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# Use of turbulent flow statistical properties for correcting erroneous velocity vectors in PIV

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## Abstract

For particular turbulent flows, PIV measurements technique provides more than 10% of spurious velocity vectors at each time step. To correct these vectors, we propose to use the Linear Stochastic Estimation (LSE) dealing with the spatial correlation tensor of the velocity. If this tensor cannot be determined in some measurement zones, Proper Orthogonal Decomposition is used to model this tensor. Both reconstruction methodologies are tested from PIV measurements performed in a Spark Ignition engine flow. We show that for coherent structures analysis, the LSE reconstruction method provides better results than classical mathematical interpolation methods. *To cite this article: Ph. Druault, Ph. Guibert, C. R. Mecanique 332 (2004).*

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## Résumé

**Utilisation de l'Estimation Stochastique pour corriger les vecteurs vitesse PIV erronés.** Les mesures PIV peuvent entraîner pour des écoulements turbulents particuliers près de 10 % de vecteurs vitesse erronés. Pour corriger ces vecteurs, on propose d'utiliser l'Estimation Stochastique Linéaire basée sur le tenseur de corrélation spatiale de vitesse. Si ce tenseur n'est pas partout accessible, la Décomposition Orthogonale aux valeurs Propres est alors utilisée pour reconstruire ce tenseur. Une validation de ces procédures de reconstruction réalisée à partir de mesures PIV en écoulement de moteur a ainsi montré l'intérêt pour l'analyse des structures cohérentes d'utiliser la LSE par rapport aux méthodes d'interpolations mathématiques couramment utilisées. *Pour citer cet article : Ph. Druault, Ph. Guibert, C. R. Mecanique 332 (2004).*

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*Keywords:* Turbulence; Coherent structures; PIV; Stochastic Estimation; Proper Orthogonal Decomposition

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## Version française abrégée

L'amélioration de la connaissance et la maîtrise de l'aérodynamique interne des moteurs à allumage commandé passe par une analyse fine des structures tourbillonnaires dites cohérentes de l'écoulement et de la partie turbulente dite incohérente. En effet, ces structures ainsi que la turbulence jouent un rôle majeur sur l'émission de polluants et sur la combustion en permettant notamment l'optimisation du mélange. Des mesures instantanées du champ de vitesse obtenues par PIV (Particle Image Velocimetry) sont alors particulièrement enrichissantes pour l'analyse de ces structures. Toutefois, sous certaines conditions d'opération, de telles mesures génèrent plus de 10 % de vecteurs vitesse erronés dus aux éventuels problèmes d'ensemencement, de déplacement de particules, de réflexions sur les parois solides ou autres. L'objectif de cette étude consiste à proposer une procédure de reconstruction de ces champs de vitesse permettant de conserver l'organisation spatio-temporelle de l'écoulement. Des méthodes d'interpolation mathématique basées sur les plus proches voisins [3,4] sont généralement utilisées. Il existe également quelques méthodes basées sur des critères plus physiques [2,5] comme le champ de vorticité ou les lignes de courant. Dans cette étude, nous proposons d'utiliser l'Estimation Stochastique Linéaire (LSE) [7] pour reconstruire ces données de vitesse. La LSE permet de reconstruire les vecteurs vitesse erronés à partir de la connaissance des vecteurs vitesse *corrects* et du tenseur de corrélation spatiale en deux points de la vitesse (voir équations (1) et (2)). Cette estimation nécessite la connaissance de ce tenseur pour tout couple de points du maillage qui peut ne pas être accessible en raison de problèmes de convergence ou d'un manque de données de vitesse en une même zone de la section de mesure. Nous proposons alors d'utiliser la Décomposition Orthogonale aux valeurs Propres (POD) pour reconstruire ce tenseur. La démarche suivie est la suivante : (i) Réalisation d'une POD à partir du tenseur de corrélation calculé directement à partir des mesures PIV brutes du champ de vitesse ; (ii) Modélisation des vecteurs propres à l'aide d'interpolations cubiques par splines ; (iii) orthonormalisation des vecteurs propres modélisés ; et (iv) reconstruction du tenseur de corrélation à l'aide de l'équation 3. A partir de la connaissance de ce tenseur sur un maillage quelconque, il est alors possible d'estimer *via* la LSE les vecteurs vitesse en tout point de ce maillage.

