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Thermodynamics modelling of plasticity and damage of argillite

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Abstract

The study of the physical and mechanical characteristics of argillites allows us to retain the state variables useful for the thermodynamic modelling of their behaviour. We built a thermodynamic potential which reproduces the nonlinear behaviour. Plasticity-damage coupling is taken into account by using an effective stress coming from strain equivalence principle. ***To cite this article: N. Conil et al., C. R. Mecanique 332 (2004).***
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Résumé

Modélisation thermodynamique de la plasticité et de l'endommagement d'une argilite. L'étude des caractéristiques physiques et mécaniques des argilites nous permet de mettre en évidence le caractère plastique et endommageable de ce type de matériau et de retenir les variables d'état nécessaires à la modélisation thermodynamique de leur comportement. Un potentiel thermodynamique qui rend compte des non linéarités du comportement est construit. Le couplage plasticité-endommagement se fait à l'aide d'une contrainte effective mise en évidence par le principe d'équivalence en déformations. ***Pour citer cet article : N. Conil et al., C. R. Mecanique 332 (2004).***
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Mots-clés : Sols ; Roches ; Plasticité ; Endommagement ; Couplage plasticité/endommagement ; Argilites

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Depuis plusieurs années, l'Agence nationale pour la gestion des déchets radioactifs (ANDRA) a été missionnée pour élaborer un concept de stockage spécifique aux déchets hautement radioactifs et à vie longue. Ainsi les travaux de reconnaissance géologiques ont mis en évidence une couche argileuse d'environ 130 mètres d'épaisseur et de grande extension située dans l'Est de la France à la frontière des départements de la Meuse et de Haute-Marne. Agée de 150 millions d'année, cette roche peu perméable, raide est une argilite qui contient 40 à 45 % de minéraux argileux. Les études des échantillons prélevés à près de 500 mètres de profondeur tendent à démontrer ses capacités de rétention des éléments qu'ils soient chimiques ou radioactifs. Les données expérimentales mettent en évidence que l'argilite est un matériau [2] :

– plastique : des déformations permanentes apparaissent très tôt,
– endommageable : on met effectivement en évidence la dégradation des propriétés mécaniques,
– anisotrope : après apparition de l'endommagement, la vitesse de décroissance du module radial de la roche est plus rapide que celle du module axial.

Une modélisation correcte du comportement des argilites de l'Est nécessitait donc la prise en compte de ces phénomènes. Dans cet article on présente un modèle qui reproduit les caractéristiques de la roche, le caractère plastique de la roche est pris en compte par l'intermédiaire d'un critère de Drücker–Prager modifié [6], un écrouissage isotrope positif et une loi non associée. Le modèle permet de reproduire l'influence de la microfissuration sur les caractéristiques mécaniques du matériau à l'aide d'une variable d'endommagement tensoriel d'ordre 2. Le couplage entre les deux phénomènes se fait à l'aide d'une contrainte effective [12], au sens de l'endommagement, mis en évidence par le principe d'équivalence en déformations. Des simulations numériques d'essais triaxiaux sont ensuite présentées [2,7]. La confrontation avec les essais expérimentaux s'avèrent être très correcte le modèle présenté reproduit donc les principales caractéristiques du matériau ainsi que la dégradation des propriétés élastiques comme le montre la Fig. 1.

1. Introduction

Argillite is a clay rock [1]. Clayey rocks are a category of sedimentary rocks, as limestone and sandstone. In the sedimentary basins 80% of the rocks are clayey. Often, clay minerals are in the faults of rocky massifs. The mechanical behaviour of the clay rocks presents some characteristics similar to soils and rocks. When there is quartz and carbonate in addition to clay minerals, the clay rocks have a 'stiff' behaviour. Laboratory experiments show that argillites are plastic, they present anisotropic damage and their hydromechanical behaviour is modified with damage [2]. To model the brittle rocks behaviour, there are several rheological methods. Many authors used plasticity theory [3] because they find this method simple to describe the materials behaviour and because this method allows reproducing the development of irreversible strains. We propose to couple the plasticity theory to the damage mechanics to describe the degradation of material's mechanical properties [2,10]. We show that this method is very adequate to describe correctly the material degradation process as the argillites.

