G and G_Q another type of the anticonvective motion is possible which is characterized by a positive deviation of the temperature in the middle of the cavity and a negative deviation of the temperature near the lateral boundaries. In the opposite case $sG < G_Q < 0$ ($0 < A_2 \leqslant A_1$) the most intensive motion is realized in the bottom layer, and the weak, induced motion with a multi-store structure appears in the top layer.



(a)

(b)

Fig. 2. (a) The streamline patterns, –– 0.0157, - - 0.00787, ··· 0, -·- −0.00787, –·– −0.0157; and (b) the fields of the temperature deviations, –– 0.0036, - - 0.0022, ··· 0.000792, –·– −0.000613, -·- −0.00202.

4. Interaction of anticonvection and thermocapillarity in multilayer systems

Now we consider the case of multilayer system. Let a rectangular cavity with rigid boundaries be filled by three immiscible viscous fluids. Indices 1 and 3 are related to the exterior layers, and index 2 is related to the middle one. The plates are kept at different constant temperatures (the total temperature drop is θ). It is assumed that surface tension coefficients on the upper and lower interfaces σ and σ_* decrease linearly with temperature $\sigma = \sigma_0 - \alpha T$, $\sigma_* = \sigma_{0*} - \alpha_* T$. The Boussinesq approximation is used for the description of convection. Let us introduce the following notation:

$$\nu_* = \nu_3/\nu_1, \quad \nu = \nu_3/\nu_2, \quad \eta_* = \eta_3/\eta_1, \quad \eta = \eta_3/\eta_2, \quad \kappa_* = \kappa_3/\kappa_1, \quad \kappa = \kappa_3/\kappa_2$$

$$\chi_* = \chi_3/\chi_1, \quad \chi = \chi_3/\chi_2, \quad \beta_* = \beta_3/\beta_1, \quad \beta = \beta_3/\beta_2$$

$$\alpha = \alpha_*/\alpha, \quad a_* = a_1/a_3, \quad a = a_2/a_3, \quad L = l/a_3$$

Here ν_m, η_m, κ_m, χ_m, β_m and a_m are, respectively, kinematic and dynamic viscosities, heat conductivity, thermal diffusivity, heat expansion coefficient and the thickness of the mth layer ($m = 1, 2, 3$).

At the interfaces, normal components of velocity vanish and the continuity conditions for tangential components of velocity and viscous stresses, temperatures and heat fluxes also apply.

We consider the model system heated from above which is characterized by the following set of parameters: $\eta = 0.2$, $\nu = 1$, $\kappa = 0.1$, $\chi = 0.1$, $\beta = 0.01$, $\eta_* = 0.04$, $\nu_* = 1$, $\kappa_* = 0.1$, $\chi_* = 0.07$, $\beta_* = 0.01$, $\alpha = 1$, $a = a_* = 1$. This choice is based on the fact that this system displays an anticonvective instability [20]. With the increase of the Grashof number, the mechanical equilibrium state becomes unstable and the steady motion is realized in the system as $G_* \geqslant 2900$. For the system under consideration, conditions for the appearance of the anticonvection are satisfied on the upper interface. The thermocapillary convection ($M \neq 0$, $G = 0$) also appears near the upper interface. Unlike the case of anticonvection, the intensities of the thermocapillary convection in both top and middle layers are of the same order. The directions of rotation coincide for anticonvective and thermocapillary motions.

The qualitatively new situation appears if both instability mechanisms act on different interfaces [21]. Such an indirect interaction is possible only in multi-layer systems. With the change in α (all the other parameters are the same), the role of two interfaces in the generation of the thermocapillary convection ($M \neq 0$, $G = 0$) also changes. If $1 < \alpha < 180$ the thermocapillary convection is generated by both interfaces, and if $\alpha > 180$, the thermocapillary motion takes place mainly near the lower interface. We take the previous system with $\alpha = 200$. With the increase of M the mechanical equilibrium state becomes unstable and the oscillatory motion appears in the system. Not far from the threshold, oscillations have a rather simple, almost sinusoidal form. For $M > 17500$ oscillations become unstable and the steady motion takes place in the system. Inclusion of G may lead to the destruction of oscillations of the thermocapillary nature (for the relatively small values of G and M) and establishment of the steady state in the system. At the larger values of G and M the new type of oscillations, essentially connected with the indirect interaction of both mechanisms of instability is realized. We would like to emphasize that even in the situation when for pure anticonvection and for pure thermocapillary convection only the steady motion takes place, the combined action of both mechanisms of instability may lead to the appearance of oscillations.

5. Conclusion

Anticonvection generated by the joint action of the external heating and heat sources (sinks) on the interface in the layers with finite thicknesses is studied. Numerical simulations of the finite-amplitude anticonvective regimes have been made for the real two-liquid system. Anticonvective structures in fluid systems subject to the anticonvective instability only in the presence of heat sources (sinks) on the interface have been obtained. We considered the system of three immiscible viscous fluids with undeformable interfaces filling a closed cavity when heating is from above. Direct and indirect interactions of anticonvective and thermocapillary mechanisms of instability are studied. The specific type of convective oscillations, essentially connected with the indirect interaction of both mechanisms of instability is discussed.

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