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C. R. Mecanique 332 (2004) 895–900
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Instability of Navier slip flow of liquids

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Received 30 December 2003; accepted after revision 14 June 2004

Available online 25 September 2004

Presented by Évariste Sanchez-Palencia

Abstract

We investigate the stability problem related to the basic slip flows of liquids in plane microchannels by using the Navier slip concept. We found that if the Navier slip parameter (N_s) equals 0.06, the critical Reynolds number (Re_{cr}) becomes 213.6. There are short-wave instabilities, however, when we further increase N_s to 0.07 or 0.08. Re_{cr} becomes 132.9 for $N_s = 0.08$ if we neglect the short-wave instability. ***To cite this article: A.K.-H. Chu, C. R. Mecanique 332 (2004).***

Résumé

Instabilité des écoulements glissant de liquides. Nous étudions la stabilité de l'écoulement de base d'un liquide dans un microcanal plan en présence de glissement aux parois, en utilisant le concept de glissement de Navier. Nous trouvons que le nombre de Reynolds critique (Re_{cr}) diminue à 213,6 quand le paramètre de glissement de Navier (N_s) augmente à 0,06. Cependant, il existe des instabilités à courte longueur d'onde quand nous augmentons le paramètre N_s à des valeurs de 0,07 et 0,08. Re_{cr} décroît à 132,9 pour $N_s = 0{,}08$ si on néglige les instabilités d'onde courte. ***Pour citer cet article : A.K.-H. Chu, C. R. Mecanique 332 (2004).***

Keywords: Fluid mechanics; MEMS (MicroElectroMechanical systems); Preconditioning; Spectral method; Degeneracies; Slip velocity

Mots-clés : Mécanique des fluides ; Stabilité ; Transition à la turbulence ; Microfluidique ; Glissement

1. Introduction

Incomplete momentum accommodation of colliding particles along an interface (say, between a solid and a fluid) will generate nonzero slip velocities at the solid wall (since the average of the bulk velocity of incoming and

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doi:10.1016/j.crme.2004.06.010

reflecting particles is not zero) has been intensively reported, due to the rapid progress of current measurement techniques [1–5]. For example, Tretheway and Meinhart [1] used micron-resolution particle image velocimetry (MPIV) to measure the velocity profiles of water flowing through 30 × 300 μm channels. The velocity profiles were measured to within 450 nm of the microchannel surface. They found that, when the microchannel surface was coated with a 2.3 nm thick monolayer of hydrophobic octadecyltrichlorosilane, an apparent velocity slip is measured just above the solid surface. This velocity is approximately 10% of the free-stream velocity and yields a slip length (the corresponding distance extrapolated from the nonzero to zero velocity in the direction normal to the wall) of approximately 1 μm [1]. They claimed that, for this slip length, slip flow is negligible for length scales greater than 1 mm, but must be considered at the micro- and nano-scales.

Craig et al. [2], however, reported direct measurements of hydrodynamic drainage forces, which show clear evidence of boundary slip in a Newtonian liquid. The degree of boundary slip was found to be a function of the liquid viscosity and the shear rate, as characterized by the slip length, and is up to ∼20 nm. This has implications for confined biological systems, the permeability of microporous media for the lubrication of nanomachines, and will be important in the microcontrol of liquid flow. These reports confirm previous theoretical and experimental attempts [6–9] ([7,8] treated weak slip flows in rarefied gases using the incompressible flow assumption, once the Mach number is much smaller than one). Note that researchers normally relate the nonzero slip velocity to the Knudsen number, defined as the ratio of the mean free path, L, of the fluid, to the characteristic length of the flow, say, d [7–9]. In fact, incomplete (energy) accommodation of colliding particles will then lead to a temperature jump along the interacting solid–fluid region [10,11].

Slip flows of gases normally occur in very-low pressure environments where the velocity-slip phenomena are common in rarefied gas dynamics [12–15] or microdomain [16,17] (see the detailed references therein). As for the latter, considering the forthcoming physical applications of micro- or nano-electromechanical systems (MEMS or NEMS [18–22]), we need to better understand the development of temporal instability in slip flows. People have already studied the stability of viscous slip flows of rarefied gases between rigid parallel plates [23] and obtained the rough flow-stability characteristics [24] (using a Monte Carlo code). The relevant stability study related to slip flows of liquids, however, is rather limited [25–28].