2. Thermodynamic potential

We use the theory of continuous media thermodynamics and adopt the small perturbations assumptions. In addition, we consider an isotherm and quasi-static evolution. Without damage the material is considered as isotropic. We use the assumption of strains partition. For damaged plastic materials the state variables are the elastic strain tensor, the set of hardening variables and the damage variable $\underline{\underline{D}}$ that will be defined later.

It is assumed that there is a partition between the hardening effects, represented by V_k, and the elasticity effects associated to damage [4]: $\psi(\varepsilon^{\mathrm{e}}, \underline{\underline{D}}, V_k) = \psi^{\mathrm{e}}(\varepsilon^{\mathrm{e}}, \underline{\underline{D}}) + \psi^{\mathrm{p}}(V_k)$. ψ^{e} is the damage thermoelastic potential and ψ^{p} is the stored energy of plastic hardening. At the initial state the material being supposed isotropic, ψ^{e} is a scalar function depending on the two symmetric tensors: $\underline{\underline{\varepsilon}}^{\mathrm{e}}$ the elastic strain tensor and $\underline{\underline{D}}$ the damage tensor.

The dissipation of the plastic strains in the argillites is mainly due to the sliding with friction, between clay minerals [5]. The dissipation due to the damage is produced by the relaxation of the internal energy, which is due to the microscopic cavities development. Thus, these two mechanisms of dissipation are represented by two different functions, that is, one plastic potential f^{p}, which is called the plastic yield function and one damage potential F, which is called the damage criterion. Then, the dissipation potential is the sum of these two potentials.

2.1. Elastoplastic model

The plastic state variables of the rocky material represent the irreversible evolutions of argillites at the macroscopic scale, the plastic strains represent the effects of the internal sliding of the clay minerals [2].

In this study, a modified Drücker–Prager criterion [6] is retained. The state variables are formulated according to the associated variables via the state laws. The problem of the plastic threshold decreases in the extension field is regulated with this expression (Eq. (1)) [2]. Nevertheless, the Drücker–Prager criterion is not adapted to argillites, because the same plastic thresholds in compression and in extension are obtained. To correct this disadvantage, it is necessary to introduce the Lode angle θ in the criterion. Generally, the plastic threshold depends on the function $h(\theta)$ for non-proportional loading. The explicit form of the function $h(\theta)$ can be determined by laboratory tests that show this effect. We retain for the function $h(\theta)$, as Chiarelli [5] and by analogy with the Mohr–Coulomb criterion, the following form: $h(\theta) = (\cos\theta - \frac{t}{\sqrt{3}}\sin\theta)$ with $-\frac{\pi}{6} \leqslant \theta \leqslant \frac{\pi}{6}$. The plastic yield function can be written then as follow:

$$f^{\mathrm{p}}(\underline{\underline{\sigma}}, \gamma_{\mathrm{p}}) = \tau h(\theta) + f(\sigma - \rho) \leqslant 0 \tag{1}$$

where ρ is the cohesive pressure of the material, τ the generalized deviatoric stress and σ the hydrostatic stress. In this study, the internal hardening variable used is the equivalent plastic shear strain γ_{p} which is often used, in the case where the plasticity is due to sliding phenomena. Furthermore, the plastic volume variation is very important in the macroscopic description of the rocks hardening behaviour; thus by analogy with metals [4], γ_{p} is written as follows, with $\underline{\underline{\varepsilon}}^{\mathrm{d}}$ the deviatoric strain tensor:

$$\gamma_{\mathrm{p}} = \int_0^t \left(\frac{2}{3}\dot{\underline{\underline{\varepsilon}}}^{\mathrm{d}} : \dot{\underline{\underline{\varepsilon}}}^{\mathrm{d}} \right) \mathrm{d}\tau \tag{2}$$

γ_{p} has a physical meaning, for an isotropic behaviour the increment $\dot{\gamma}_{\mathrm{p}}$ is, within one multiplier factor, equal to the increment of plastic volumetric strain:

$$\dot{\varepsilon}_{\mathrm{v}}^{\mathrm{p}} = \mathrm{tr}(\dot{\underline{\underline{\varepsilon}}}^{\mathrm{p}}), \qquad \dot{\varepsilon}_{\mathrm{v}}^{\mathrm{p}} = f^* \dot{\gamma}_{\mathrm{p}} \tag{3}$$

The parameter f^* represents the material dilatancy coefficient. f^* depends only on the hardening variable (Eq. (2)).

