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Stability of a hypersonic shock layer on a flat plate

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Abstract

Stability of a hypersonic shock layer on a flat plate is examined with allowance for disturbances conditions on the shock wave within the framework of the linear stability theory. The characteristics of the main flow are calculated on the basis of the Full Viscous Shock Layer model. Conditions for velocity, pressure, and temperature perturbations are derived from steady Rankine–Hugoniot relation on the shock wave. These conditions are used as boundary conditions on the shock wave for linear stability equations. The growth rates of disturbances and density fluctuations are compared with experimental data obtained at ITAM by the method of electron-beam fluorescence and with theoretical data of other authors. ***To cite this article: A.A. Maslov et al., C. R. Mecanique 332 (2004).***

Résumé

Stabilité d'une couche de choc hypersonique sur une plaque plane. On examine la stabilité d'une couche de choc hypersonique sur une plaque plane en tenant compte des perturbations de l'onde de choc dans le cadre de la théorie de la stabilité linéaire. Les caractéristiques du flux principal sont calculées à l'aide du modèle complet de la couche de choc visqueuse. Des conditions sur les perturbations de la vitesse, de la pression et de la température sont dérivées de la relation stationnaire de Rankine–Hugoniot sur les ondes de choc. Ces conditions sont utilisées comme conditions aux limites sur l'onde de choc pour les équations de stabilité linéaire. Les vitesses de croissance des perturbations et des fluctuations de densité sont comparées avec des données expérimentales obtenues à l'ITAM par la méthode de fluorescence par faisceau d'électrons a et avec les résultats théoriques d'autres auteurs. ***Pour citer cet article : A.A. Maslov et al., C. R. Mecanique 332 (2004).***

Keywords: Computational fluid mechanics; Hypersonic flow; Boundary layer; Shock layer; Stability; Simulation

Mots-clés : Mécanique des fluides numérique ; Flux hypersonique ; Couche limite ; Couche de choc ; Stabilité ; Simulations

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1. Introduction

Investigations of stability of boundary and shock layers at high Mach numbers ($M \geqslant 10$) and moderate Reynolds numbers ($\mathrm{Re}_{x\infty} = 10^4–10^5$) are important because these conditions are encountered on the leading edges of the wing and fuselage of promising hypersonic aircraft. For the viscous shock layer, there are currently few experimental measurements of density fluctuations, temperature, and mass flow rate on the test-section walls [1–4].

Experimental measurements of the growth rate of disturbances in the shock layer on a flat plate by the method of electron-beam fluorescence of nitrogen were first performed in [5].

Theoretical investigations of stability of high-velocity boundary layers are also scarce. In such papers, linear problems of stability for a supersonic self-similar boundary layer are usually solved [6]. There are only few numerical solutions for the equations of stability of a hypersonic boundary layer [7].

Because of the difficulties associated with computing hypersonic boundary layers within the linear stability problem, asymptotic methods were developed ($\mathrm{Re}, M \to \infty$) [8,9].

In classical problems of the linear stability theory, the presence of the shock wave (SW) was ignored, since the SW was located far from the boundary-layer edge. In these cases, free-stream conditions (decaying disturbances) were used or asymptotic conditions outside the boundary layer were imposed.

In hypersonic flows, however, where the boundary-layer thickness is comparable with the distance between the body and SW, the presence of the SW should be taken into account. Therefore, it is necessary to replace the free-stream conditions by the corresponding conditions on the SW. The paper [10] was the first attempt to consider the SW effect on stability of the boundary layer on a wedge in the hypersonic limit (i.e., the SW is almost parallel to the wall and $\gamma - 1$ is very small). In a similar formulation (but with the use of all conditions on the shock wave for disturbances), the problem of stability for a hypersonic ($M_\infty = 8$) boundary layer on a wedge and on a cone was solved in [11]. The stabilizing effect of the SW on the first and second modes of disturbances was demonstrated.

In this Note, a hypersonic viscous gas flow in the shock layer on a flat plate is considered. The velocity, temperature, density, and pressure profiles calculated on the basis of the Full Viscous Shock Layer (FVSL) equations are tested for stability within the framework of the linear stability theory with allowance for conditions on the shock wave.