Ces procédures de reconstruction ont été validées en utilisant des mesures PIV réalisées pendant la phase d'admission dans le cylindre d'un moteur à allumage commandé. Le plan de mesure correspond à celui du tumble c'est-à-dire le plan contenant l'axe du cylindre où l'écoulement a un mouvement de rotation. Le système PIV La-Vision a été utilisé. La configuration expérimentale ainsi que le dispositif de mesure sont similaires à ceux détaillés dans [10]. Les outils de post traitement permettant d'annuler les vecteurs vitesse erronés sont détaillés dans [10] et ont montré qu'environ 10 % des vecteurs obtenus étaient faux. Les procédures de reconstruction du champ de vitesse ont alors été mises en œuvre. De plus, durant la phase de mesure d'un plan particulier, des problèmes de réflexion sont apparus nécessitant la reconstruction des corrélations car celles-ci n'étaient pas statistiquement convergées. Une illustration des différentes procédures de reconstruction proposées dans ce travail est donnée sur les Figs. 1 et 2. Des différences notables ont été observées entre les résultats issus de la LSE et des méthodes classiques d'interpolation par splines notamment lorsqu'une zone assez large contient des vecteurs erronés confirmant les résultats de [11]. Dans cet article, on présente une analyse comparative où les résultats issus de ces deux méthodes d'interpolations semblent similaires (voir Fig. 2 (c) et (d)). Toutefois, les champs de vorticité issus respectivement de ces deux champs de vitesse mettent en évidence de notables différences (Fig. 2 (e) et (f)). Ce résultat montre donc que pour une analyse fine de la structuration d'un tel écoulement, il est indispensable de bien reconstruire les vecteurs vitesse erronés en prenant en compte la physique de l'écoulement par l'intermédiaire des corrélations spatiales de vitesse. Par ailleurs, une telle approche peut également être étendue à la génération de vitesse sur un maillage quelconque et aux mesures PIV tridimensionnelles (3 composantes–3 directions) nécessitant des interpolations.

## 1. Introduction

Gas motion within the engine cylinder is one of the major factors that controls the combustion process as well as the fuel air mixing, and also has a significant impact on heat transfer. The initial flow pattern is set up by the intake

process and gives the important features of gaz motion (coherent structures) which drive the overall location of the mixture and the specific mixture strategy. It has also been demonstrated that turbulence is mainly generated during the compression process due to transfer energy from rotating structures. Thus, the analysis of the instantaneous flow coherent structures and the flow turbulent incoherent part in a confined Spark Ignition (SI) engine is of great importance for engine efficiency. The Particle Image Velocimetry (PIV) measurement method seems well adapted for current investigations. PIV is a powerful technique for determining the instantaneous velocity field in a flow-section by performing a correlation analysis on an image of particles transported by the flow. This measurement technique requires some efficient post-processing algorithms, including the detection of the incorrect or spurious velocity vectors and the removal of these vectors in order to enhance PIV performance. Indeed, even with highly homogeneous seeding, insufficient local density of particle-images may occur or a non-uniform particle displacement distribution within the interrogation area or some out-of-plane velocity or meshing bias. Moreover, when a solid boundary is located inside the interrogation window (as for SI engine flow measurements), the PIV measurement requires particular attention due to the possible problems of reflections and of dirty windows, leading spurious velocity vectors being obtained.

Many procedures used to detect erroneous vectors have been already tested in the past and generally they are based on image correlation errors or/and on filtering procedures and/or on physical velocity threshold measurements (modulus, direction, velocity range, etc.) ([1–3] among others). This analysis will not be discussed in this article. In this study, we focus on methods permitting the reconstruction of the instantaneous erroneous velocity field on the mesh grid of the PIV window or eventually on any mesh.

Usually, mathematical tools such as spline interpolation from the 8 neighbourhood [3] or adaptative Gauss window [4] are used. Algorithms based on physical criteria such as the vorticity field or streamlines [2,5] can also be used for substituting erroneous vectors. Artificial neural networks [6] have also been developed for such an investigation. Here we propose a new reconstruction method: the Linear Stochastic Estimation (LSE), dealing with the two point spatial correlation tensor of the velocity. LSE is then implemented to reconstruct the instantaneous fluctuating velocity field  $u'(X, t)$  deduced from the Reynolds decomposition ( $u(X, t) = \overline{U(X)} + u'(X, t)$ , where an overline indicates the conventional time average).