By taking into account the experimental results obtained with tests carried out on East argillites, Chiarelli [2] proposed the following increasing positive function: $f(\gamma_{\mathrm{p}}) = f_{\mathrm{m}} - (f_{\mathrm{m}} - f_0)\,\mathrm{e}^{-b\gamma_{\mathrm{p}}}$. f_0 is the initial plastic threshold when $\gamma_{\mathrm{p}} = 0$ and f_{m} the value of $f(\gamma_{\mathrm{p}})$ on the elastic domain limit; b is a material parameter.

To take into account the dilatancy behaviour of the argillites a non-associated law is used. To reproduce this dilatancy the following plastic potential [2] which gives us the flow direction is retained:

$$g^{\mathrm{p}}(\underline{\underline{\sigma}}, \gamma_{\mathrm{p}}) = \tau h(\theta) + f^* \sigma \tag{4}$$

Therefore the flow rule is written as follows:

$$\underline{\underline{\dot{\varepsilon}}}^{\mathrm{p}} = \dot{\lambda}^{\mathrm{p}}\left(\frac{3\underline{\underline{\sigma}}^{\mathrm{d}}}{2\tau}h(\theta) + \frac{f^*}{3}\underline{\underline{1}}\right) \tag{5}$$

The expression of the potential (Eq. (4)), which allows us to obtain the flow rule, enables us to verify the dilatancy coefficient definition (Eq. (3)), within one multiplier factor (the Lode angle function). Another way to write the flow rule is:

$$\begin{cases} \dot{\varepsilon}_{\mathrm{v}}^{\mathrm{p}} = \dot{\lambda}^{\mathrm{p}}\dfrac{\partial g^{\mathrm{p}}}{\partial \sigma} \\ \dot{\gamma}_{\mathrm{p}} = \dot{\lambda}^{\mathrm{p}}\dfrac{\partial g^{\mathrm{p}}}{\partial \tau} \end{cases} \quad \Rightarrow \quad \begin{cases} \dot{\varepsilon}_{\mathrm{v}}^{\mathrm{p}} = \dot{\lambda}^{\mathrm{p}} f^* \\ \dot{\gamma}_{\mathrm{p}} = \dot{\lambda}^{\mathrm{p}} h(\theta) \end{cases}$$

To have the same formulation that Eq. (3) and to simplify parameters identification, it is assumed that $\delta = \frac{f^*}{h(\theta)}$ where δ is the dilatancy coefficient which determines the rate of volumetric plastic strain:

$$\delta = \frac{\dot{\varepsilon}_{\mathrm{v}}^{\mathrm{p}}}{\dot{\gamma}_{\mathrm{p}}}$$

In the case of the dilatancy, δ is positive and in the case of the contractancy δ is negative. The experimental results [5] allow us to retain the following formulation for the dilatancy coefficient: $\delta(\gamma_{\mathrm{p}}) = \delta_{\mathrm{m}} - (\delta_{\mathrm{m}} - \delta_0)\,\mathrm{e}^{-b'\gamma_{\mathrm{p}}}$ where δ_{m}, δ_0 and b' are material parameters. Nevertheless, although the plastic model is very judicious, it does not allow us to reproduce the material degradations, i.e. microcracks and microcavities effects on the material behaviour. Thus, to reproduce microcracks effects on the mechanical characteristics of the material, i.e. the decrease of the material's stiffness, a damage variable must be introduced to the model. Afterwards, the extension of the elastoplastic model to a damage plastic model is presented.

