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# Validation of averaged equations for thermal vibrational convection in near-critical fluids

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**Abstract**

Within an averaging approach, the governing equations and effective boundary conditions describing both the average and pulsation motion of a near-critical fluid subjected to high-frequency vibrations are obtained. Vibrations induce the non-homogeneities in average temperature. Owing to these non-homogeneities, the average flows can be generated even in isothermal cavity under weightlessness. These flows are examined for 1D and 2D configurations. The direct numerical simulations fulfilled earlier confirm the averaged model, we obtain the same flow structures by essentially smaller requirements for computational time. ***To cite this article: A.Vorobev et al., C. R. Mecanique 332 (2004).***


**Résumé**

**Validation d'équations moyennées pour l'étude de la convection vibrationnelle dans les fluides supercritiques.** La procédure de moyennisation nous permet d'obtenir les équations du mouvement d'un fluide supercritique soumis à des vibrations de haute fréquence et les conditions aux limites associées. La vibration génère des non-homogénéités du champ de température moyen. En raison de ces non-homogénéités, l'écoulement moyen existe même au sein d'une cavité isotherme et en micropesanteur. Cette nouvelle formulation nous a permis d'analyser de façon commode des structures d'écoulement 1D et 2D. Les résultats obtenus à partir de la formulation des équations moyennées sont en accord avec des résultats de simulation numériques directe précédemment obtenus et nécessitant des temps de calculs excessifs. ***Pour citer cet article : A.Vorobev et al., C. R. Mecanique 332 (2004).***


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## Version française abrégée

Les vibrations de hautes fréquences sont connues pour induire un écoulement moyen [1]. On se propose dans ce travail d'analyser le mécanisme à l'origine des instabilités convectives au sein des fluides critiques lorsque les vibrations sont à l'origine de la non-homogénéité du champ de température.

Le premier objectif est l'obtention du modèle des équations moyennées permettant de décrire les écoulements générés, en micropesanteur, par les vibrations au sein d'une cavité isotherme remplie de fluide au voisinage du point critique. Le forts couplages thermoacoustiques dans les couches limites thermiques modifient les conditions aux limites dynamiques pour les équations dans le coeur. Le modèle est ensuite utilisé pour étudier le quasi-équilibre dans la couche plane infinie et pour analyser l'écoulement stationnaire dans une cavité carrée. L'objectif final est de pouvoir comparer les résultats obtenus à partir de cette approche aux résultats de simulation numérique directe des équations de Navier–Stokes, pour les fluides supercritiques, obtenus précédemment dans la même situation [2]. Ce qui permet de valider cette nouvelle approche.

Au voisinage de son point critique le fluide fortement compressible est assimilé à un fluide newtonien et ses écoulements sont régis par les équations de Navier–Stokes. Ce formalisme est simplifié si l'on considère les ordres de grandeurs et les échelles de temps et d'espaces des différents termes intervenant dans ces équations et que l'on applique une méthode d'échelles multiples.

L'hypothèse des hautes fréquences conduit à $a \sim \delta \ll L \ll \lambda$, où $a$ désigne l'amplitude des vibrations, $\delta$ l'épaisseur de la couche limite, $L$ une échelle de longueur associée aux dimensions de la cavité, $\lambda$ est la longueur d'onde de la célérité du son. En utilisant ces différentes hypothèses, on montre qu'il est possible d'étudier séparément l'écoulement dans la partie centrale et l'écoulement dans la couche limite.

Il est important d'accéder au comportement du système pour des échelles de temps supérieures à l'échelle du temps associée à la vibration. L'approche basée sur la moyennisation nous permet de décomposer le système d'équations régissant l'écoulement en deux parties, un système permettant l'analyse des champs moyens et un autre pour l'analyse des fluctuations.

En microgravité, nous analysons l'action des vibrations de hautes fréquences, agissant dans une direction fixée sur une cavité totalement isotherme renfermant un fluide au voisinage de son point critique. La faible non-uniformité de la température qui en résulte est uniquement due à la vibration. L'ordre de grandeur de cette faible inhomogénéité de température nous permet de définir le paramètre de Boussinesq. En utilisant la procédure de moyennisation on obtient les équations pour les champs moyens et les équations pour les fluctuations.

En utilisant la formulation obtenue, nous avons résolu le cas de la couche infinie, en microgravité et soumise à des vibrations perpendiculaires aux plans de la couche de fluide. Pour la situation de quasi équilibre, nous avons obtenu les champs de température et de vitesse associés aux fluctuations. Le champ de vitesse obtenu est identique à celui obtenu au niveau de la référence [5].