## 2. Linear Stochastic Estimation

The Stochastic Estimation was initially introduced by Adrian [7] in order to provide a conditional estimate of the large scale structures present in turbulent flows. Such an estimation uses the conditional information (here the instantaneous fluctuating velocity field) known at  $X_r = (x_r, y_r, z_r)$  locations called reference locations in conjunction with its statistical properties to estimate the information at surrounding locations  $X = (x, y, z)$ . In this study, the condition is different at each time step due to the random character of the erroneous instantaneous velocity vector. Moreover, due to 2D PIV measurements the third direction  $z$  is not taken into account. Then a LSE of the instantaneous fluctuating velocity component  $u_i^{\text{est}}(t)$  at  $X$  location is expressed as follows:

$$u_i^{\text{est}}(X, t) = \sum_{k=1}^{n_c} \sum_{r=1}^{n_r} A_{ik}(X, X_r) u'_k(X_r, t) \quad (1)$$

where  $n_c$  is the number of velocity components taken into account ( $n_c = 2$ ) and  $n_r$  indicates the number of reference points where the *correct* velocity field is known. For each equation, the coefficients  $A_{ik}(X, X_r)$  are determined using the following equation:

$$A_{ik}(X, X_r) \overline{u'_i(X) u'_j(X)} = \overline{u'_k(X_r) u'_j(X)} \quad (2)$$

where  $\overline{u'_i(X) u'_j(X)}$  and  $\overline{u'_k(X_r) u'_j(X)}$  are the Reynolds tensor and the two-point correlation tensor, respectively. LSE procedure is able to interpolate or extrapolate the velocity field on any mesh providing that the correlation

tensor is known on the required mesh. If the correlation tensor cannot be accessed everywhere due to problems of statistical convergence, we have to reconstruct this tensor before performing LSE procedure.

### 3. Modeling of the two point spatial correlation tensor

The modeling of the spatial correlation tensor can be performed with Hermite polynomials [8]. However, due to the possible important lack of data and the particularities of confined engine flows, such modeling is not adapted and it cannot provide satisfactory results. We then propose to use the Proper Orthogonal Decomposition (POD) [9] for reconstructing the spatial correlation tensor on any mesh. The POD approach used for studying coherent structures in turbulent flows decomposes the fluctuating velocity field into a basis set of optimal in the mean square sense, orthogonal eigenfunctions  $\Phi^{(n)}(X)$ . This approach leads to an integral eigenvalue problem whose kernel is the spatial correlation tensor that can be expressed as follows:

$$\overline{u'_i(X_1)u'_j(X_2)} = \sum_{n=1}^{N_{\text{mod}}} \lambda^{(n)} \Phi_i^{(n)}(X_1) \Phi_j^{(n)}(X_2) \quad (3)$$

where  $\lambda^{(n)}$  are the eigenvalues and  $N_{\text{mod}}$  is the total number of POD eigenfunctions. The implemented correlation reconstruction procedure is now detailed. We previously observed that it needs at least 600 instantaneous PIV velocity fields to obtain a statistically well converged spatial correlation tensor. In this sense, when we have not enough velocity fields due to local erroneous velocity vectors, we put to zero the corresponding value of the calculated tensor. From this original correlation tensor, a POD is performed. Resulting POD modes then contain zero values at several grid points. Due to the limited number of these grid points, each POD eigenfunction is reconstructed with the spline interpolation method. Finally, all reconstructed POD modes are orthonormalized and the spatial correlation tensor is estimated from Eq. (3) using all modes. Note that here we propose to use all POD eigenfunctions in order to limit the errors, even if we observed that the reconstructed spatial tensor arising from the modelled POD modes corresponding to 90% of the total kinetic energy is quasi exactly the same as the one obtained with all modes.

### 4. Application to PIV measurements in a SI engine flow

Such reconstruction methodologies have been tested from PIV measurements obtained in a SI engine flow. This flow test case is particularly suited to current methodology due to the large velocity gradients and the great number of time-space scales present in such confined flow, leading to an important number (around 10%) of spurious vectors for particular operating conditions.

A double exposure image processed with LaVision PIV system is used. The full twin time sequence PIV system and its post processing procedures are similar to those detailed in [10]. Briefly, the PIV algorithm is cross correlation based on an initial interrogation window of  $32 \times 32$  pixels with a step width of 16, yielding 50% overlap. Images are then classically post-processed by eliminating erroneous vectors in putting them to zero. Finally velocity vectors are available on a regular mesh of  $33 \times 36$  points. We focus on the early stage of the admission (Fig. 1(a)) by performing PIV measurements at different stages of this early stage. More than 700 instantaneous vectors velocity field is obtained for each measurement section. Before using LSE procedure, we calculate the spatial correlation tensor from the *correct* velocity field. We only implement the correlation tensor reconstruction procedure in one measurement plane because of the many reflections occurring during these measurements, leading us to obtain zero values for the correlation tensor (Fig. 1(b)). POD methodology is then used to model this tensor. Firstly, each POD mode is reconstructed with mathematical tools. As an illustration, the second POD mode relative to the streamwise velocity component  $u_1$  is given on Fig. 1 (c) and (d). Secondly, following Eq. (3), the spatial correlation