2.2. Damage plastic model

In 1958 Kachanov [7] introduced formally the damage concept. This concept was made known by Lemaitre who introduced the strains equivalence principle [4]. The concept introduces the notion of the degradation internal variable D. The micromechanical analyses [8] show that the use of a second order tensor allows us to characterize in a rigorous way the microcracks distribution in the two-dimensional case, and give a good approximation in the three-dimensional case. In the case of parallel and plane microcracks family, with n_i the unit normal at the family i, the microcracks direction can be reproduced by the tensorial product $(n \otimes n)_i$. Furthermore, to take into account the importance of any family i, a cracking density, $d_i(S)$ is introduced. It is a scalar function which is proportional at the extension of the decohesion surface produced by the microcracks opening in the representative elementary volume. Then, the damage variable can be formulated by the following relation:

$$\underline{\underline{D}} = \sum_i d_i(S)(n \otimes n)_i$$

The effective stress concept allowed the development of the isotropic damage mechanics [4]. Thus, an induced anisotropic damage thermodynamic model is retained which allows us to obtain easily the effective stress definition.

The free energy formulation of the material requires several assumptions: the material is supposed initially isotropic and the induced anisotropy is caused by the initiation and propagation of the oriented microcracks under loading; for a constant damage state, the material has a linear elastic behaviour; lastly we make the assumption of low density microcracks without interaction. To avoid the incompatibility problem between the physics and the thermodynamics, two damage variables [11] are introduced. Thus, the particular form of the Gibbs energy proposed by Lemaitre et al. [12] is adopted, knowing that the Helmotz energy can be obtained with a classical way: $\psi^{\mathrm{e}} = \psi^{\mathrm{e}*} - \underline{\underline{\sigma}} : \underline{\underline{\varepsilon}}$.

The potential is decomposed into one deviatoric part, which depends on the damage variable $\underline{\underline{D}}$, and one hydrostatic part which depends on another variable d_H.

$$d_H = g(D), \qquad D = \frac{1}{3}\operatorname{tr}\underline{\underline{D}}$$

The Gibbs energy is written with E and ν respectively the Young modulus and the Poisson coefficient of the isotropic, linear, elastic nondamaged material and $\underline{\underline{\sigma}}^{\mathrm{d}}$ the deviatoric stress tensor and σ the hydrostatic stress.

$$\rho\psi^{\mathrm{e}*} = \frac{1+\nu}{E}\underline{\underline{H}}\,\underline{\underline{\sigma}}^{\mathrm{d}}\underline{\underline{H}}\,\underline{\underline{\sigma}}^{\mathrm{d}} + \frac{3(1-2\nu)}{2E}\frac{\sigma^2}{1-d_H}$$

with

$$\underline{\underline{H}} = (\underline{\underline{1}} - \underline{\underline{D}})^{-1/2}$$

The identification law $d_H = g(D)$ shows that a good approximation could be the following:

$$d_H = \eta D$$

where η is a material parameter which characterizes its sensibility to the hydrostatic stress.

The elastic constitutive law can be expressed in function with effective stress tensor:

$$\underline{\underline{\varepsilon}}^{\mathrm{e}} = \rho\frac{\partial\psi^{\mathrm{e}*}}{\partial\underline{\underline{\sigma}}} = \frac{1+\nu}{E}\underline{\underline{\tilde{\sigma}}} - \frac{\nu}{E}\operatorname{tr}\underline{\underline{\tilde{\sigma}}}\,\underline{\underline{1}}$$

This law introduces naturally the symmetrical effective stress which is independent on the elastic parameters. This effective stress will be used for the coupling with the plasticity.

$$\underline{\underline{\tilde{\sigma}}} = (\underline{\underline{H}}\,\underline{\underline{\sigma}}^{\mathrm{d}}\underline{\underline{H}})^{\mathrm{d}} + \frac{\sigma}{1-d_H}\underline{\underline{1}} \tag{6}$$

After inversion, the elastic stress tensor had the following expression:

$$\underline{\underline{\sigma}} = \frac{E}{1+\nu}\left[\left(\underline{\underline{H}}^{-1}(\underline{\underline{\varepsilon}}^{\mathrm{e}})^{\mathrm{d}}\underline{\underline{H}}^{-1}\right)^{\mathrm{d}} + \frac{\underline{\underline{D}}^{\mathrm{d}}}{3(1-D_H)}\underline{\underline{H}}^{-1}(\underline{\underline{\varepsilon}}^{\mathrm{e}})^{\mathrm{d}}\underline{\underline{H}}^{-1}\right] + \frac{E(1-d_H)}{3(1-2\nu)}\operatorname{tr}\underline{\underline{\varepsilon}}^{\mathrm{e}}\underline{\underline{1}}$$

As has been observed for the concrete [2], the microscopical observations carried out on various rocks under different solicitations, show that the damage evolution is closely connected at the extension strains according to some privileged directions. Thus, this induces a dilatancy and anisotropy. The study of various results [5,9,13] allows us to retain the following variable as the motor of the induced anisotropic damage: $\hat{\varepsilon} = \sqrt{\underline{\underline{\varepsilon}}^{+} : \underline{\underline{\varepsilon}}^{+}}$.