Nous avons ensuite analysé, le cas d'une cavité carrée isotherme remplie de fluide au voisinage du point critique et soumise á des vibrations de direction parallèle à l'un des cotés de la cavité en situation de microgravité. L'origine de l'écoulement dans cette situation est attribuée uniquement à l'hyper compressibilité du fluide. Nous avons montré que l'écoulement moyen n'est possible que si l'intensité des vibrations est suffisamment élevée.

En raison de l'importance du temps de calcul nécessaire pour mener à bien la simulation directe, seuls les champs fluctuants ont été décrits [2]. Seuls donc ces écoulements ont fait l'objet de comparaison avec nos résultats. Ce qui nous permet de valider le modèle élaboré uniquement pour ce qui est des champs fluctuants. Nous montrons également que ce résultat peut-être obtenu à partir de la résolution d'équations linéaires simples du premier ordre (Fig. 1).

Cette nouvelle formulation nous permet d'accéder à la description des champs moyens de l'écoulement, en résolvant numériquement, par une méthode aux différences finies les équations obtenues. La structure des isothermes et des lignes de courant est représentée sur Fig. 2.

## 1. Introduction

High-frequency vibrations are known to induce the average flows in non-uniformly heated fluid (the so-called thermal vibrational convection [1]). The present paper is devoted to the investigation of specific mechanism of convective instability in a near-critical fluid, when only vibrations induce the temperature non-homogeneities.

We derive the averaged model describing the vibrational flows of near-critical fluid filling an isothermal cavity subjected to translational vibrations in weightlessness. Later on, the derived mathematical model is applied for consideration of quasi-equilibrium in a plane layer, and for analysis of the steady flows in a cylindrical cavity of square cross-section. The final objective is a comparison of the results with direct numerical simulations of the full Navier–Stokes equations for compressible medium [2]. In this way, we are able to verify the averaging approach.

## 2. Assumptions

In the vicinity of the liquid–gas critical point, the thermodynamic parameters and transport coefficients exhibit strong anomalies, e.g., isothermal sound speed $a_T$ tends to zero; thermal expansion coefficient $\beta$, heat capacities $c_p,\ c_V$, and thermal conductivity $\kappa$ diverge. The singularity exponent for each parameter is different, e.g., the thermal diffusivity $\chi = \kappa/\rho c_p$ tends to zero. We also assume that the singularities of isochoric heat-capacity $c_p$, sound speed $c$, and dynamical viscosity $\eta$ are not strong for the considered distance from the critical point; these coefficients are assumed constant.

Consideration of fluid behaviour is fulfilled within a hydrodynamic approach. A near-critical fluid is considered to be a compressible Newtonian fluid, its motion is described by the Navier–Stokes equations.

These equations are simplified if we consider the orders of magnitudes of typical time and space scales and then apply the multiple-scale method. Namely, the frequency of vibrations $\omega$ is such that $a \sim \delta \ll L \ll \lambda$, where $a$ is the amplitude of vibrations, $\delta$ is the boundary layer thickness (viscous layer, $\delta_\nu = \sqrt{\nu/2\omega}$ or thermal, $\delta_\chi = \sqrt{\chi/2\omega}$; $\nu = \eta/\rho$ is kinematic viscosity, and $\omega$ is frequency of vibrations), $L$ is a typical size of geometry, $\lambda$ is the wavelength of sound. Based on these assumptions, hydrodynamic fields in the bulk and in the boundary layer can be considered separately.

The behaviour of a system on timescales certainly higher than the vibration period needs to be known, $\omega \gg (\chi/L^2, \nu/L^2)$. The averaging description enables us to decompose the governing system of equations into two parts: for average and pulsation fields. All physical fields $F$ are represented as a sum of average $\overline{F}$ and pulsation $\widetilde{F}$ components. For average and pulsation components of the velocity we use different notations, namely $\vec{u}$ and $\vec{w}$.

The vibration period and the typical time scale of the piston effect [3] are related as $\omega\tau_{\mathrm{PE}} \sim (L/\gamma\delta_\chi)^2$ (where $\gamma = c_p/c_V$ is the specific-heat ratio); this combination is assumed to be finite.

## 3. Governing equations and effective boundary conditions

We consider the influence of translational harmonic vibrations ($\vec{w}_0 = a\omega\vec{j} = a\omega\vec{j}_0\cos(\omega t)$ is pulsation velocity; $\vec{j}_0$ is a unit vector) on a near-critical fluid filling a container with isothermal boundaries. Weightlessness is assumed.