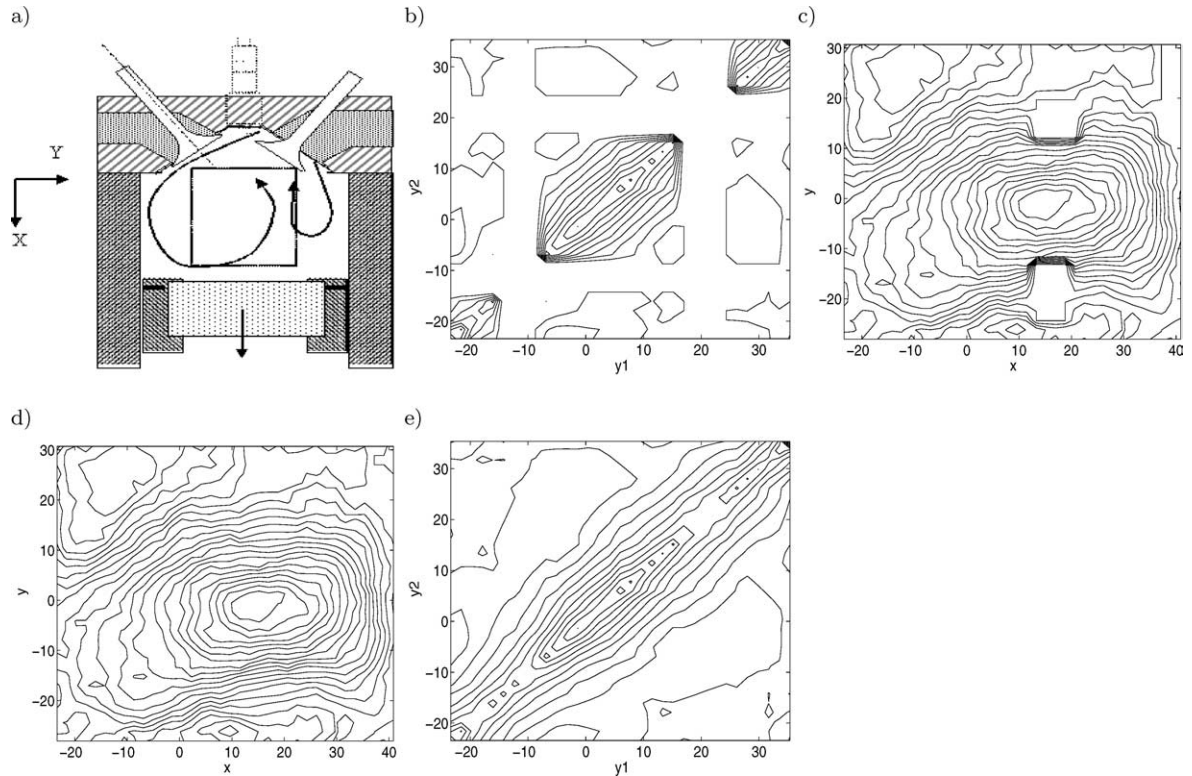


Fig. 1. (a) Engine flow pattern in the tumble plane – early stage of the admission ; (b) Transverse correlation tensor  $R_{u_1 u_1}(y_1, y_2)$  for the streamwise location  $x = 15.15$  – Equally spaced isocontours from  $-1$  to  $10$ ; (c) Original POD eigenfunction  $\Phi_{u_1}^{(2)}$  – Equally spaced isocontours from  $-0.03$  to  $0.075$ ; (d)  $\Phi_{u_1}^{(2)}$  reconstruction (same isocontours as (c)); (e)  $R_{u_1 u_1}(y_1, y_2)$  reconstruction based on all POD eigenfunctions (same isocontours as (c)).

Fig. 1. (a) Ecoulement étudié dans le plan du tumble – phase d’admission; (b) Corrélation transversale  $R_{u_1 u_1}(y_1, y_2)$  pour la position longitudinale  $x = 15,15$ ; (c) Deuxième mode propre POD  $\Phi_{u_1}^{(2)}$ ; (d) Modélisation de  $\Phi_{u_1}^{(2)}$ ; (e) Reconstruction de  $R_{u_1 u_1}(y_1, y_2)$  à partir de tous les modes POD reconstruits.