$\underline{\underline{\varepsilon}}^{+}$ is the positive part (or extension part) of the total strains, given by a spectral decomposition, introduced by Ortiz [14] and completed by Ju [13]; its formulation is

$$\underline{\underline{\varepsilon}}^{+} = \sum_{k=1}^{3}\varepsilon_k H(\varepsilon_k) u_k \otimes u_k$$

where $H(\varepsilon_k)$ is the Heaviside function of the k^{th} eigenvalue (principal strain) of the strain tensor and u^k is the eigenvector associated to ε_k.

Then, it is assumed that the damage appears when the equivalent strain reaches some threshold depending on the present damage state. Thus the damage criterion can be written [5]:

$$F(\hat{\varepsilon}, \underline{\underline{D}}) = \hat{\varepsilon} - (r_0 + r_1 \operatorname{tr}\underline{\underline{D}}) = 0$$

r_0 and r_1 are two material parameters; r_0 is the damage initial threshold and r_1 characterizes the degradation of the material during the damage process.

Then, the damage evolution law can be written:

$$\begin{cases} \dot{\underline{\underline{D}}} = 0 & \text{if } F < 0 \text{ or if } F = 0 \text{ and } \dot{F} < 0 \\ \dot{\underline{\underline{D}}} = \dot{\lambda}^{\mathrm{d}} \dfrac{\partial F}{\partial \hat{\varepsilon}} & \text{if } F = 0 \text{ and } \dot{F} = 0 \end{cases}$$

that is:

$$\begin{cases} \dot{\underline{\underline{D}}} = 0 & \text{if } F < 0 \text{ or if } F = 0 \text{ and } \dot{F} < 0 \\ \dot{\underline{\underline{D}}} = \dot{\lambda}^{\mathrm{d}} \dfrac{\underline{\underline{\varepsilon}}^{+}}{\hat{\varepsilon}} & \text{if } F = 0 \text{ and } \dot{F} = 0 \end{cases}$$

Consequently, the directions of the damage increment are given by the directions of the positive strains increment. In other words and in the radial case at least, the evolution directions of $\underline{\underline{D}}$ are the same that the directions of the positive evolutions of $\underline{\underline{\varepsilon}}$. This result is also found in the literature [15].

The consistency condition $\dot{F}(\hat{\varepsilon}, \underline{\underline{D}}) = 0$ allows us to determine the damage multiplier $\dot{\lambda}^{\mathrm{d}}$:

$$\dot{\lambda}^{\mathrm{d}} = \frac{\underline{\underline{\varepsilon}}^{+} : \underline{\underline{\underline{\underline{P}}}}^{+} : \dot{\underline{\underline{\varepsilon}}}}{r_1 \operatorname{tr} \underline{\underline{\varepsilon}}^{+}}$$

with $\underline{\underline{\underline{\underline{P}}}}^{+}$ fourth-order projection tensor.

Then the coupling with the plasticity will be easy by using the effective stress (Eq. (6)). The elastic criterion and the plastic yield criterion will depend on the damage, using the effective stress, replacing $\underline{\underline{\sigma}}$ by $\underline{\underline{\tilde{\sigma}}}$ in the nondamaged material criterion. Thus, it is admitted that only the stress is replaced, but the hardening variables are not modified by the damage in the criterion. Furthermore, we make the assumption that the Lode angle is not affected by the damage.