2. Formulation of the problem

At high Mach numbers and moderate Reynolds numbers, a thick boundary layer is formed on the body; its thickness can be compared with the distance from the body to the shock wave. Therefore, the FVSL model is a good approximation for such flows. The flow past a flat plate within the framework of this model was considered in [12,13]. In the present work, the FVSL model was used to compute the characteristics of the mean flow on a flat plate. The shock-wave position in FVSL computations was set on the basis of experimental data. The boundary conditions on the SW were the generalized Rankine–Hugoniot conditions.

The points (experimental data) [12] and the solid curve (FVSL computation) in Fig. 1(a) show the mean density profiles normal to the body in the cross section $x = 0.078$ m for the following test conditions: $\mathrm{Re}_1 = 6 \times 10^5\ \mathrm{m}^{-1}$, $M_\infty = 21$, $T_w/T_0 = 0.26$, $L = 360$ mm, and $T_0 = 1150$ K. The FVSL solution is in good agreement with the experimental values of density. The boundary layer occupies $\frac{3}{4}$ of the shock layer, and the inviscid flow region is only $\frac{1}{4}$, i.e., the SW is located close to the boundary-layer edge. The computations also show that the velocity (Fig. 1(b)) and temperature (Fig. 1(c)) are almost constant in the inviscid part of the shock layer, whereas the pressure (and, hence, the density) normal to the surface changes twofold exactly in the inviscid region behind the SW (Fig. 1(b)). The same figure shows the density, velocity, and temperature profiles obtained by solving the equations of a self-similar boundary layer for a flat plate ($\times$-dashed curves) for the same test conditions. The boundary-layer profiles differ from the FVSL solution, which involves more significant differences in stability characteristics.

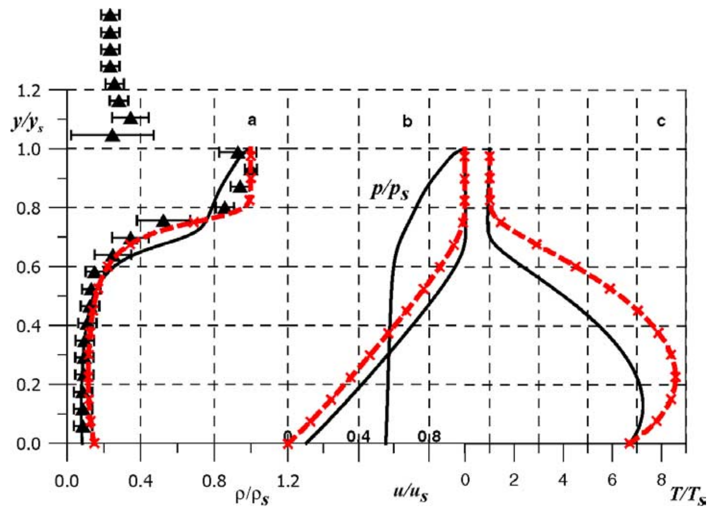

Fig. 1. The characteristics of the mean flow on a flat plate in the cross section $x = 0.078$ m for $\mathrm{Re}_1 = 6 \times 10^5\ \mathrm{m}^{-1}$, $M_\infty = 21$, $T_w/T_0 = 0.26$, $L = 0.36$ m, and $T_0 = 1150$ K. — FVSL computation; -×-×- self-similar boundary layer; ▲ experimental data [12].

The shock-layer profiles computed by the FVSL model and the self-similar boundary-layer profiles were then tested for stability within the framework of the locally parallel approximation. The shock-layer stability was calculated using the Dan and Lin system of linear equations [14].