The linear stability analysis for this flow, which considers infinitesimal perturbations (ψ', the perturbed stream function, could be decomposed into normal modes, each component varying with time as $e^{\sigma t}$ for some complex number $\sigma = s + i\omega$; s and ω are real numbers) to the basic-flow stream function ψ_0, starts by substituting $\psi_0 + \psi'$ into the 2D incompressible Navier–Stokes equations together with nonzero slip conditions and obtains the Orr–Sommerfeld (O–S) equation after linearization [24,29]. To check the stability characteristics, the O–S equation should be solved for slip flows and a curve of marginal stability will be found. This curve comes from the mode $s = 0$, while if $s > 0$ for a mode, then the corresponding disturbance will be amplified, growing exponentially with time (the instability happens).

The previous numerical approach showed that a plane Poiseuille flow is stable if the Reynolds number is less than $Re_{\mathrm{cr}} \sim 5772$ (where the no-slip approach was adopted and the result was obtained by Orszag using the spectral method [29]). These results do not agree with experimental studies, which show that Poiseuille flow may become unstable at Reynolds numbers as low as ∼360 (partly due to the coarseness of their micron channels, cf. Pfahler et al. [18,19]) or ∼1000 [30–32]. As interface or wall noise normally occur in MEMS or NEMS applications, we propose here to study the influence of the Navier's slip velocity [33,34] (applicable to liquids) in the stability of slip flows. In fact, this approach might also be valid to gases [35,36].

In this Note, we use the verified numerical code [35–37] to investigate the linear stability problem linked to those basic slip flows of liquids in a microdomain. Our results show that the critical Reynolds number Re_{cr} becomes 213.6 (when the Navier slip parameter N_s is 0.06; here, $N_s = \mu S/d$ is the dimensionless Navier slip parameter; S is a proportionality constant as $u_s = S\tau$, τ, the shear stress of the bulk velocity, u_s, the dimensional slip velocity; for a no-slip case, $S = 0$, but for a no-stress condition. $S = \infty$, μ is the fluid viscosity, d is one half of the channel width) which is much less than the conventional case [29]. Re_{cr} could be further lowered to 132.9 or 165.3, however, if we neglect the short-wave instability which occurs at $N_s = 0.08$ or 0.07.

2. Formulation

Following the usual method of linearized stability theory, we have $v_i(x_i,t) = \bar{v}_i(x_i) + v_i'(x_i,t)$, and similarly, $p(x_i,t) = \bar{p}(x_i) + p'(x_i,t)$ for the velocity and pressure terms in the incompressible Navier–Stokes equations. Then by substituting these into the dimensionless 2D Navier–Stokes equation, and eliminating the pressure terms, the linearized equation or so-called O–S equation, which governs the variation of the disturbances is:

$$(D^2-\alpha^2)^2\phi = \mathrm{i}\alpha Re\big[(\bar{u}-\lambda)(D^2-\alpha^2)\phi - (D^2\bar{u})\phi\big], \tag{1}$$

where $D \equiv \mathrm{d}/\mathrm{d}y$, $Re = \rho u_{\max} d/\mu$ is the Reynolds number based on a half channel-width, d, and u_{max} is the maximum velocity of the basic flow (at the center-line). ρ is the density of the fluid. $\bar{u} = 1 - y^2$ is the (mean) basic velocity profile of the flow ($-1 \leqslant y \leqslant 1$) [29]. The stream function for the disturbance, Ψ, such that $u' = -\partial\Psi/\partial y$, $v' = -\partial\Psi/\partial x$, is assumed to have the form $\Psi(x,y,t) = \phi(y)\exp[\mathrm{i}\alpha(x-\lambda t)]$ in the usual normal-mode analysis, α is the wave number (real) and λ is $\lambda_r + \mathrm{i}\lambda_i$. This is a kind of Tollmien–Schlichting transversal wave, λ_i is called the amplification factor, and α equals to $2\pi\Lambda^{-1}$, where Λ is the wave length of the Tollmien–Schlichting perturbation [24]. In the usual temporal stability problem, in which the growth or decay of a disturbance in time is considered, we take α and Re to be real and then treat the (complex) wave-speed λ as the eigenvalue parameter of the problem.