Then, Eqs. (1) and (4) are written as follows:

$$f^{\mathrm{p}}(\underline{\underline{\tilde{\sigma}}}, \gamma_{\mathrm{p}}) = \tilde{\tau} h(\theta) + f(\tilde{\sigma} - \rho) \leqslant 0$$
$$g^{\mathrm{p}}(\underline{\underline{\tilde{\sigma}}}, \gamma_{\mathrm{p}}) = \tilde{\tau} h(\theta) + f^* \tilde{\sigma}$$

Thus the equations of the problem written with indicial notation are:

$$\varepsilon_{ij} = \varepsilon^{\mathrm{e}}_{ij} + \varepsilon^{\mathrm{p}}_{ij}, \qquad \varepsilon^{\mathrm{e}}_{ij} = \frac{1+\nu}{E}\tilde{\sigma}_{ij} - \frac{\nu}{E}\tilde{\sigma}_{kk}\delta_{ij}$$
$$\tilde{\sigma}_{ij} = \left(H_{ik}\sigma^{\mathrm{d}}_{kl}H_{lj}\right)^{\mathrm{d}} + \frac{\sigma}{1-d_H}\delta_{ij}, \qquad H_{ij} = (\delta_{ij} - D_{ij})^{1/2}, \qquad d_H = \eta D = \frac{\eta}{3}D_{kk}$$
$$\dot{\varepsilon}^{\mathrm{p}}_{\mathrm{v}} = \dot{\lambda}^{\mathrm{p}} f^*, \qquad \dot{\gamma}_{\mathrm{p}} = \dot{\lambda}^{\mathrm{p}} h(\theta), \qquad \dot{D}_{ij} = \dot{\lambda}^{\mathrm{d}} \frac{\varepsilon^{+}_{ij}}{\hat{\varepsilon}}$$

The analytical determination of the plastic multiplicator $\dot{\lambda}^{\mathrm{p}}$ can be more laborious if we do not use the effective stress concept, this concept allows to avoid this difficulty. The consistancy condition, by taking into account the effective stress will be written:

$$\frac{\partial f^{\mathrm{p}}}{\partial \underline{\underline{\tilde{\sigma}}}} : \dot{\underline{\underline{\tilde{\sigma}}}} + \frac{\partial f^{\mathrm{p}}}{\partial \gamma_{\mathrm{p}}}\dot{\gamma}_{\mathrm{p}} = 0.$$

However, by definition, the effective stress [4,12] can be written: $\underline{\underline{\tilde{\sigma}}} = \underline{\underline{\underline{\underline{E}}}}^{0} : \underline{\underline{\varepsilon}}^{\mathrm{e}}$, with $\underline{\underline{\underline{\underline{E}}}}^{0}$ the fourth-order initial stiffness tensor.

By deriving this expression the following expression is obtained:

$$\dot{\underline{\underline{\sigma}}} = \underline{\underline{\underline{\underline{E}}}}^0 : \dot{\underline{\underline{\varepsilon}}}^{\mathrm{e}} = \underline{\underline{\underline{\underline{E}}}}^0 : (\dot{\underline{\underline{\varepsilon}}} - \dot{\underline{\underline{\varepsilon}}}^{\mathrm{p}}) = \underline{\underline{\underline{\underline{E}}}}^0 : \left(\dot{\underline{\underline{\varepsilon}}} - \dot{\lambda}^{\mathrm{p}} \frac{\partial g^{\mathrm{p}}}{\partial \underline{\underline{\sigma}}}\right)$$

Thus we can determine $\dot{\lambda}^{\mathrm{p}}$:

$$\dot{\lambda}^{\mathrm{p}} = H(f^{\mathrm{p}}) \left\langle \frac{\frac{\partial f^{\mathrm{p}}}{\partial \underline{\underline{\sigma}}} : \underline{\underline{\underline{\underline{E}}}}^0 : \dot{\underline{\underline{\varepsilon}}}}{\frac{\partial f^{\mathrm{p}}}{\partial \underline{\underline{\sigma}}} : \underline{\underline{\underline{\underline{E}}}}^0 : \frac{\partial g^{\mathrm{p}}}{\partial \underline{\underline{\sigma}}} - \frac{\partial f^{\mathrm{p}}}{\partial \gamma_{\mathrm{p}}} h(\theta)} \right\rangle$$

This expression does not depend on $\dot{\underline{\underline{\underline{\underline{E}}}}}(\underline{\underline{D}})$ and thus it is easier to calculate.