Vibrations induce the small non-homogeneities in temperature. Based on these non-homogeneities we construct the Boussinesq parameter $\beta\theta_w = a^2\omega^4 L^2/(4a_T^2) = \epsilon \ll 1$ [4]. Having applied the averaging procedure [1], the problem for average fields are obtained:

$$\begin{aligned}
&\frac{1}{Pr}\left(\frac{\partial \vec{u}}{\partial t} + (\vec{u}\nabla)\vec{u}\right) = -\nabla\Pi + \Delta\vec{u} + 2\big(\overline{(\vec{w}^{(1)} + \vec{w}^{(2)})\cdot\vec{j}}\big)\nabla T + \frac{\sigma-1}{2}Ra_a(\vec{j}_0\cdot\vec{r})^2\nabla T \\
&\nabla\cdot\vec{u} = 0, \qquad \frac{\partial T}{\partial t} + (\vec{u}\nabla)T = \Delta T + \frac{\partial}{\partial t}\langle T\rangle, \quad \langle T\rangle = \frac{1}{V}\int_V T\,\mathrm{d}V
\end{aligned} \tag{1}$$

Here $\overline{(\ldots)}$ means time averaging over period of vibrations; $\vec{r}$ is a non-dimensional radius-vector, $\Pi$ is a modified pressure.

The equations are similar to the usual equations of vibrational convection [1]; however, the new (last) term in momentum equation appears due to compressibility. In the equation of heat-transfer there appears a term describing the changes in thermodynamic part of pressure [5].

We divide the pulsation field $\vec{w}$ into two parts: $\vec{w}^{(1)}$ (related to the non-homogeneities in pulsation velocity due to temperature variations) and $\vec{w}^{(2)}$ (related to the effects of compressibility). The problem for pulsation fields is as follows:

$$\begin{aligned}
&\nabla\times\vec{w}^{(1)} = Ra_v\nabla T\times\vec{j}, \qquad \nabla\cdot\vec{w}^{(1)} = 0, \qquad \vec{w}_n^{(1)} = 0 \\
&\nabla\times\vec{w}^{(2)} = 0, \qquad \nabla\cdot\vec{w}^{(2)} = -Ra_a(\vec{j}\cdot\vec{r}), \qquad \vec{w}_n^{(2)} = Ra_{\mathrm{PE}}(\vec{j}_0\cdot\vec{r})\big(\cos(\omega t) + \sin(\omega t)\big)
\end{aligned} \tag{2}$$

The additional pulsation field is generated in the boundary layer (for perfectly heat-conductive walls).

The boundary conditions for average fields can be written:

$$T = -2(\vec{j}_0\cdot\vec{r})^2, \qquad \vec{u} = 0 \tag{3}$$

The effective boundary condition for average temperature results in fundamentally new mechanism for generation of the average flow. Usually vibrations can bring to average flow through the vibrational force or through the Schlichting boundary condition [1]. Both these mechanisms do not work for an isothermal cavity subjected to uniform vibrations. However, vibrations are able to induce the non-homogeneities in temperature, which result in the onset of average convective flows.

Governing equations (1), (2) and boundary conditions (3) are written down in non-dimensional form. Units of length, time, average velocity and temperature are $L$, $L^2/\chi$, $\chi/L$, $\theta_w = a^2\omega^4L^2/(4a_T^2\beta\cdot a_T^2)$. The scale for $\vec{w}^{(1)}$ and $\vec{w}^{(2)}$ is $2\nu\chi/(a\omega\beta\theta L^2)$. The following non-dimensional parameters are introduced:

$$\begin{aligned}
&\beta\theta_w = \frac{a^2\omega^4L^2}{4a_T^4}, \qquad Pr = \frac{\nu}{\chi}, \qquad \sigma = \rho_c\left(2\frac{p_{\rho T}}{p_T} - \frac{S_{\rho T}}{S_T}\right), \qquad Ra_v = \frac{1}{2}\frac{(a\omega L)^2}{\nu\chi}(\beta\theta_w)^2 \\
&Ra_a = \frac{1}{2}\frac{(a\omega L)^2}{\nu\chi}\beta\theta_w\left(\frac{\omega L}{c}\right)^2, \qquad Ra_{\mathrm{PE}} = \frac{1}{2}\frac{(a\omega L)^2}{\nu\chi}\beta\theta_w\left(\frac{\omega L}{c}\right)^2\gamma\frac{\delta_\chi}{L}
\end{aligned}$$

Here we use Prandtl number $Pr$, the new parameter accounting for the model of gas $\sigma$, vibrational Rayleigh number $Ra_v$, 'acoustic' $Ra_a$ and 'near-critical' $Ra_{\mathrm{PE}}$ analogues of the vibrational Rayleigh number. In these expressions, $\rho_c$ is critical density; we denote also $p_T = (\partial p/\partial T)_\rho$, $p_{\rho T} = \partial^2 p/(\partial\rho\partial T)$, etc.