By means of replacing the variables $z_1 = f, z_2 = f', z_3 = \alpha\phi, z_4 = \pi/\gamma M_S^2, z_5 = \theta, z_6 = \theta'$, this system of linear stability equations reduces to a system of six ordinary differential equations $z_i' = \sum_{j=1}^{6} a_{ij} z_j$ solved by the Runge–Kutta method. Outside the shock layer, for $y > y_S$, all flow parameters are constant, and the solution of the system of linear stability equations is presented in the form $z = \sum_{i=1}^{6} A_i \exp(\lambda_i y)$, where λ_i are the eigenvalues of the problem, which have the following values with acceptable accuracy:

$$\lambda_{4,1} = \pm\sqrt{\mathrm{i}\alpha R(1-c)}, \qquad \lambda_{5,2} = \pm\sqrt{\mathrm{i}\alpha R\sigma(1-c)}, \qquad \lambda_{6,3} = \pm\alpha\sqrt{1 - M_S^2(1-c)^2}$$

The classical approach usually involves the condition of disturbance decay outside the boundary layer, and rapidly growing solutions (corresponding to $\lambda_4, \lambda_5, \lambda_6$) are left out of consideration. In hypersonic flow, where the boundary-layer thickness is comparable with the shock-layer thickness, decay of disturbances in the inviscid portion of the shock layer, because of its small thickness, can be insufficient for neglecting disturbances reflected from the shock wave. In the present work, the decay condition is replaced by the condition of disturbance propagation only within the shock layer, with linearized Rankine–Hugoniot equations being satisfied on the shock wave. Therefore, all six particular solutions, corresponding to $\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6$, are considered. The general solution of the system of stability equations, determined by the sum $Z_i = \sum_{j=1}^{6} C_j A_i^{(j)} \mathrm{e}^{\lambda_j y}$ $(i = 1, \ldots, 6)$, should satisfy six boundary conditions on the plate surface and on the SW:

$y = 0$: $\quad f = \phi = \theta = 0$

$y = \Delta y$: linearized conditions on the SW for dimensionless amplitude f (velocity u), ϕ (velocity v), θ (temperature), and π (pressure) of disturbances:

$$f = A\phi, \quad A = -\frac{\overline{u_S} - \cos^2\beta}{\mathrm{tg}\,\beta\cos^2\beta}, \quad \pi = Q\phi,$$

$$Q = (1 - A\,\mathrm{tg}\,\beta)\frac{1/(\overline{\rho_S}\,\overline{u_S}\gamma M_\infty^2) + \sin^2\beta/(\overline{\rho_S}\,\overline{u_S}) - \overline{u_S}}{\mathrm{tg}\,\beta\overline{u_S}}, \quad \theta = S\phi, \quad S = -A\overline{u_S}\,\overline{u_S}\frac{(\gamma-1)M_\infty^2}{c_{ps}\overline{T_S}}$$

Here Δy is the thickness of the shock layer normalized to the transverse scale of similarity $\delta^* = \sqrt{\mu_S^* x^* / u_S^* \rho_S^*}$, ρ is the density, u and v are the velocity components in the x and y directions, respectively, p is the pressure, β is the SW slope counted from the x axis, γ is the ratio of specific heats in the free stream, and M_∞ is the free-stream Mach number; the subscript S indicates the parameters behind the SW, and the barred quantities refer to the mean flow characteristics (solution of FVSL equations). The problem formulation is described in more detail in [15]. The boundary conditions on the SW stated above, however, differ from those in [15] by the absence of v_S. This is more correct because the stability problem is solved in a quasi-parallel approximation. Since only the density, temperature, and pressure change significantly on the SW, elimination of v_S does not affect the results.

3. Results

The points in Fig. 2 show the experimentally measured growth rates of disturbances in the shock layer on a flat plate (the cross section $x = 0.078$ m) for $\mathrm{Re}_1 = 6 \times 10^5\ \mathrm{m}^{-1}$, $M_\infty = 21$, $T_w/T_0 = 0.26$, $L = 360$ mm, and $T_0 = 1150$ K [15]. This corresponds to the following parameters of the stability problem:

– Reynolds number $R = \sqrt{\mathrm{Re}_{1,x}} = 272.7$;
– Mach number behind the shock wave $M_S = 10.2$;
– Surface temperature normalized to the temperature behind the SW, $T_w = 6.7$;
– $\Delta y = 57.68$ the thickness of the shock layer.