A necessary, yet insufficient, condition to validate the model is to check the positivity of the dissipation $D = \underline{\underline{\sigma}} : \dot{\underline{\underline{\varepsilon}}} - \rho_0 \dot{\psi} \geqslant 0$. That is: $\underline{\underline{\sigma}} : \dot{\underline{\underline{\varepsilon}}}^{\mathrm{p}} + Y_D : \dot{\underline{\underline{D}}} - A_k \cdot \dot{V}_k \geqslant 0$. With V_k, the set of the internal variables, A_k, the associated variables and $\underline{\underline{Y}}^{\mathrm{d}}$ the associated thermodynamic force $\underline{\underline{Y}}^{\mathrm{d}}$.

We know that: $\underline{\underline{\sigma}} : \dot{\underline{\underline{\varepsilon}}}^{\mathrm{p}} - A_k \cdot \dot{V}_k \geqslant 0$. However, the no-negativity of $Y_D : \dot{\underline{\underline{D}}}$ is not obligatory. From an algorithmic point of view, we must verified the sign of $Y_D : \dot{\underline{\underline{D}}}$ for each step and for each integration point.

3. Simulation of loading-unloading cycles

The proposed model contains 12 parameters in total, this parameters can be order in three groups: the first group is introduced for the description of the elastic behaviour of the damaged material. It contains the initial elastic parameters (E, ν) and the parameter η, the second group contains the parameters relating to the damage evolution criterion r_0, r_1, the last group concerns the plasticity, it contains the parameters which describe the material plastic behaviour $(f_{\mathrm{m}}, f_0, \rho, b, \delta_{\mathrm{m}}, \delta_0, b')$.

The model parameters can be determined by two triaxial tests with loading-unloading cycles. The parameters identification [2] was carried out with the tests performed by Chiarelli [5].

We have simulated classical triaxial tests in drained condition and the numerical results have been compared with the experimental results [5]. The simulations of the loading-unloading cycles allow us to note that the damage elastoplastic model reproduces very correctly the experimental results. Indeed, these comparisons (Fig. 1) show a very good general agreement. The model describes strictly the principal characteristics of the material, i.e. the degradation of the elastic properties and the apparition of the permanent strains. Thus, the consequence of the induced anisotropy is correctly described.

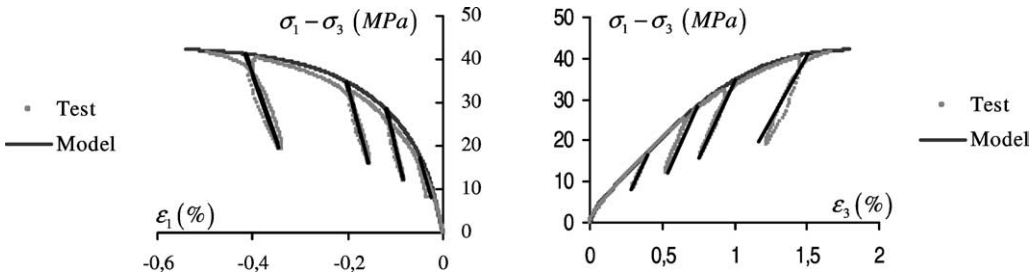

Fig. 1. Triaxial compression with loading-unloading cycles under 10 MPa confining pressure: comparison between experimental results [5] and numerical simulations [2].

4. Conclusions

In this paper, the formulation of a plastic damage model written in the thermodynamic frame of the generalized standard materials, which reproduces the damage behaviour of the claystones and particularly the anisotropic behaviour induced by the damage, is presented. The plastic behaviour takes into account, the dependence in the mean stress of the material plastic parameters (friction angle, dilatancy angle), the hardening and resoftening character of the material, and the dilatancy–contractancy stage of the volumetric behaviour. The model was validated by typical triaxial loading tests, for different confining pressure. This model is three-dimensional. A strong point of this model is that it makes it possible to easily represent the coupling of damage-plasticity [2,12].

Also, the saturated porous media mechanics permits to take into account very easily in the model the pore pressure variations [2]. This method seems to be the more judicious to reproduce the fluid–solid interactions like the damage influence on the rock hydromechanical characteristics.

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