The detailed derivation of Eqs. (1), (2) and effective boundary conditions (3) is given in [4].

System (1), (3) is applied for consideration of a fluid behaviour in an isothermal plane layer and isothermal cylindrical cavity of square cross-section. The model of a near-critical fluid described in [2,3] is used. Namely, van der Waals state equation, isochoric heat capacity and isotropic sound speed, and the following law for heat conductivity: $\kappa = \kappa_0(1 + \kappa_1 t^{-0.5})$, where $\kappa_0 = 10^{-2}\ \mathrm{W\,m^{-1}\,K^{-1}}$, $\kappa_1 = 0.75$, $t = (T - T_c)/T_c$ (values of $\kappa_0$, $\kappa_1$ for $CO_2$). Critical parameters for $CO_2$ ($T_c = 314$ K, $\rho_c = 468\ \mathrm{kg\,m^{-3}}$, $p_c = 7.38$ MPa; dynamical viscosity, $\eta_0 = 3.5\times10^{-5}\ \mathrm{kg\,m^{-1}\,s^{-1}}$) are applied.

## 4. One-dimensional results

Let us consider a plane layer filled with a near-critical fluid, subjected to transversal (along $z$-axis) vibrations. The profiles of pulsating temperature and velocity in quasi-equilibrium (i.e., in a state when $\partial/\partial t = 0$, and there is no average motion, $\vec{u} = 0$) are as follows:

$$\begin{aligned} &\beta\widetilde{T} = -\frac{a}{L}\left(\frac{\omega L}{c}\right)^2 \gamma z \sin(\omega t) \\ &w_z = \frac{1}{2}a\omega\left(\frac{\omega L}{c}\right)^2\left[\left(\frac{1}{4} - z^2\right)\cos(\omega t) + \gamma\frac{\delta_\chi}{L}\big(\cos(\omega t) + \sin(\omega t)\big)\right], \qquad w_x = 0 \end{aligned} \tag{4}$$

$x$ is a longitudinal coordinate. The velocity profile coincides with the result given in [3].

We made the estimations for the following physical situations (the same as in [2]): (a) $\Delta T = 15$ mK, $A = 0.1\ \text{m} \times \text{s}^{-2}$, $f = 5$ Hz, (b) $\Delta T = 0.1$ K, $A = 10\ \text{m} \times \text{s}^{-2}$, $f = 3$ kHz and (c) $\Delta T = 0.2$ K, $A = 0.1\ \text{m} \times \text{s}^{-2}$, $f = 20$ Hz. Here $\Delta T$ is a distance from the critical point, $A = a\omega^2$ is acceleration of oscillations, and $f = \omega/2\pi$ is frequency of oscillations. The thickness of the layer is $L = 1$ cm.

The amplitudes of the pulsation temperature oscillations, according to formula (4), are (a) $|\widetilde{T}| = 1.8 \times 10^{-6}$ K, (b) $|\widetilde{T}| = 1.8 \times 10^{-4}$ K, (c) $|\widetilde{T}| = 1.8 \times 10^{-6}$ K; the amplitude of temperature oscillations is determined by amplitude and frequency of the vibrational forcing.

## 5. Two-dimensional results (square cavity)

Let us consider an isothermal square container filled with a near-critical fluid under weightlessness. The container is subjected to translational vibrations along $z$-axis.

There are neither gravity nor external heating; the only reason for fluid motion is hyper-compressibility of a near-critical fluid. The effect of compressibility may cause the perceptible pulsation and especially the average flows only for sufficiently strong vibrations.

Let us estimate the non-dimensional parameters for the above-enumerated cases. The values of Prandtl number are: (a) $Pr = 73$, (b) $Pr = 47$, (c) $Pr = 33$. The vibrational Rayleigh number: (a) $Ra_v = 4.3 \times 10^{-10}$, (b) $Ra_v = 6.1 \times 10^{-7}$, (c) $Ra_v = 2.3 \times 10^{-17}$. The 'acoustic' Rayleigh number: (a) $Ra_a = 1.3 \times 10^{-7}$, (b) $Ra_v = 3.0$, (c) $Ra_v = 2.0 \times 10^{-7}$. The 'near-critical' Rayleigh number: (a) $Ra_{\text{PE}} = 2.4 \times 10^{-7}$, (b) $Ra_{\text{PE}} = 7.5 \times 10^{-2}$, (c) $Ra_{\text{PE}} = 3.7 \times 10^{-10}$.