To consider the effect of the nonzero slip velocity (existing along the solid walls), as the bulk fluid is separated from the solid boundaries by a thin layer of fluid of lower viscosity, we adopt in the following the approximate approach which Navier [34] used. In this thin layer, velocity field is almost linear and the shear stress is proportional to the velocity. They are both continuous in the whole flow regime. To take the leading order approximation into account, with the dimensionless form, we then have the slip velocity at the wall (the subscript w means at the wall)

$$u_w = N_s \frac{\mathrm{d}\bar{u}}{\mathrm{d}y}\Big|_w, \tag{2}$$

where N_s is the (dimensionless) Navier slip parameter. For the case of flows in plane microchannels, we have $\bar{u} = \mp N_s\,\mathrm{d}u/\mathrm{d}y$ (cf. [7–9] or [16,17]), $v = 0$ at $y = \pm 1$.

The basic slip flow now has this form: $\bar{u}(y) = (1 - y^2 + 2N_s)$. Boundary conditions for ϕ in Eq. (1) remain the same as that in [29,35–37]: $\phi(\pm 1) = D\phi(\pm 1) = 0$, since we are only interested in the neutral stability boundary for the primary slip flow of liquids.

We adopt the Chebyshev polynomial expansion approach to approximate the O–S equation and boundary conditions. After that, we solve the eigenvalue problem [29] by using the verified code [35–37] which uses the spectral method [38].

3. Results and discussions

The preliminary verification for our numerical code was made [35–37] by comparing with the bench-mark results of Orszag (obtained in 1971). After that, we can obtain (using double-precision machine accuracy) the detailed complex spectra ($\lambda_r + \mathrm{i}\lambda_i$) for general slip cases and plot the neutral boundary curve for different cases as shown in Fig. 1. We tune the slip parameter N_s to be 0, 0.001, 0.01, 0.05, and 0.06. The corresponding critical Reynolds numbers (Re_{cr}) are 5772, 5282, 2480, 290.5, and 213.6, respectively. It seems the boundary effect due to the slip velocity [33,34] will degrade the flow stability significantly. The possible physical reasoning for this early instability compared to the no-slip case is that there are unbalanced conservation laws for the translation and angular momentum (relevant to vorticity production) along the solid–fluid interface once the slip velocity is not zero therein. Resonance or amplification of noise waves along this abrupt interface could be easily triggered due to whatever unknown mechanism.

The critical Reynolds number might shift to the much lower values, $\approx$165.3 and 132.9, when the Navier slip parameter N_s increases to 0.07 and 0.08 even though we found some strange spectra related to the short-wave

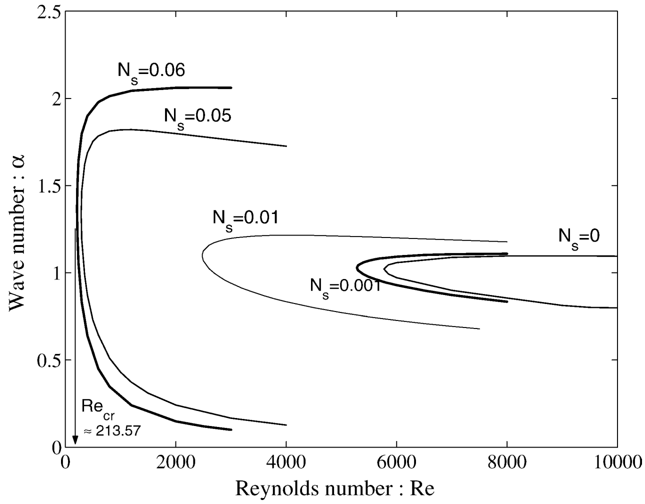

Fig. 1. Comparison of the Navier-slip effects due to $N_s = 0.001, 0.01, 0.05$, and 0.06 on the neutral stability boundary of the plane Poiseuille flow. The critical Reynolds number of the flow, $Re_{\mathrm{cr}} \sim 5772, 2480, 213.6$ for the different cases $N_s = 0, 0.01, 0.06$, respectively. N_s is the slip parameter introduced by Navier [34].

Fig. 1. Comparaison des effets de glissement dûs aux valeurs $N_s = 0{,}001$, 0,01, 0,05, et 0,06 sur la courbe de stabilité neutre de l'écoulement de Poiseuille plan. Le nombre de Reynolds critique de l'écoulement, $Re_{\mathrm{cr}} \sim 5772$, 2480, 213,6 pour les cas $N_s = 0$, 0,01, 0,06, respectivement. N_s est le paramètre de glissement introduit par Navier [34].