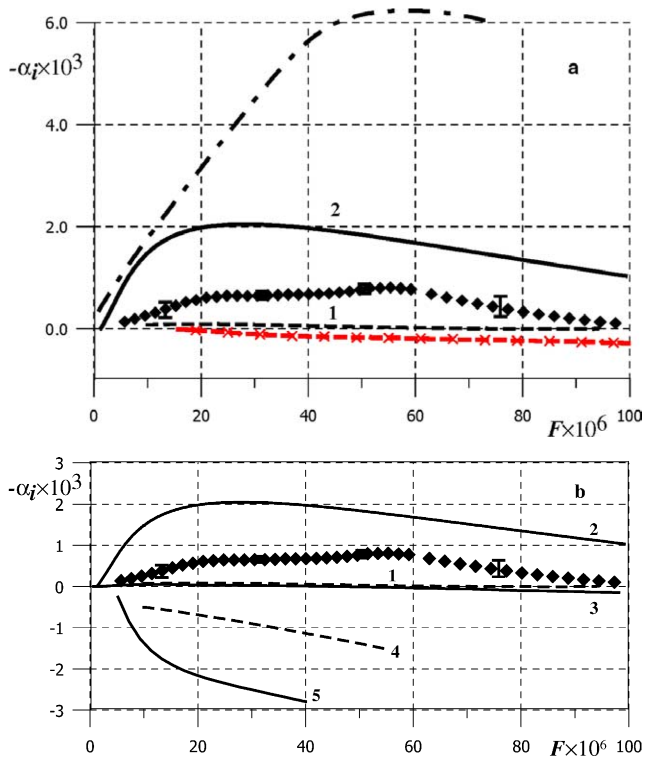

Fig. 2. The growth rate of disturbances versus frequency in the shock layer on a flat plate ($R = 272.7$; $M_S = 10.2$; $T_w = 6.7$; $\Delta y = 57.68$). 1,4- - - solutions without allowance for the SW; 2,3,5— solutions with allowance for the SW; -·-·- asymptotic theory [9]; -×-×- self-similar boundary layer; ♦ experimental data [12].

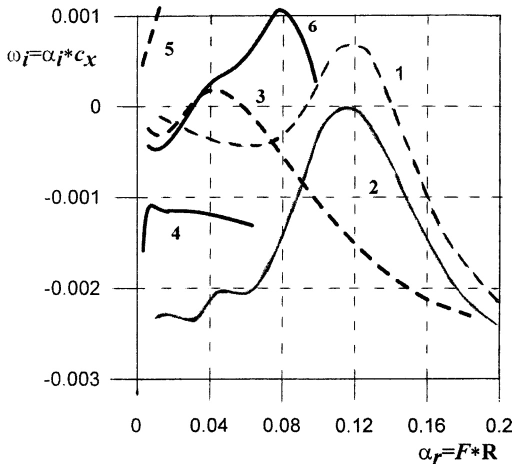

Fig. 3. Time-dependent growth rate of disturbances versus the wavenumber for $M_S = 6.72$, $R = 305$, $\Delta y = 34.26$, and $T_w = 8.65$. 1,2 solutions on a 5° wedge [11]; 3,4,5,6 solutions on a flat plate at an angle of attack of 5°; - - - - without allowance for the SW; — with allowance for the SW.

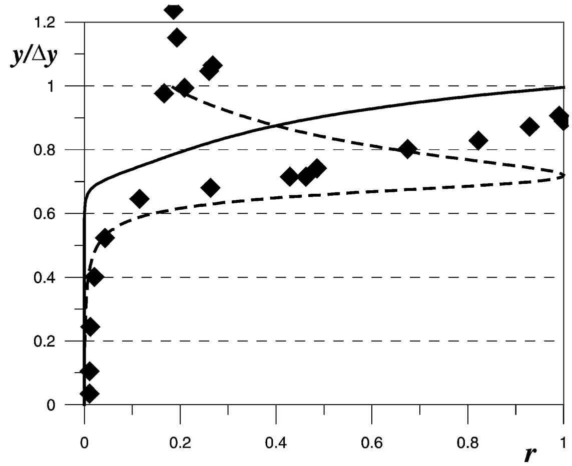

Fig. 4. Density fluctuations in the shock layer on a flat plate for $R = 272.7$, $M_S = 10.2$, $T_w = 6.7$, and $\Delta y = 57.68$. — computation with SW capturing; - - - - without SW capturing; ♦ experimental data [12].