The generation of average flows is possible only for the case (b) when vibrations are sufficiently strong. If gravity or external temperature gradients are added, the average flows become not negligible for more moderate vibrations.

The estimations also say how Eqs. (2), (3) can be further simplified. The only parameter proportional to the square of the small quantity $\beta\theta_w$ is vibrational Rayleigh number $Ra_v$. The estimations make clear that we can neglect the non-homogeneities in pulsation velocity denoted by $\vec{w}^{(1)}$.

In [2], in the part devoted to a square cavity, only pulsation fields for parameters of case (c) were obtained. Thus, we are able to justify the Eqs. (2) and boundary conditions (3) for pulsation fields. The results obtained with the help of finite-difference method are given in Fig. 1. This figure exactly reproduces the similar figures given in [2]. The $x$-component of the pulsation velocity is at $x = -\frac{1}{2}$, $z = -\frac{1}{4}$, $\omega t = 0$: $|w_x| = 1.1 \times 10^{-9}\ \text{m} \times \text{s}^{-1}$.

Finally the average fields for the case (b) were obtained numerically by finite-difference method. The structures of temperature and stream-function fields are plotted in Fig. 2. The resulting steady average flow has a four-vortex pattern. To obtain the dimensional values of the average temperature and velocity, the values from Fig. 2 must be multiplied by $a^2\omega^4L^2/4a_T^4\beta = 2.1 \times 10^{-7}$ K and $\chi/L = 2.3 \times 10^{-7}\ \text{m} \times \text{s}^{-1}$.

The amplitudes of average velocity and temperature non-homogeneities are small; nevertheless, they grow approaching the critical point, and they give not so small non-homogeneities in density and non-small governing

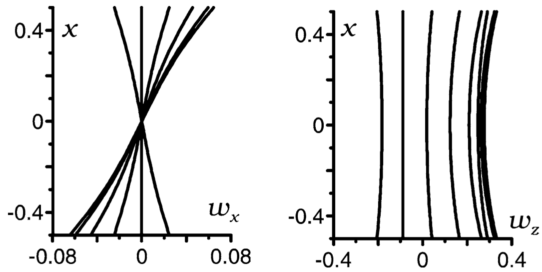


Fig. 1. Profiles of $x$-, transversal, (at $z = 0.25$) and $z$-, longitudinal, components (at $z = 0$) of the pulsation velocity at $\omega t = 0, \frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}, \frac{5\pi}{8}, \frac{3\pi}{4}, \frac{7\pi}{8}$. $\Delta T = 0.2$ K, $A = 0.1\ \mathrm{m\,s^{-2}}$, $f = 20$ Hz.

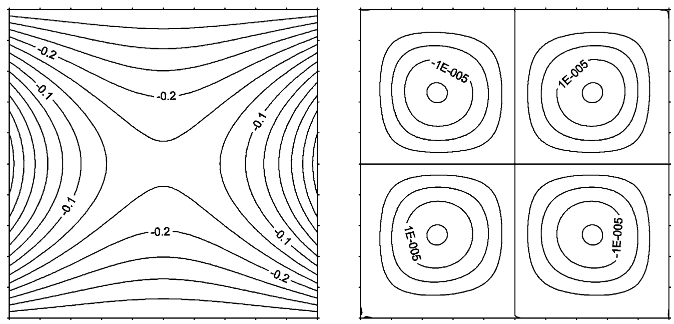


Fig. 2. Average temperature and velocity fields. $\Delta T = 0.1$ K, $A = 10\ \mathrm{m\,s^{-2}}$, $f = 3$ kHz.

non-dimensional parameters. In addition the structures of the flows obtained are close to the experimental results [6] of the 'ALICE' project. The convective slot subjected to transversal vibrations was filled with a near-critical fluid. A heater was set up in the centre fixing the background temperature. Under high-frequency vibrations, the average four-vortex flows (as in Fig. 2) were observed.

## 6. Conclusion

The description of vibrational convective flows induced only in hyper-compressible near-critical fluids is given. We derive the averaged equations and effective boundary conditions. Based on the model formulated we have examined two specific problems, already considered in [2] by direct numerical simulations of the full Navier–Stokes equations for a compressible viscous fluid.

Owing to restrictions for computational time only pulsating flows for moderate vibrational forcing (in 2D case, up to 20 Hz) were described in [2]. We showed that the same results for pulsating flows could be obtained from simple non-viscous linear equations. Moreover, the averaging approach allows us to consider more complex configurations and to study the structure of average fields.

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