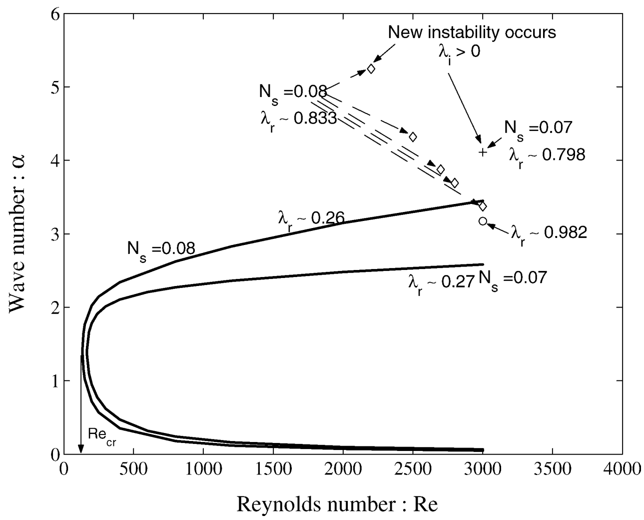

Fig. 2. Comparison of Navier-slip effects for $N_s = 0.07$ and 0.08 on the neutral stability boundary of the plane Poiseuille flow. $Re_{\mathrm{cr}} \sim 165.3$ and 132.9 for cases $N_s = 0.07$ and 0.08, respectively, if we neglect the strange short-wave (larger wave number) instability ($\lambda_i > 0$). λ_r is the phase speed of the disturbance [24,29].

Fig. 2. Comparaison des effets de glissement pour $N_s = 0{,}07$ et 0,08 sur la courbe de stabilité neutre d'un écoulement de Poiseuille plan. $Re_{\mathrm{cr}} \sim 165{,}3$ et 132,9 pour les cas $N_s = 0{,}07$ et 0,08, respectivement, si on néglige les instabilités d'onde courte $\lambda_i > 0$ (grands nombres d'onde). λ_r est la vitesse de phase de la perturbation [24,29].

instability (which has a certain origin, qualitatively similar to those long-wave spectra reported in [37]). We plot these in Fig. 2. We can observe the upper branch of the neutral stability boundary (solid curves of larger wave numbers α) keeps increasing as the Reynolds number increases for $N_s = 0.07$ or 0.08. Those of smaller N_s cases, which were shown in Fig. 1, however, will finally decrease, even if the Reynolds number keeps increasing. These are relevant to the long-wave instability. During the search for these least stable modes (long wave) [24,29], we noticed that there are similar spectra reported in [37], which have near-zero λ_i as we lower the Reynolds number and, meanwhile, raise the wave number. The (λ_i) finally become positive as $N_s = 0.07$ ($\lambda_r \sim 0.798, 0.982$ as illustrated) or 0.08 ($\lambda_r \sim 0.83$ as illustrated). We thus termed them 'modes of short-wave instability' as their wave numbers are higher than those found at neutral boundaries (e.g., the phase speed of the disturbances λ_r are close to around 0.27 and 0.26 for $N_s = 0.07$, and 0.08, respectively) for the same Reynolds numbers. From their corresponding length scales, we interpret that these short- and long-wave instabilities are due to the increasing Navier slip along the liquid-wall interface. However, the detailed reasoning which could explain such strange earlier instabilities (smaller critical Reynolds numbers for different N_s) remains unknown.

We noticed that in real microfludic or nanofludic devices the apparent effect of slip velocities cannot be neglected. The challenge is how to use a simplified approach to simulate microscopically the velocity-slip effect for a wide range of physical parameters related to the real solid–fluid interface. There are, in fact, other crucial issues such as randomness, nonhomogeneous slip velocities, thermal effects (e.g., compressibility), etc. Our results, however, could serve as a baseline or starting point for these more complicated problems.

We finally conclude that boundary noise (the Navier slip velocities) will premature any instability mechanism considering the temporal growth of the disturbances. Either the range of wave numbers relevant to the propagating small-amplitude disturbance wave or the Navier slip parameters as well as Reynolds numbers of the basic slip flows must be carefully selected for the optimal flow control usage in scientific applications of MEMS or NEMS. Further interesting issues are related to the nonlinear stability [39,40] of these slip flows.

Acknowledgements

The author is partially supported by the China Post-Dr. Science Foundation (Grant No.: 1999-17). The author thanks the referees for their valuable comments.

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