For these conditions, Fig. 2(a) shows the data [9] obtained by an asymptotic method (dash-and dotted curve) and the growth rate of disturbances in a self-similar boundary layer on a plate ($\times$-dashed curve). Then, using the algorithm described above, we obtained growth rate coefficients α_i without allowance for the bow SW (dashed curve 1) and with allowance for the SW (solid curve 2). Fig. 2(a) reveals a destabilizing effect of the shock wave. However, this contradicts the data of [10,11] that demonstrated a stabilizing effect of the shock wave on a wedge flow. Let us consider possible reasons of this contradiction.

Fig. 3 shows the growth rates of disturbances ω_i in time versus the wavenumber α_r computed without the SW effect (dashed curve 1) and with the SW effect (solid curve 2) for the flow on a 5° wedge, taken from [11]. For flow over wedges, the mean flow is obtained by the boundary layer equations with the edge conditions taken as the flow conditions behind the shock.

For comparison, stability of the shock layer on a plate at an angle of attack of 5° was computed in the present work ($M_\infty = 8$, $\mathrm{Re}_L = 4 \times 10^6$, adiabatic surface). The mean characteristics were computed within the FVSL model. In the cross section $x = 0.15$, the parameters behind the SW ($M_S = 6.72$, $R = 305$, $\Delta y = 34.26$, $T_w = 8.65$) corresponded to the parameters of [11] behind the SW on a 5° wedge ($M_S = 6.8$, $R = 300$, $\Delta y = 30.7$). The equations of stability without allowance for the SW on a flat plate at incidence yielded two solutions, in contrast to [11]. One of them, solution 3 in Fig. 3, almost coincides with a similar solution 1 on a wedge with a small shift in terms of frequency, and the other one is the strongly unstable solution 5. Emergence of solution 5 is, apparently, related to the difference in the mean profiles of FVSL and self-similar boundary layer equations (similar to Fig. 1).

Similarly, computations of the stability problem with shock-wave capturing yielded not only a stable solution, as was obtained in [11] (curve 4) but also an unstable solution (curve 6). Solution 6 arrived from the region of strongly unstable solutions (curve 5) under the influence of the SW-induced stabilizing effect as well.

Thus, there is no contradiction. One only has to trace all disturbances, including rapidly growing ones. Computations of the stability characteristics without allowance for the bow shock wave yielded a weakly unstable solution 1 and stable solution 4 (Fig. 2(b)). Under the SW influence, solution 4 is stabilized and yields solution 5; weakly unstable solution 3 is close to the solution of the stability problem without shock capturing, but it also demonstrates the stabilizing effect of the shock wave. Because of the SW presence, however, solution 2 also arrives from the region of strongly unstable solutions. This is an unstable solution, which is yet in better qualitative agreement with the experimental data.

Fig. 4 shows the distributions of density fluctuations: the dashed curve 1 is the solution of stability equations without allowance for the SW, and the solid curve 2 is the solution with the SW taken into account. In the case of the solution without the SW, the maximum of fluctuations is located under the SW, in the transition region from the viscous to inviscid part of the shock layer, i.e., at the boundary-layer edge. In the solution with allowance for the SW, the maximum of fluctuations is shifted to the SW. This shift is caused by allowance for rapidly growing disturbances interacting with the SW. A comparison with the experimental distribution of density fluctuations (points) [12] shows that the solution with allowance for the SW is closer to experimental data.

4. Conclusions

Calculation of the mean flow within the framework of FVSL (with allowance for the viscid-inviscid interaction) leads to better agreement of the stability characteristics with experimental data.

It is shown that the interaction of disturbances and the SW results in:

(1) branching of the solutions of the linear stability problem, i.e., more than one solution were obtained for a comparatively narrow range of frequencies;
(2) stabilizing effect of the SW all disturbances in the shock layer. In real evolution of disturbances, one should take into account strongly unstable modes, which are usually skipped in the classical stability theory;
(3) shifting of the maximum of fluctuations to the SW, which is caused by rapidly growing disturbances.

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