is modelled using the reconstructed POD modes (see Fig. 1(e) for a selected streamwise location). Finally, LSE is used to reconstruct the lack of velocity vectors for each measurement section. This LSE reconstruction is compared to the classical spline interpolation method. Spline interpolation provides *satisfactory* results except if the distance between two consecutive *correct* vectors is large, confirming previous results [11]. Fig. 2 gives an illustration of an instantaneous reconstruction velocity field. We voluntary chose an instantaneous velocity field where splines interpolation seems to well work. But even if Fig. 2 (c) and (d) seem quite similar; some noticeable differences are observed on both resulting vorticity field representations (Fig. 2 (e) and (f) respectively) particularly on the intensity and the gradient of these vorticity fields. Note that spatial derivatives are calculated with a second order centered differences scheme. Knowing that instantaneous structures are of importance for the engine efficiency, it is then essential to reconstruct the velocity field, preserving the flow organization. Finally, the LSE procedure has the advantage in providing a better physical velocity field reconstruction and can be easily extended to the reconstruction of the velocity field on any mesh and to the three dimensional-three components PIV measurements needing some velocity field interpolations.

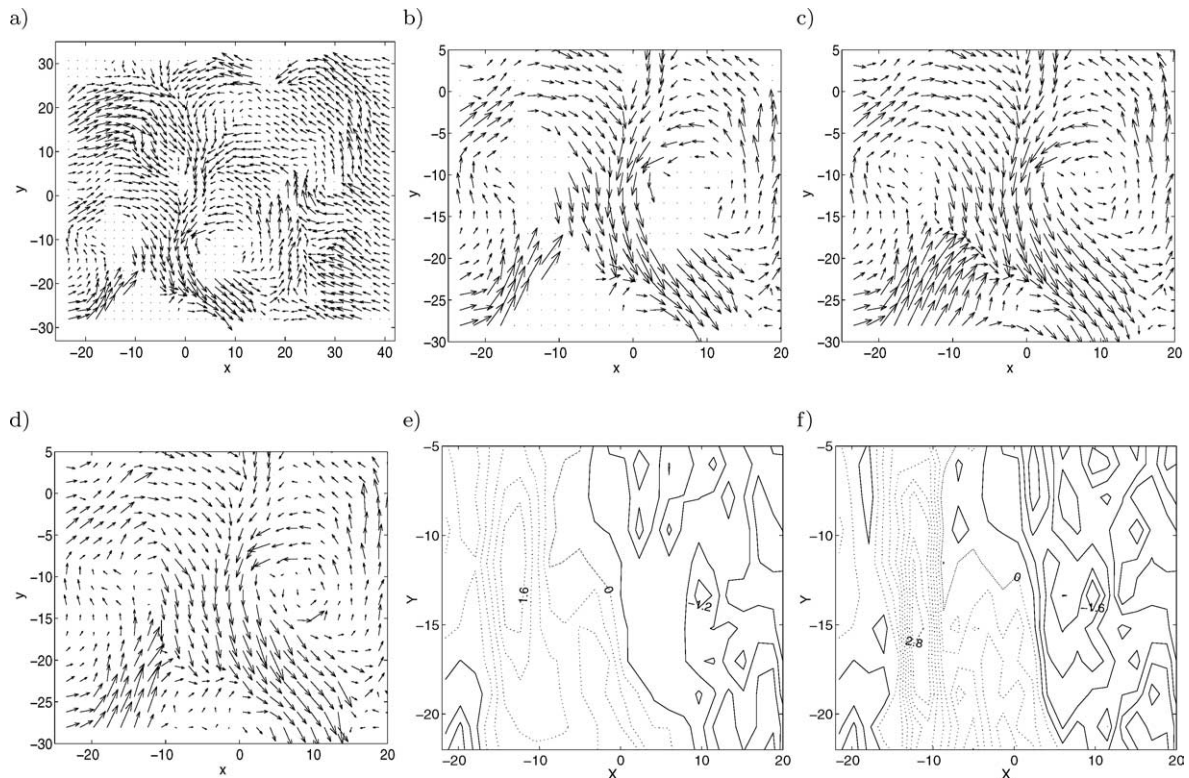


Fig. 2. Instantaneous vector velocity field in the tumbling plane : (a) original field with erroneous vector put to zero ; (b) zoom of (a) ; (c) spline interpolation velocity field ; (d) LSE reconstruction velocity field ; (e) and (f) Isocontours of the vorticity fields arising from (c) and (d), respectively. Contour increment is 0.4 and positive contours are dashed.

Fig. 2. (a) Champ de vitesse instantané brut, les vecteurs erronés ont été mis à zéro ; (b) zoom de la figure (a) ; (c) Interpolation cubique par splines du champ de vitesse ; (d) Reconstruction du champ de vitesse par LSE ; (e) et (f) Isocontours des champs de vorticité issus respectivement des figures (c) et (d).

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